CHAPTER 12

THE USE OF RANGE TABLES FOR THE CONTROL OF GUNFIRE FROM SHIPS. THE DETERMINATION OF BALLISTIC CORRECTIONS.

1201. From our previous study of naval gun sights (Chapter XII, *Naval Ordnance, 1933*), we have learned that the calibration of the range scale of a given gun is based on Columns 1 and 2 of that gun's range table. It is evident, then, that the gun, when properly aimed, will give the range set on its range scale only under the conditions which have been assumed for the computation of the range table, as defined in article 801. Under any other conditions the range given by the gun will vary from that set on its range scale, and in Chapters 10 and 11 we have learned how to determine the changes in range due to variations from any of the standard conditions set forth in article 801, except the last two (i.e., items (g) and (h)). With regard to item (h), viz., the assumption that the gun and target are in the same horizontal plane, it has already been shown in article 813 that the requirements of this condition are practically fulfilled when the gun is laid in elevation with respect to the line of sight, as is usually the case in naval gunnery; also, that if the gun should be laid with respect to the horizontal, as may be the case in indirect fire, the required corrections are obtainable from formulas (803) and (804), although such corrections are of small importance in comparison with other problems that enter into this type of fire. The single remaining assumption (item (g)), viz., that the earth is motionless, will be dealt with further in Chapter 13.

1202. The algebraic sum of the several changes in range due to variations from standard conditions, evidently determines the error in range that would result if the gun were fired with the actual target distance set on its range scale. In order to compensate for this error, we must therefore set the sights for a range which is the algebraic sum of the target distance and of these several changes in range. The range actually set on the gun's range scale, or the *sight-bar range*, then is an artificial value by means of which the gun's angle of elevation is adjusted in order that the gun may, under the non-standard conditions, give a range equal to the actual target distance. It is essential to a proper understanding of ballistic corrections to appreciate that the sight-bar range does not in any sense define the actual range of the trajectory. The following example will make this clear. Let us suppose that, for the firing of a 16″2600 f.s. battery, the actual target distance is known to be 10,000 yards, which, under standard conditions, requires that the guns be laid at an angle of elevation of 4°56.′8 (see 16″2600 f.s. table in *Range and Ballistic Tables, 1935*). Let us suppose further that due to variations from standard conditions the guns, if laid as indicated above, would give a range 500 yards short of the desired 10,000 yards ((−) 500 yards being the algebraic sum of the several changes in range, as determined from the range-table columns). From Column 2 (b) of the range table we find that in order to compensate for this reduction in range, we must increase the angle of elevation by 18.′0; that is, we must set the sights for an angle of elevation of 4°56.′8+18.′0=5°14.′8. The latter can be accomplished very simply, of course, by setting on the sights the sight-bar range 10,500 yards, i.e., the range-

Sight-bar range

table range which corresponds to the angle of elevation of 5°14.′8. The point to be grasped is that the guns will now give a range of 10,000 yards, and that the setting 10,500 on their sights is, in fact, only an artificial value by means of which the required angle of elevation of 5°14.′8 is obtained. This fact must be considered in connection with the choice of the range on which ballistic corrections are to be based, as will appear presently.

1203. The unqualified term *range* is commonly used to denote either distances, or settings on the sights of the gun, and ordinarily the proper sense of the term can be inferred from the connections surrounding its use. In our present discussions, however, it will frequently be necessary to distinguish specifically between range in the sense of distance, and range in the sense of a sight-setting, and we shall, for this purpose, use the following terms.

(a) *Target distance.* This denotes the actual distance from the gun to the target.

(b) *Sight-bar range.* This denotes the setting on the range scale of the gun, and it should equal the target distance only when the conditions of firing agree in all respects with (or are equivalent to) the standard conditions assumed for the range table.

In practice, various approximations of the target distance may be available, such as the *present range*, which is the best estimate of the target distance as determined at the time of firing, and the *navigational range*, which is the best estimate of the same distance as determined for post-firing analysis. Similar distinctions are made also in connection with the sight-bar range. Such distinctions are required chiefly for the purpose of post-firing analysis. It is to be understood that the terms defined above are general in nature; that is to say, the target distance may be merely an assumed value, or it may be based on a range-finder range taken just prior to firing, or it may be based on a careful plot of data available after the firing; and the sight-bar range, of course, partakes of the limitations of the target distance on which it is based.

1204. The algebraic sum of the drift and other lateral deviations (i.e., those due to wind, motion of gun, and motion of target), evidently determines the lateral error that would result if the bore of the gun were set in the vertical plane containing the gun and target. In order to compensate for this lateral error we must offset the bore with respect to this vertical plane, and we accomplish this by offsetting the line of sight laterally with respect to the bore so that when the line of sight is brought on the target the bore has the required lateral offset. For example, if the algebraic sum of all the lateral deviations is 100 yards to the right, the bore must be directed at a point 100 yards to the left of the target in order that the trajectory may terminate at the target; that is, the *line of sight* must be offset with respect to the *bore* by an angle which will cause the latter to point 100 yards to the left of the target when the line of sight is on the target. If the distance to the target is 10,000 yards, this angle then must be, in this case, the angle whose tangent equals $\frac{100}{10,000}$; or, in any case, the line of sight must be offset laterally with respect to the bore by an angle whose tangent equals the algebraic sum of all the lateral deviations divided by the target distance.

The lateral angle between the line of sight and the bore is called the *deflection*, and it is set by means of a deflection scale. (The mechanical arrangements

Deflection

for setting deflection are shown in Chapter XII, *Naval Ordnance, 1933*.) For convenience, the deflection scale is graduated in terms of an arbitrary angular unit known as the *mil*, the latter being the angle which is subtended by an arc equal to one one-thousandth of the arc's radius, and hence being equal to .001 radian (approximately 3′44, or 3′26″). Within the limits of deflection that are likely to occur in practice, a very close approximation of the angular deflection corresponding to a given lateral deviation can then be obtained by dividing the latter by one one-thousandth of the target distance and using the result so obtained as the deflection in mils.* For example, let us suppose that the lateral deviation is 1000 yards, and the target distance 10,000 yards (which represents a rather extreme case). The deflection actually required in this case is $\tan^{-1}\frac{1000}{10,000}=5°\,42′6$. If we use, instead, $\frac{1000}{10}=100$ mils, we are then actually using a deflection of $100\times3′44=5°44′$, corresponding to a lateral deviation of 1004 yards at the given target distance of 10,000 yards. The error, even in this extreme case, is slight, and in ordinary cases it is practically insignificant. It may be stated, therefore, that *the deflection in mils is obtained by dividing the lateral deviation by one one-thousandth of the target distance.*

Definition of the mil

How deflection scales are marked

Deflection scales are marked so that parallelism between the line of sight and bore is indicated by the setting 100 (formerly the setting 50 was used to indicate parallelism, and in antiaircraft equipment the setting 500 is now so used), and so that the bore of the gun is directed to the *right* if the deflection setting is *raised* (i.e., numerically increased) and to the *left* if the deflection setting is *lowered* (i.e., numerically decreased), assuming that the line of sight remains pointed at the same object. An easily remembered rule for determining the proper direction in which to set the sights in deflection follows from the above; for in order to move the point of fall to the right we must raise the deflection setting (*right, raise*), and in order to move the point of fall to the left we must lower the deflection setting (*left, lower*). This rule applies, of course, to the direction in which the *correction* is to be made, and not to the direction in which the *error* is found. For example, if the algebraic sum of all the lateral deviations is 100 yards to the *right*, and the target distance is 10,000 yards, we evidently must apply a correction of $\frac{100}{10}=10$ mils to the *left*, and, in accordance with the above rule,

Rule for setting deflection

Definition of the term "scale"

the required deflection setting is then $100-10=90$. For the sake of convenience and brevity in fire-control communication, the setting that is to be applied to the deflection scale is (in the U. S. Navy) always referred to as the *scale*; in the above example, then, the *scale* is 90.

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* The mil is sometimes defined as being the angle whose tangent equals .001, which is practically equal to .001 radian. The U. S. Army uses a *mil* which is defined by the relation 6400 mils $=360°$, as compared with the corresponding relation $2000\,\pi$ mils $=360°$, or 6283.2 mils $=360°$, which applies to the graduations of U. S. Navy deflection scales.

Formerly deflections scales in the U. S. Navy were graduated in *knots* instead of in mils, but this convention was discontinued at about the time of the World War. Very few scales of this type are still in service; an explanation of their use, if required, may be found in article 323, *The Groundwork of Practical Naval Gunnery, 1915*.

1205. In order that the sights of a gun may be set correctly, we must determine the effects of the variations from the several standard conditions upon which a range table is based (art. 801). The range-table columns give the *errors* in the point of fall that correspond to variations of given amounts in the several conditions for which standard values have been assumed. The errors may be in range, or they may be lateral.

We shall refer to errors in range as *range errors*, and to the corresponding corrections as *range corrections*. If the error is such as to increase the range, we shall call the *error over* and its corresponding *correction down*. If the error is such as to decrease the range, we shall call the *error short* and its corresponding *correction up*.

We shall refer to lateral errors (or deviations) in yards as *deflection errors in yards*, and the corresponding corrections as *deflection corrections in yards*. If the error is such as to move the point of fall to the right, we shall call the *error right* and its corresponding *correction left*. If the error is such as to move the point of fall to the left, we shall call the *error left* and its corresponding *correction right*.

Ballistic correction in range

1206. The algebraic sum of all the range errors, calling overs (+) and shorts (−), is the total range error, and the latter with sign reversed is called the *ballistic correction in range*. The algebraic sum of the target distance and the ballistic correction in range, is the *sight-bar range*.

The algebraic sum of all the deflection errors in yards, calling right (+) and left (−), is the total deflection error in yards, and the latter with sign reversed, and after having been converted to mils (by dividing it by one one-thousandth of the target distance in yards), is called the *ballistic correction in deflection*. The algebraic sum of 100 (or other setting which indicates zero deflection) and the ballistic correction in deflection, is the *scale*.

Ballistic correction in deflection

The ballistic correction in range is usually prefixed by Up or Down, as "Up 500," "Down 500," etc. Algebraic signs may also be used; for example, "Up 500" may also be stated as "+500," and "Down 500" as "−500." Similarly, the ballistic correction in deflection is usually prefixed by Right or Left, as "Right 5," "Left 5," etc. Or, with algebraic signs, "Right 5" may also be stated as "+5," and "Left 5" as "−5." Both of these ballistic corrections together are sometimes referred to simply as *the ballistic*. For example, we say the ballistic is "Up 500-Right 2," or "Down 500-Left 6," etc.

1207. As we proceed to the problem of determining the individual items that constitute the ballistic corrections in range and in deflection, a question arises as to the range on which the various corrections are to be based. Let us assume, for example, that a battery of 16″2600 f.s. guns is to be fired at a target which is at a distance of 31,200 yards, and under conditions which are in accord with the standards on which the range table is based with the exception that the initial velocity is 50 f.s. less than standard. We find from the 16″2600 f.s. range table that the velocity reduction of 50 f.s. causes a reduction in range of $5 \times 180 = 900$ yards, whence, in accordance with the rules already stated, we determine that a sight-bar range of $31{,}200 + 900 = 32{,}100$ yards is required. The question arises, in this case, whether the drift shall be found with the target distance* of 31,200 yards, or with the sight-bar range 32,100 yards.

Choice of the range on which ballistic corrections are to be based

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* It is to be understood, of course, that we are dealing here, as well as elsewhere throughout the following discussions, with the trajectory whose point of fall actually occurs at the

That the drift in this case cannot be found directly from the range table either with the target distance or with the sight-bar range, becomes evident when we consider that the required trajectory agrees neither with that corresponding to the range-table range of 31,200 yards, nor with that corresponding to the range-table range of 32,100 yards. By using the sight-bar range 32,100 yards we are giving the gun the angle of elevation of 26°20′.1, which corresponds to this range in the range table, but nevertheless we still expect the gun to give a horizontal range of only 31,200 yards. The trajectory which is required to give the horizontal range of 31,200 yards in this case is, in fact, one which corresponds to the arguments $\phi = 26°20'.1$ and $V = 2550$ f.s., as compared with the range-table trajectory (for the same horizontal range) which corresponds to the arguments $\phi = 25°00'.0$ and $V = 2600$ f.s. The drift that actually corresponds to the sight-bar range 32,100 yards can be found by solving the drift formula (901), using the values $X = 31{,}200$ yards, $\phi = 26°20'.1$, and $\omega = 37°48'$, the required value of ω having been found from the A.L.V.F. Tables* with the arguments $\phi = 26°20'.1$, $V = 2550$ f.s., and Log $C = 1.12707$ (the Log C for this example is the same as that already found in article 815). The solution shows that the drift in this case is 783.7 yards, as compared with the range-table values 739.7 yards corresponding to the target distance 31,200 yards, and 809.4 yards corresponding to the sight-bar range 32,100 yards.

By assuming other likely variations from standard conditions, we find a similar situation, namely, that the correct value of the drift cannot be obtained from the range table directly either with the target distance or with the sight-bar range, but that the correct value lies somewhere between those obtained from the range table, respectively, with the target distance and with the sight-bar range. Ordinarily a very good approximation of the drift can be obtained by entering the range table with the mean of the target distance and sight-bar range. In the above example this mean range is 31,650 yards, and the corresponding drift from the range table is 773.7 yards, which differs by less than 10 yards from the correct value.

It can likewise be shown that very good approximations of the lateral deviations due to wind, motion of gun, and motion of target are obtainable with the mean of the target distance and sight-bar range. For example, the time of flight corresponding to the conditions of the above example is found from the A.L.V.F. Tables (with $\phi = 26°20'.1$ and $V = 2550$ f.s.) to be 59.62 seconds, which is greater than the range-table value of 57.79 seconds corresponding to the target distance 31,200 yards, but less than the range-table value of 60.39 seconds corresponding to the sight-bar range 32,100 yards. Column 18 depends directly on this T which lies between the values for target distance and sight-bar range. Column 16 depends on this T, and also on a value of X which is equal to the target distance and on a value of ϕ which corresponds to the sight-bar range. Column 17 is, in effect, a combination of Columns 16 and 18 (art. 1120). In all of these cases, therefore, the value of the effect must lie between the range-table values corresponding, respectively, to target distance and sight-bar range. The mean of target distance and sight-bar range, although less satisfactory for some of these columns than for others, as will be evident from the different relations upon which the several

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target, that is to say, the trajectory whose horizontal range is equal to the target distance. Changes in the actual horizontal range of the trajectory should, for the purposes of these discussions, be regarded as the equivalent of changes in the target distance.

* It is necessary to use second differences in interpolating with respect to ϕ in these tables (see art. 712).

columns depend, nevertheless generally gives better approximations than are obtainable with the target distance or sight-bar range themselves.

1208. Similar reasoning is applicable also in the case of range errors. Let us continue with the example already stated above, viz., 16″2600 f.s. battery, target distance 31,200 yards, velocity reduction 50 f.s., sight-bar range 32,100 yards, and angle of elevation 26°20′.1 corresponding to the latter. The change of range due to the 50 f.s. velocity reduction was determined from the value in Column 10 for the target distance 31,200 yards (i.e., $5\times180=900$ yards). The question now is whether we shall make a further approximation by taking the value from Column 10 for the sight-bar range 32,100 yards, which would then give a change in range of $5\times186=930$ yards, and hence a new sight-bar range of 32,130 yards. But the value $\Delta X_V=930$ yards, as found by this process, corresponds to the arguments $\phi=26°20'.1$ and $V=2600$ f.s., whereas the arguments are actually $\phi=26°20'.1$ and $V=2550$ f.s. The new value of ΔX_V found with the sight-bar range is therefore too great, and the second approximation of the sight-bar range (i.e., 32,130 yards) is also too great. It will generally be the case for range, as well as for deflection, that the true value of a ballistic correction lies between the values found from the range table, respectively, with target distance and sight-bar range.

Considering that in any actual case the range is affected by a combination of variations in V and C and of wind, motion of gun, and motion of target, it would be an endless task to determine the relative merits of the use, respectively, of target distance and sight-bar range for finding individual ballistic corrections from the range table under the great variety of such combinations that is likely to occur in practice. A fairly comprehensive analysis based on likely combinations of variations in V and C was made recently,* and it showed that in some cases the use of the sight-bar range for finding ballistic corrections improves matters, while in other cases it makes matters worse (the analysis was applied to both range and deflection). It was shown, moreover, that it is exceedingly difficult to make generalizations as to the relative merits of the target distance and the sight-bar range as arguments for the determination of ballistic corrections from the range table, either for range or for deflection.

A further consideration that enters into the problem is the fact that the values in the range-table columns from which ballistic corrections are to be determined are themselves, in many cases, rather broad approximations. Column 12, for example, gives the *mean* of the changes in range due to plus and minus 10% variations in air density; the use of this column involves error both because the column is based on a mean as to the sign of the variation, and because variations to which the column is to be applied in practice may be much smaller than that on which the column is based. The use of Column 10 also involves errors of this nature (see arts. 1004 and 1009). This is true, to varying degrees, of all of the columns from which ballistic corrections are determined (except Column 6). In view of these limitations of the range-table columns themselves, as well as of the accuracy with which most of the physical values that are to be applied to them can be obtained in practice (e.g., air density, velocity variations, wind, target speed), and of the uncertainty that usually exists as to the correct determination of the target distance itself, it seems altogether illogical to resort to such elabora-

*Ref. Letter of Special Board on Naval Ordnance (Bureau of Ordnance file S 72 4/49/184, of 1 Feb. 1930).

tions in the determination of ballistic corrections as are involved in basing them on successive approximations of the sight-bar range. For post-firing analyses, which can be based on relatively more accurate data than are generally available at the time of firing, somewhat greater elaboration in the determination of ballistic corrections may be justifiable. But even under the conditions of a deliberate post-firing analysis, it is doubtful whether any useful end is served by attempting any refinement beyond that of determining the several *deflection* corrections with the mean of the correct target distance and the correct sight-bar range. Any theoretical improvement in the *range* corrections that can be gained by basing them on a mean between target distance and sight-bar range is generally of small consequence in comparison with the uncertainty that remains in the value of the target distance itself, even considering the best value of the latter that is obtainable for post-firing analysis.

1209. The best that can be said, in the way of a conclusion based on the above discussions, is that due caution must be exercised in laying down any general rules that are to govern the choice of the range argument on which ballistic corrections are to be based, and that attempts to improve the ballistic corrections by basing them on successive approximations of the sight-bar range may, in fact, serve merely to introduce additional error. A broad view of the situation, giving due weight to practical considerations as well as to theoretical considerations, leads to the conclusion that the preponderance of advantages lies on the side of the simplest practice of all, namely, that of basing all ballistic corrections on the *target distance, and this practice will be followed in the examples given in this textbook.* Various other practices for the determination of ballistic corrections are prescribed, from time to time, in the official instructions governing such matters, and the latter must be followed in the computation and analysis of ballistic corrections in connection with the preparation of target-practice reports.

Use of target distance for determining ballistic corrections

It is to be noted, however, that no matter what practice is followed in determining the deflection error in yards, the conversion of the latter to mils must always be based on the *target distance,** for the value of the mil is a function of actual distance and not of sight-bar range.

We shall now consider the various other details involved in the use of the range-table columns for determining the individual items that constitute the ballistic corrections in range and in deflection.

1210. Column 10 gives the change in range corresponding to a velocity variation of the magnitude stated in the heading of the column. Simple proportion may be used for finding changes of range corresponding to any velocity variations that are likely to be occasioned by the combined effects of erosion and variations in powder temperature, but this column is not applicable at all to the changes in velocity occasioned by reduced charges (arts. 1004 and 1006). The matter of signs has already been discussed in article 1002. There should be no confusion as to signs, for an *increase* in initial velocity always causes an *increase* in range, i.e., a *plus error*, or an *over;* and a *decrease* in initial velocity always causes a *decrease* in range, i.e., a *minus error*, or a *short.*

Use of Column 10

Variations from the standard initial velocity are due principally to two causes, viz.,

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* See also note on page 158.

Variations in powder temperature

(a) *Variations in the temperature of the powder.* The standard initial velocity is based upon the standard powder temperature of 90°F., and it is assumed that a variation of ±1° from this standard temperature causes a variation of ±2 f.s. in the initial velocity, for all guns. This relation is an approximate one, but serves conveniently as an average for all guns and all likely temperature variations. Variations in excess of about 20°F. from the standard should be avoided, however, by control of magazine temperatures.

Erosion

(b) *Erosion.* The gradual enlargement of the bore (especially at the origin), due to the wear incident to firing, results in a reduction of the initial velocity that is given by the standard charge. The amount of the velocity loss due to erosion is determined experimentally at the proving ground; it varies for different guns, and for a given gun it varies with the weight of charge. The proving ground furnishes *erosion curves* for the several guns and for their several designed charges (i.e., full and reduced charges). Two types of such curves are now generally furnished for the determination of velocity loss due to erosion, viz., (1) a plot of velocity loss against *bore enlargement*, and (2) a plot of velocity loss against *rounds fired*. Since bore enlargement is the immediate source of the velocity loss that is occasioned by erosion, the velocity-loss-bore-enlargement relation provides the more positive means for the determination of erosion losses. The velocity-loss-rounds-fired relation is necessarily less positive in its application to individual guns, since it does not provide for the differences in wear that the same number of rounds may cause in different guns of the same type. Guns should be star-gauged as frequently as practicable for the determination of bore enlargement. (This requires the services of equipment and specially trained personnel now carried only at navy yards and on some fleet tenders.) In the absence of up to date bore-enlargement data, the erosion loss should be carried forward from that indicated by the last star-gauging, by applying to the latter the additional loss indicated by the rounds-fired erosion curve for the number of rounds fired since that star-gauging.

The following example illustrates the use of Column 10.

Given: The 16″2600 f.s. gun; temperature of powder 82°F.; velocity loss due to erosion 28 f.s.

Find: From the range table, at 10,000 and 30,000 yards, the range errors due to the variations from the standard initial velocity.

Velocity variation due to powder = (−)8×2 = (−)16 f.s.
Velocity variation due to erosion = (−)28 f.s.
Total velocity variation = (−)44 f.s.

From Column 10, at 10,000 yards,
(−)44/10×67 = (−)295 yards

From Column 10, at 30,000 yards,
(−)44/10×173 = (−)761 yards

1211. Column 11 gives the change in range corresponding to a variation in weight of projectile of the magnitude stated in the heading of the column. Simple proportion may be used for finding changes of range corresponding to any likely variations from the designed weight of the projectile on which the range table is based. The matter of signs has already been discussed in article 1016. It is to be noted that the sign of the range *error* is usually, but not always, contrary to the sign of the variation in projectile weight; this has already been explained in article 1019.

Use of Column 11

Occasion for the use of Column 11 arises but rarely, for variations in weight of projectile are held within narrow limits. This column has little practical use except for the purpose of determining the amount of dispersion in range that may be attributed to such variations in projectile weight as are allowed by manufacturing tolerances. For example,

Given: Assuming a tolerance of ± 6 lbs. in the manufacture of projectiles for the 16″2600 f.s. gun.

Find: From the range table, at 10,000 and 30,000 yards, the range errors corresponding to the maximum variation in projectile weight that is occasioned by the given tolerance.

From Column 11, at 10,000 yards,

$$(\pm)6/10 \times (\mp)\ 19 = \underline{(\mp)\ 11 \text{ yards}}$$

From Column 11, at 30,000 yards,

$$(\pm)6/10 \times (\mp)15 = \underline{(\mp)9 \text{ yards}}$$

1212. Column 12 gives the change in range corresponding to a variation of $\pm 10\%$ from the standard air density.* Simple proportion may be used for finding changes of range corresponding to any variations in air density that are likely to occur in practice (art. 1009). The matter of signs has already been discussed in article 1007. There should be no confusion as to signs, for an *increase* in air density always causes a *decrease* in range, i.e., a *minus error*, or a *short;* and a *decrease* in air density always causes an *increase* in range, i.e., a *plus error*, or an *over*. For reasons that have been outlined in articles 420–425, the ballistic density should be used in determining the effects of variations from the standard air density.

Use of Column 12

As has already been explained in article 1014, a table of multipliers (Table V, *Range and Ballistic Tables, 1935*) is provided for use with Column 12, these multipliers being simply the ratios between any given variation in air density and the tabular variation of 10% on which Column 12 is based. It is important to note that the arguments assumed for Table V are the *surface density factor* (δ) and the maximum ordinate, and that the value of a multiplier as given in the body of the table represents the resultant of the operations, (1) of converting the surface density factor (δ), by means of Table IV, to the ballistic density (δ_b) *assumed* to correspond to the given surface density and maximum ordinate; and (2) of finding the ratio between the variation expressed by this ballistic density and the 10% variation on which Column 12 is based (see example in article 1014). That is to say, the multipliers in Table V embody the

Use of ballistic density

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* Column 12 is also useful for determining the effects of variations in other factors which enter into the ballistic coefficient. Thus, for example, this column gives also the change in range due to a variation of $\pm 10\%$ in the coefficient of form. The sign, as given by the column for a density variation, is also correct for a variation in coefficient of form, since δ and i both appear in the denominator of the ballistic coefficient.

determination of the ballistic density by means of Table IV, and thus render unnecessary the use of the latter table. But if the ballistic density has already been determined from actual aloft soundings, it is then necessary to eliminate that portion of Table V which represents the operation of converting surface density to ballistic density, and this requires that the multiplier be found with the ballistic density and *zero* maximum ordinate (see art. 1015). The following rules may therefore be stated.

Rules for determination of multiplier for Column 12

I. *If only surface observations are available, the multiplier for Column* 12 *is determined by entering Table V with the surface density factor* (δ) *as obtained from Table III, and with the maximum ordinate corresponding to the given range.*

II. *If the ballistic density has been determined from actual aloft observations (as by an aerological party), the multiplier for Column* 12 *is determined by entering Table V with this ballistic density and zero maximum ordinate.*

1213. The following examples illustrate the use of Column 12 under each of the procedures stated above.

I. *Given:* The 16″2600 f.s. gun, surface air temperature 48°F., surface barometric pressure 30.″55.

Find: From the range table, at 10,000 and 30,000 yards, the range errors for the ballistic densities which are *assumed* to correspond to the given surface conditions.

From Table III, with 48° and 30.″55,

$$\delta = 1.059$$

From the range table (Col. 8),

At 10,000 yds., $y_s = 709$ ft.
At 30,000 yds., $y_s = 12{,}001$ ft.

From Table V,

For $\delta = 1.059$ and $y_s = 709$ ft., $M = (-).58$*
For $\delta = 1.059$ and $y_s = 12{,}001$ ft., $M = (-).41$*

From Column 12, at 10,000 yards,

$$(-).58 \times 165 = (-)96 \text{ yards}$$

From Column 12, at 30,000 yards,

$$(-).41 \times 1160 = (-)476 \text{ yards}$$

II. *Given:* The 16″2600 f.s. gun, and the following ballistic densities, as determined for the stated ranges by the aerological party; at 10,000 yards, $\delta_b = .944$; at 30,000 yards, $\delta_b = .972$.

Find: From the range table, at 10,000 and 30,000 yards, the range errors corresponding to the given ballistic densities.

From Table V, entering with $y_s = 0$ in both cases,

For $\delta_b = .944$, $M = (+).56$
For $\delta_b = .972$, $M = (+).28$

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* It is to be noted that the equivalent of three decimal places in the density factor is given by two decimal places in the multiplier; the latter therefore should not be carried beyond two places.

From Column 12, at 10,000 yards,

$$(+).56 \times 165 = (+)92 \text{ yards}$$

From Column 12, at 30,000 yards,

$$(+).28 \times 1160 = (+)325 \text{ yards}$$

It is to be noted that the sign of the multiplier for Column 12, as given by Table V, always denotes the sign of the range *error* corresponding to the conditions to which the multiplier pertains (art. 1015). The plus sign is, of course, to be understood when no sign is tabulated.

1214. Columns 13, 14, and 15 give the changes in range corresponding, respectively, to 10-knot* components of wind, motion of gun, and motion of target in the line of fire. Columns 16, 17, and 18 give the lateral deviations corresponding, respectively, to 10-knot* components of wind, motion of gun, and motion of target perpendicular to the line of fire. Simple proportion may be used for finding the changes in range or lateral deviations corresponding to any values of these components that are likely to occur in practice.

Use of Columns 13–18

With regard to the signs that are to be used with these columns, it is evident that in the case of wind and motion of gun the direction of the error agrees with the direction of the component causing it. That is to say, wind or motion of gun *in* the direction of fire causes *plus errors*, or *overs;* wind or motion of gun contrary to the direction of fire causes *minus errors*, or *shorts;* and wind or motion of gun to the *right* or *left* with respect to the direction of fire causes, respectively, *plus errors* (*rights*) or *minus errors* (*lefts*). In the case of target motion, however, the direction of the error is opposite to the direction of the component causing it. Target motion *in* the direction of fire causes *minus errors* (*shorts*); target motion *contrary* to the direction of fire causes *plus errors* (*overs*); and target motion to the *right* or *left* with respect to the direction of fire causes, respectively, *minus errors* (*lefts*) or *plus errors* (*rights*).

1215. Since Columns 13–18 are based on components in the line of fire and perpendicular to the line of fire, it is necessary, in any case, to resolve the wind, motion of gun, and motion of target into such components. This can be done very simply, as illustrated in Figure 27. In this figure the position of the firing vessel is at A, and its course and speed are represented by the vector $G = AB$; the position of the target is at C, and its course and speed are represented by the vector $T = CD$; the direction and velocity of the wind are represented by the vector $W = EF$. The line of fire is AC; θ_G is the angle between the firing ship's course and the line of fire, θ_T the angle between the target's course and the line of fire, and θ_W the angle between the direction of the wind and the line of fire. G, T, and W have, respectively, the components G_X, T_X and W_X *in* the line of fire, and G_Z, T_Z, and W_Z *perpendicular* to the line of fire. It will be clear from the figure that $G_X = G \cos \theta_G$, $T_X = T \cos \theta_T$, and $W_X = W \cos \theta_W$; in other words, the components *in* the line of fire are in each case equal to the vector times the *cosine* of the angle included between the vector and the line of fire. Also, $G_Z = G \sin \theta_G$, $T_Z = T \sin \theta_T$, and $W_Z = W \sin \theta_W$, or the components *across* the line of fire are in each case equal to the vector times the *sine* of the angle included between the vector and the line of fire.

Determination of components of wind, motion of gun, and motion of target

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* In the older range tables these columns were based on 12-knot components, instead of 10-knot components.

A convenient procedure, then, is to measure always the angles included between the directions of firing ship, target, and wind, and of the line of fire, for the components in the line of fire (range components) may then always be found with the cosines of these angles, and the components perpendicular to the line of fire (deflection components) always with their sines. For the purpose of determining the proper signs of the several components, no better or more convenient rule can be offered than

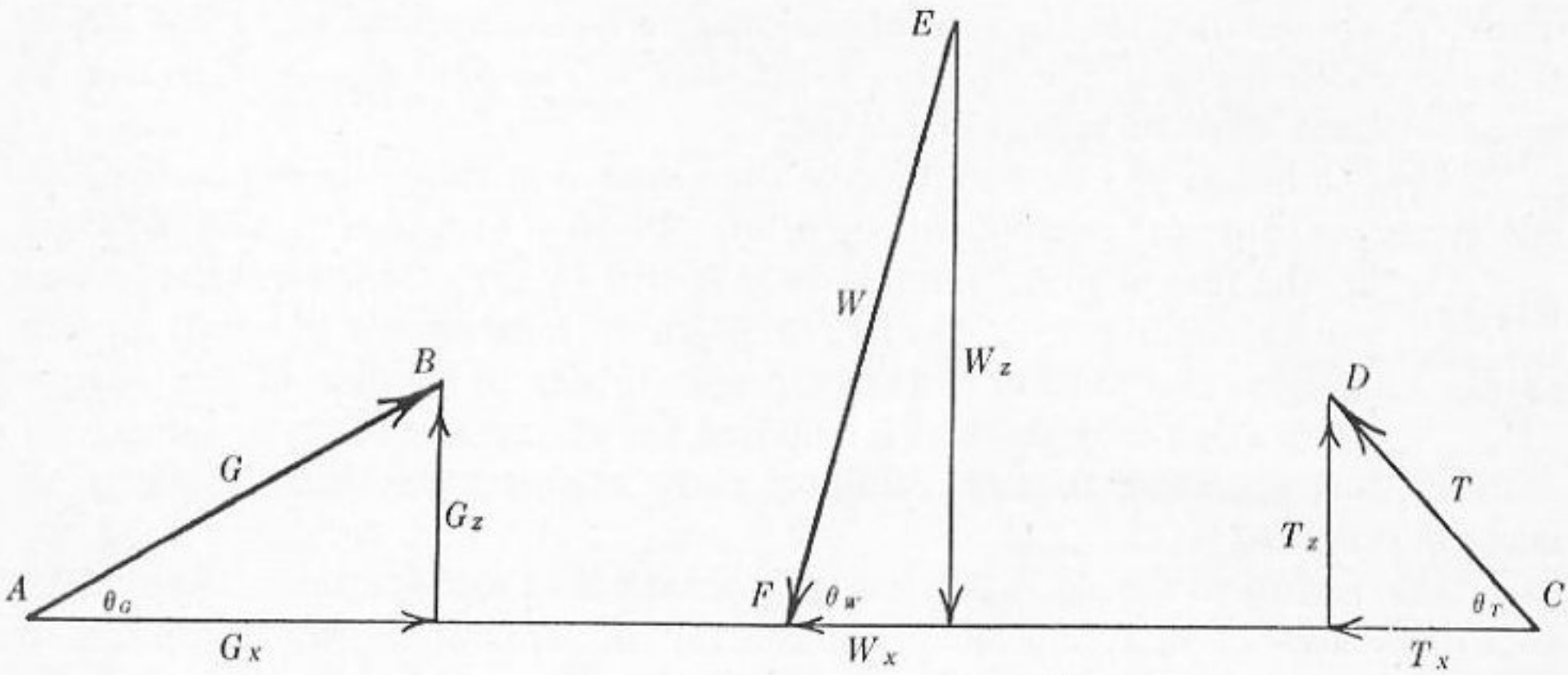


FIGURE 27

to draw a diagram similar to that in Figure 27; this avoids the troublesome procedure of deducing the signs of the components from the signs of the trigonometric functions on which they depend, and hence of considering the quadrants in which the several angles may lie.

1216. Wind, motion of gun, and motion of target are always measured in *knots* at sea. Hence we shall always express the vectors W, G, and T in knots. Let us assume for the vectors and included angles of Figure 27 the following values:

$$W=20,\ \theta_W=75°; \qquad G=15,\ \theta_G=30°; \qquad T=10,\ \theta_T=45°.$$

Applying the rule stated above, and referring to Figure 27 for the directions of the components, we have

$$W_X=20\cos 75°=5.2 \text{ Against or } (-)$$
$$W_Z=20\sin 75°=19.3 \text{ Right or } (+)$$

$$G_X=15\cos 30°=13.0 \text{ With or } (+)$$
$$G_Z=15\sin 30°=7.5 \text{ Left or } (-)$$

$$T_X=10\cos 45°=7.1 \text{ Against or } (-)$$
$$T_Z=10\sin 45°=7.1 \text{ Left or } (-)$$

The directions stated are the directions of the components themselves with respect to the direction of fire, and not the directions of the resultant errors in the fall of shot.

The several errors in the point of fall, for the 16″2600 f.s. gun at 10,000 yards, are now found as follows.

From
- Col. 13, with $W_x = 5.2$; $\frac{5.2}{10} \times 19 =$ 10 yds. short or (−)
- Col. 14, with $G_x = 13.0$; $\frac{13.0}{10} \times 58 =$ 75 yds. over or (+)
- Col. 15, with $T_x = 7.1$; $\frac{7.1}{10} \times 75 =$ 53 yds. over or (+)

From
- Col. 16, with $W_z = 19.3$; $\frac{19.3}{10} \times 10 =$ 19 yds. right or (+)
- Col. 17, with $G_z = 7.5$; $\frac{7.5}{10} \times 65 =$ 49 yds. left or (−)
- Col. 18, with $T_z = 7.1$; $\frac{7.1}{10} \times 75 =$ 53 yds. right or (+)

It will be observed that the several errors agree as to sign with their respective components except in the case of target motion, in which the signs of the errors are contrary to those of the components (art. 1214).

1217. In referring to wind we have heretofore assumed a *true wind*, i.e., a wind whose velocity and direction are measured with respect to the ground; the values in Columns 13 and 16 are based on such a wind. However, the wind that is actually felt and directly measured on a moving ship is an *apparent wind* which is the resultant of the true wind and of the ship's motion (and hence of the gun's motion). The course and speed of the ship being known, it is a simple problem in the resolution of forces to determine true wind from apparent wind, and the problem can be solved in various ways, of which a graphical solution is generally the most convenient. Mechanical devices based on the graphical solution are sometimes available, or they can be constructed without difficulty. Table 32, *Bowditch*, converts apparent wind to true wind, but not in units convenient for our present purposes.

Distinction between true wind and apparent wind

It is sometimes convenient to base ballistic corrections directly on the apparent wind, i.e., without first converting the latter to true wind, and this can be done readily, as will appear from the following. It has already been shown (arts. 1117, 1118, 1120) that the effect of a given component of gun motion is the equivalent of the effect of an equal component of target motion* less the effect of an equal component of wind. It is evident, then, that the correction for a given component of gun motion differs from that for an equal component of target motion only in that it takes into account the effect on the trajectory of the wind which is set up by the gun's motion (the target's motion, of course, does not affect the trajectory itself). But the wind that is set up by the gun's motion (and hence ship's motion) is the same as that which, in combination with the true wind, makes up the apparent wind which is felt on the ship. It follows, then, that if we are to base the ballistic correction for wind on the *apparent wind*, we must omit from the ballistic correction for gun motion that portion which is designed to account for the wind set up by the gun's motion, since the latter has already been accounted for by being included in the apparent wind. But we have found above that a correction for gun motion differs from a correction for target motion only in that

* For the purposes of this relation, the effect of target motion is to be considered as being of the same sign as the effect of gun motion.

the former includes the effect of the wind set up by the gun's motion; that is to say, a ballistic correction for gun motion which omits this wind effect is the equivalent of a ballistic correction for target motion. Hence it follows that if the ballistic correction for wind is to be based on the apparent wind, the ballistic correction for gun motion must be based on values designed to apply to target motion.

1218. The conclusion stated above can be verified as follows. Let us assume that a ship is steaming on course north at a speed of 10 knots, that a true wind is blowing from north at a velocity of 10 knots, and that the battery of 16″2600 f.s. guns is to be fired abeam to starboard at a stationary target distant 10,000 yards from the ship. We find from Column 16 of the range table that the lateral deviation due to the 10-knot component of *true wind* blowing *from the left* is 10 yards *to the right;* and from Column 17 that the lateral deviation due to the 10-knot component of gun motion *to the left* is 65 yards *to the left;* hence the resultant lateral deviation is 55 yards *to the left.* Now the lateral deviation due to gun motion which we have found from Column 17 (i.e., 65 yards) is, in fact, based on the consideration that the 10-knot sidewise motion imparted to the projectile by the moving gun would amount to a lateral deviation of the projectile, during the time of flight, of 75 yards (Col. 18), provided that this sidewise motion were not opposed by the air; and on the further consideration that this sidewise motion is, however, opposed by the 10-knot wind which this sidewise motion creates against itself, and that the lateral deviation is therefore reduced by 10 yards (Col. 16), whence the resultant deviation becomes $75-10=65$ yards (which agrees with the value found from Column 17). But under the given condition of a *true wind* of 10 knots, the *apparent wind* is now 20 knots, and the projectile will experience this 20-knot apparent wind in its flight. For if we resolve the lateral motion of the projectile as before we find, in this case, that the 10-knot sidewise motion initially imparted to the projectile by the motion of the gun is now opposed not only by the 10-knot wind which this sidewise motion creates against itself, but also by the 10-knot true wind which is blowing, that is, altogether, by an apparent wind of 20 knots. Then the 75-yard deviation which the projectile would suffer if its sidewise motion were unopposed is, in this case, reduced by the effect of a 20-knot opposing wind, or by 20 yards (Col. 16), whence the resultant deviation is $75-20=55$ yards, which agrees with the result previously found. In the former case we have used the 10-knot component of true wind with Column 16 and the 10-knot component of gun motion with Column 17; in the latter case we have used the 20-knot component of apparent wind with Column 16 and the 10-knot component of gun motion with Column 18; the same result has been obtained in both cases. And what has been found here to be true for components perpendicular to the line of fire is likewise true for components in the line of fire.*

1219. The following rules may therefore be stated:

Rules for use of true wind and apparent wind

I. *If true wind is to be used in determining the ballistic corrections, the range-table columns are to be used as follows:*

For wind	*Columns 13 and 16*
For motion of gun	*Columns 14 and 17*
For motion of target	*Columns 15 and 18*

II. *If apparent wind is to be used in determining the ballistic corrections, the range-table columns are to be used as follows:*

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* See also note adjacent to art. 1119.

For wind ..... *Columns 13 and 16*
For motion of gun ..... *Columns 15 and 18*
For motion of target ..... *Columns 15 and 18*

1220. The meaning and determination of the *ballistic wind* have already been explained in articles 1114–1116. The process of determining the components of wind in and perpendicular to the line of fire and applying them to the range-table columns, is the same whether the wind is a ballistic wind or merely a surface wind. The greater accuracy that is afforded by the use of the ballistic wind rather than merely the surface wind depends entirely on the fact that the former is a better measure of the wind itself than is the latter, and not on any difference as to the use of these two kinds of wind in connection with the range table.

Use of ballistic wind

The aerological party determines the *ballistic true wind*, i.e., a ballistic wind measured with respect to the ground. The *ballistic apparent wind* can be determined from the *ballistic true wind* by combining the latter vectorially with the true course and speed of the firing ship. For example, if the ballistic true wind is 10 knots from north, and if the firing ship is steaming at 10 knots on course north, the ballistic apparent wind is 20 knots from north. The use of true wind (whether ballistic wind or surface wind) is preferable to the use of apparent wind, since the apparent wind changes with every change in course or speed of the firing ship, whereas the true wind is not affected by the maneuvers of the firing ship. But many range-keepers are so designed that it is necessary to use with them ballistic corrections based on apparent wind,* and it should be borne in mind, when such range-keepers are used, that the ballistic true wind determined by the aerological party must be converted to the corresponding ballistic apparent wind by the firing ship itself.

1221. It is sometimes convenient, especially at target practices, to use as point of aim some well-defined feature of the target which is sufficiently displaced from the desired point of impact to require the application of a *point of aim correction.* When the latter is designed to compensate for a displacement in the point of aim in elevation, it is called the *point of aim correction in range;* and when designed to compensate for a displacement in the point of aim in train, it is called the *point of aim correction in deflection.* The following examples illustrate the determination of point of aim corrections.

Point of aim correction

I. *Given:* A night target practice is to be fired by a 5″3150 f.s. battery at ranges varying from about 4500 yards to about 2500 yards; the allowed target area is to be 20 feet high, and is to be represented by the lower half of a target which is 40 feet high. The point of aim is to be at the top edge of the whole target.
Find: From the range table, the point of aim corrections in range at 4500 yards and at 2500 yards.

The allowed target has a height of 20 feet, and its center, which is the desired point of impact, is therefore 10 feet above the water. The point of aim, being at the top of the whole target, is 40 feet above the water, or 30 feet above the desired point of impact. From Column 19 of the range table we find that the following changes in the sight-bar range are required to lower the point of impact 30 feet.

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* This is the case with any range-keeper which corrects for motion of gun on the basis of Columns 15 and 18 of the range table, as will be evident from the rules stated in art. 1219.

$$\text{At 4500 yards, } \frac{(-)\,30}{12} \times 100 = \underline{(-)\,250 \text{ yards}}.$$

$$\text{At 2500 yards, } \frac{(-)\,30}{5} \times 100 = \underline{(-)\,600 \text{ yards}}.$$

The results found, viz., (−) 250 yards and (−)600 yards, are the point of aim corrections in range, respectively, at 4500 yards and at 2500 yards;* it is to be noted that the signs as given pertain to the *corrections*, and not to errors.

II. *Given:* A target practice is to be fired by a 16″2600 f.s. battery at a range of about 25,000 yards; the target is to be 140 feet long and 40 feet high, and it is expected that it will lie practically perpendicular to the line of fire; the point of aim in train is to be at the left-hand edge of the target.

Find: The point of aim correction in deflection at 25,000 yards. Also determine from the range table how much difference in range is occasioned by shifting the point of aim in elevation from the top edge of the target to the bottom edge (or waterline).

The point of aim in train is 140/2 = 70 feet, or about 23 yards, to the left of the desired point of impact. At 25,000 yards 1 mil = 25 yards, hence the point of aim *correction* in deflection is practically <u>right 1 mil.</u>

From Column 19 of the range table we find that the difference in range occasioned by shifting the point of aim in elevation 40 feet amounts to about

$$\frac{40}{140} \times 100 = \underline{29 \text{ yards}}.$$

1222. Column 19 can also be used for finding the height of impact above the waterline, in the vertical plane of the target, that corresponds to a given distance of point of fall beyond the target, or vice versa. For example, in the case of the 16″2600 f.s. gun at 10,000 yards, a shot which falls 120 yards beyond the target should have passed through the vertical plane of the target at a height of about $\frac{120}{100} \times 31 = 37$ feet above the waterline. Or, if the target is 40 feet high, a shot which passes through the top of the target should fall at a distance of about $\frac{40}{31} \times 100 = 129$ yards beyond the target.†

1223. We have now examined the details involved in the use of the several range-table columns which are provided for the determination of the effects of

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* Although it is not uncommon practice to use the top of the 40-foot target as the point of aim in elevation, it is often overlooked that at the close ranges which are likely to occur at night firings this may often require the use of sight-bar ranges which are materially less than the target distance.

† At very short ranges, such as occur at Short Range Battle Practice, the use of Column 19 for such purposes is subject to considerable inaccuracy. This will be dealt with in detail in Appendix B.

Practical methods for determining ballistic corrections

variations from the standard conditions on which the range table is based. Our next step will be to examine the principal features of practical methods by means of which all of these details may be handled expeditiously in the determination of the sight-bar range and scale. In actual practice, some of the elements of the ballistic corrections in range and in deflection are usually handled automatically by mechanical devices, such as the range-keeper; or some of them may be determined from graphical devices, such as alinement charts. However, intelligent use of such devices generally demands a thorough understanding of the entire problem of determining the several ballistic corrections on which the sight-bar range and scale depend. For this reason, if for no other, it is advantageous to acquire a close familiarity with the direct solution by computation of the several elements which enter into the determination of the sight-bar range and scale. Moreover, a post-firing computation of all elements of the ballistic corrections is required to be submitted with each target-practice report, whether or not some or all of these elements were determined at the time of firing by mechanical or graphical devices.

1224. A standard form, known as Sheet 10 of the target-practice report forms, is provided for the computation of ballistic corrections in connection with

Sheet 10

the preparation of target-practice reports. As a target-practice report form, Sheet 10 constitutes a post-firing analysis of the ballistic corrections in range and in deflection, and includes a complete record of the computations of the several elements composing these corrections and of the various conditions on which they depend. However, this sheet serves also as a convenient form for the pre-firing computation of those portions of the ballistic corrections which are not handled by the range-keeper or other devices, and it is often so used. In any event, it constitutes a summary of the various elements composing the ballistic corrections, and a guide as to the provisions which exist in various range-keepers for the automatic determination and application of some of these elements.

Since the notation and arrangement of data on Sheet 10 are sometimes confusing to the beginner, we shall use, for the illustrative problems that are to follow, a form in which the principal features of Sheet 10 are outlined in a somewhat more elementary fashion than on the sheet itself. The details involved in the various operations expressed on Sheet 10 will be obvious from an inspection of the form of solution given here.*

1225. A space is provided at the side of Sheet 10 for drawing a diagram representing the direction and speed of own ship and target, and the direction and

Graphical representation of wind, motion of gun, and motion of target

velocity of the wind. This diagram is constructed by representing the motions of own ship, target, and wind as vectors with respect to the line of fire. The line of fire is represented by a line running from left to right across the diagram, and the own-ship vector is drawn at the left-hand end of the diagram, the target vector at the right-hand end, and the wind vector in the middle. We shall now examine the details involved in constructing the several vectors.

1226. The own-ship vector must make with the line of fire the angle between the direction of motion of own ship and the direction of the line of fire. This is

Own-ship vector

equal to the *relative bearing of the target.* Usually we know this relative bearing directly, from observation. If we know, instead, the *true* bearing

* A sample of Sheet 10, and further details concerning its use, are not given here, because the sheet is changed from time to time. It is expected, however, that copies of the current Sheet 10, together with appropriate supplemental instructions relative to details which are not covered here, will be issued to midshipmen for study and for the solution of problems.

of the target, we must subtract therefrom the true course of own ship to obtain the relative bearing of the target; if this operation results in a negative angle, we must add 360° to the latter to get the corresponding positive angle. (Relative bearing is always measured clockwise, from ahead, through 360°.) To draw the own-ship vector on the diagram we must lay off the whole angle of relative bearing *anticlockwise* from the line of fire. It is a great aid in checking the correctness of the diagram to imagine oneself located at the own-ship origin and looking down the line of fire. The following examples illustrate the features just discussed:

(a) Relative bearing of target = 45°
Own ship's speed = 12 knots

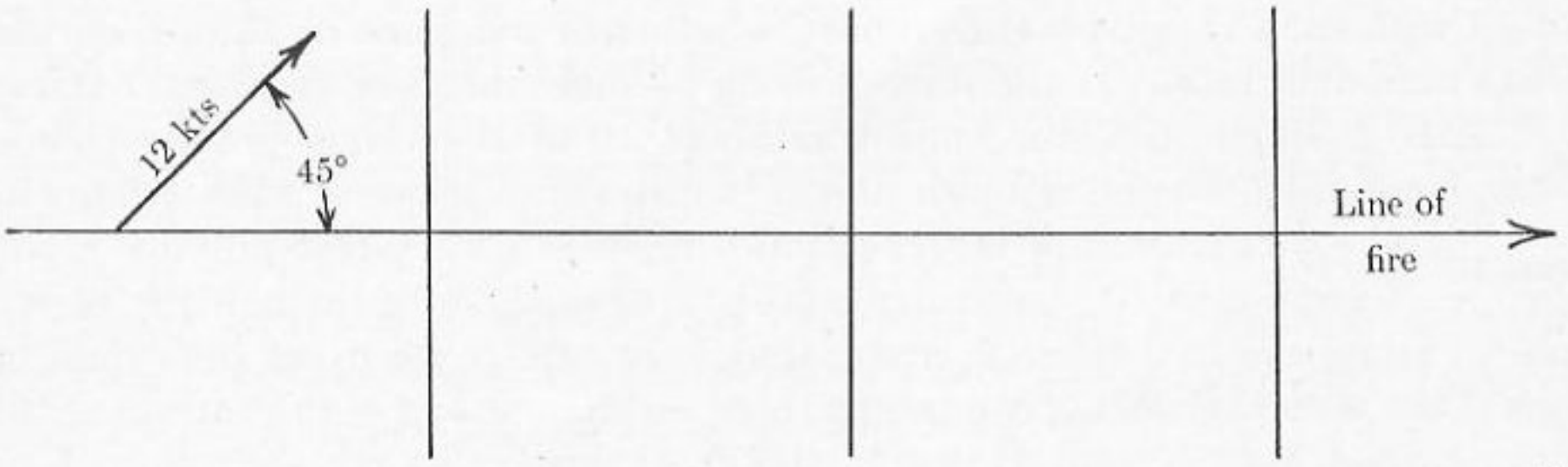


FIGURE 28

(b) True bearing of target = 50°
Own ship's true course = 200°
difference = (−)150°
(+)360°
Relative bearing of target = 210°
Own ship's speed = 12 knots

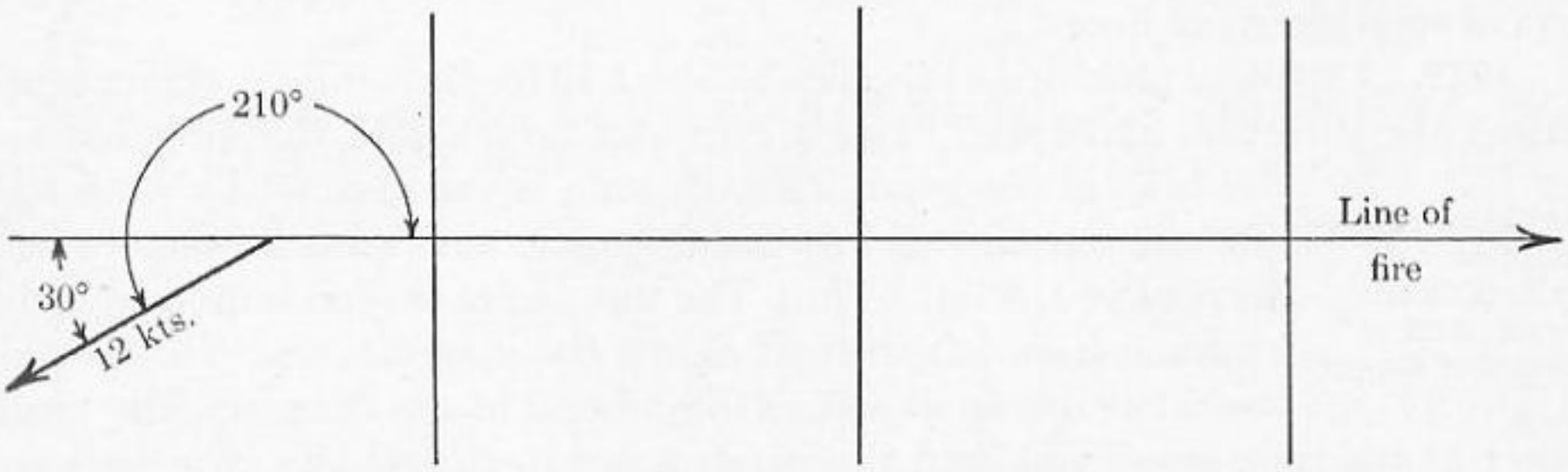


FIGURE 29

Target vector

1227. The target vector must make with the line of fire the angle between the direction of motion of the target and the direction of the line of fire. This is equal to the *target angle*. We often know the target angle directly, as for example when the spotter observes it. If the target angle is not directly known but the target's true course is known, we must determine the tar-

get angle by referring the target's true course to the true direction of the line of fire. It is to be noted that the target angle is actually the relative bearing of own ship from the target. The operation of deducing target angle from true course and bearing is therefore similar to that of deducing relative bearing of target from true course and bearing. In the present case we must subtract the *target's true course* from the *reverse of the true bearing of the target*, adding 360°, if necessary, to get a positive angle. (Target angle, like relative bearing, is always measured clockwise, from ahead, through 360°.) To draw the target vector on the diagram we must lay off the whole target angle *anticlockwise* from the *line of fire reversed*. In plotting target angle it is a great aid to have in mind the conception of looking from the target back toward own ship. Examples:

(a) Target angle = 45°
Target's speed = 15 knots

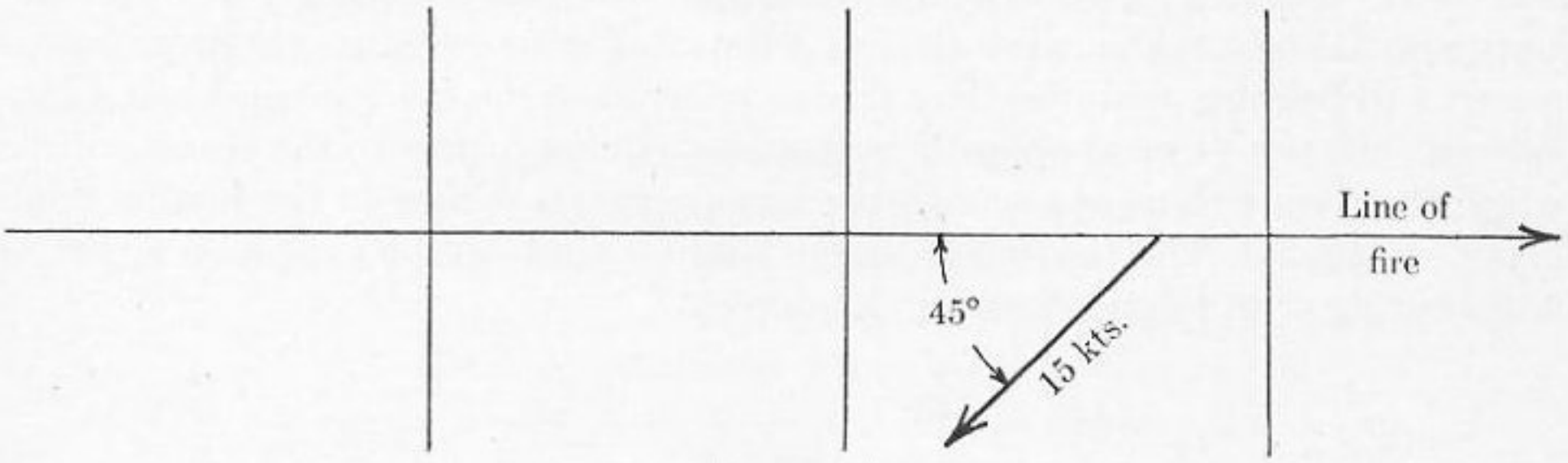


FIGURE 30

(b)

True bearing of target	=	230°
Reverse of true bearing of target	=	50°
True course of target	=	200°
difference	=	(−)150°
		(+)360°
Target angle	=	210°
Target's speed	=	15 knots

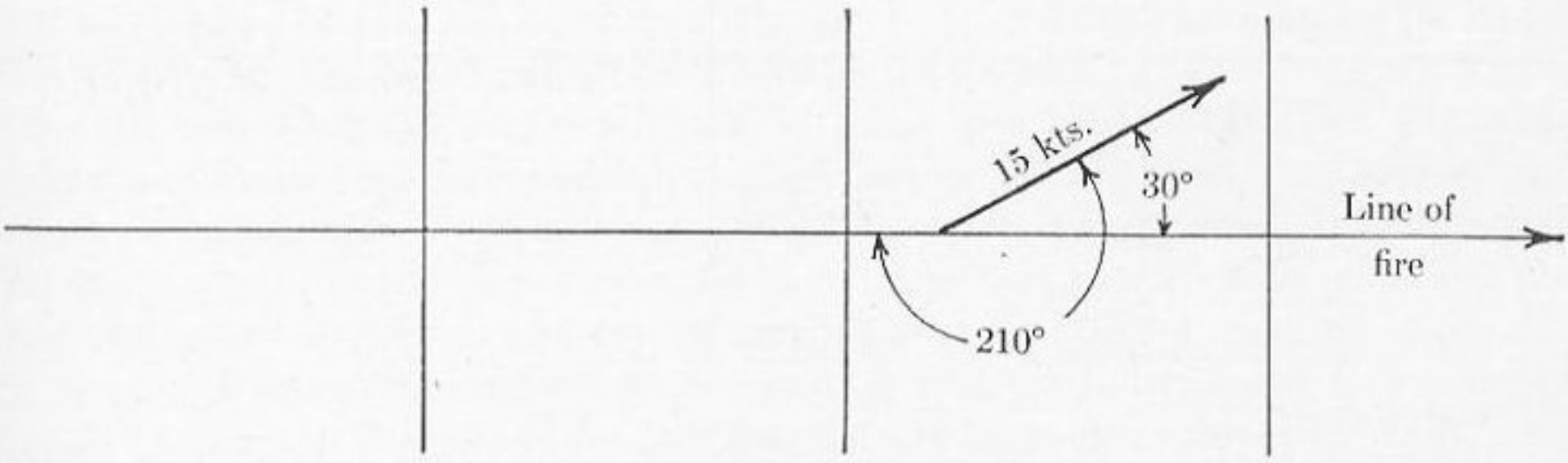


FIGURE 31

1228. The wind vector must make with the line of fire the angle between the direction *toward* which the wind is blowing, and the direction of the line of fire;

Wind vector

this is known as the *wind angle*. The wind angle is measured clockwise, from the direction *toward* which the wind is blowing, through 360°. However, it is the universal practice in stating wind directions to state the direction *from* which the wind blows (*e.g.*, a north wind blows *from* the north, not *toward* the north). This must be borne carefully in mind when plotting the wind vector. The term *bearing of the wind* is often used (as for example on sheet 10), and it also signifies the direction *from* which the wind blows. The wind angle is therefore determined by subtracting the *reverse of the bearing of the wind* from the *bearing of the target*, adding 360°, if necessary, to get a positive angle. To draw the wind vector we must lay off the whole wind angle *anti-clockwise* from the line of fire. The bearing of the wind may, of course, be either a true bearing or a relative bearing, and it is immaterial whether the wind angle is found from the relative bearings of wind and target or from their true bearings, but care must be taken not to use true bearing of wind with relative bearing of target, or vice versa. The relative bearing of wind corresponding to a given true bearing of wind can, of course, be obtained by subtracting from the latter the true course of own ship. Care must also be taken, when dealing with wind, not to confuse the term *true* as referred to bearing, with the term *true* as referred to the kind of wind being considered. The term *true* as opposed to the term *relative* relates to the *bearing* of the wind; the term *true* as opposed to the term *apparent* relates to the *kind* of wind being considered. The bearing of either kind of wind can be expressed either in true bearing or in relative bearing. Examples:

(a) Relative bearing of wind	=	90°
Relative bearing of target	=	45°
Reverse of relative bearing of wind	=	270°
difference	=	(−)225°
		(+)360°
Wind angle	=	135°
Wind velocity	=	10 knots

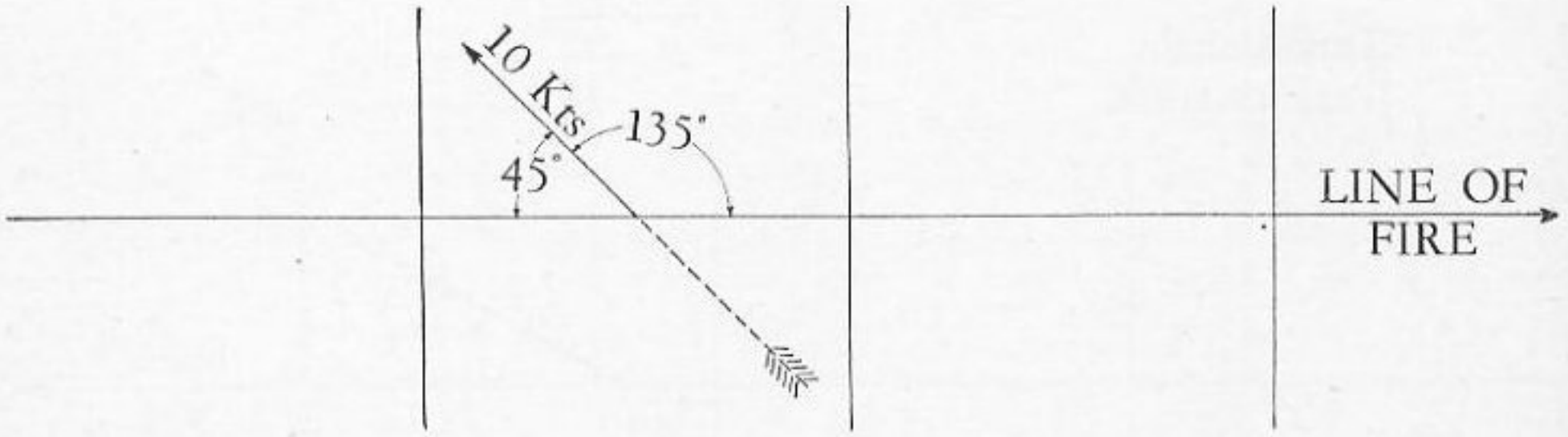


FIGURE 32

(b) True bearing of wind	=	10°
True bearing of target	=	340°
Reverse of true bearing of wind	=	190°
Wind angle	=	150°
Wind velocity	=	10 knots

1230. Some prefer, when dealing with true courses and bearings, to make a navigational plot of the set-up, since this kind of diagram serves not only for the purpose of showing the vectors in relation to the line of fire, but also for the purpose of reducing to a simple graphical process the entire problem of determining the required angles. Figure 35 illustrates the application of this method to the following problem.*

Navigational diagram

True course of own ship	= 30°
Speed of own ship	= 15 knots
True bearing of target	= 60°
True course of target	= 285°
Speed of target	= 10 knots
True bearing of wind	= 345°
Velocity of wind	= 20 knots

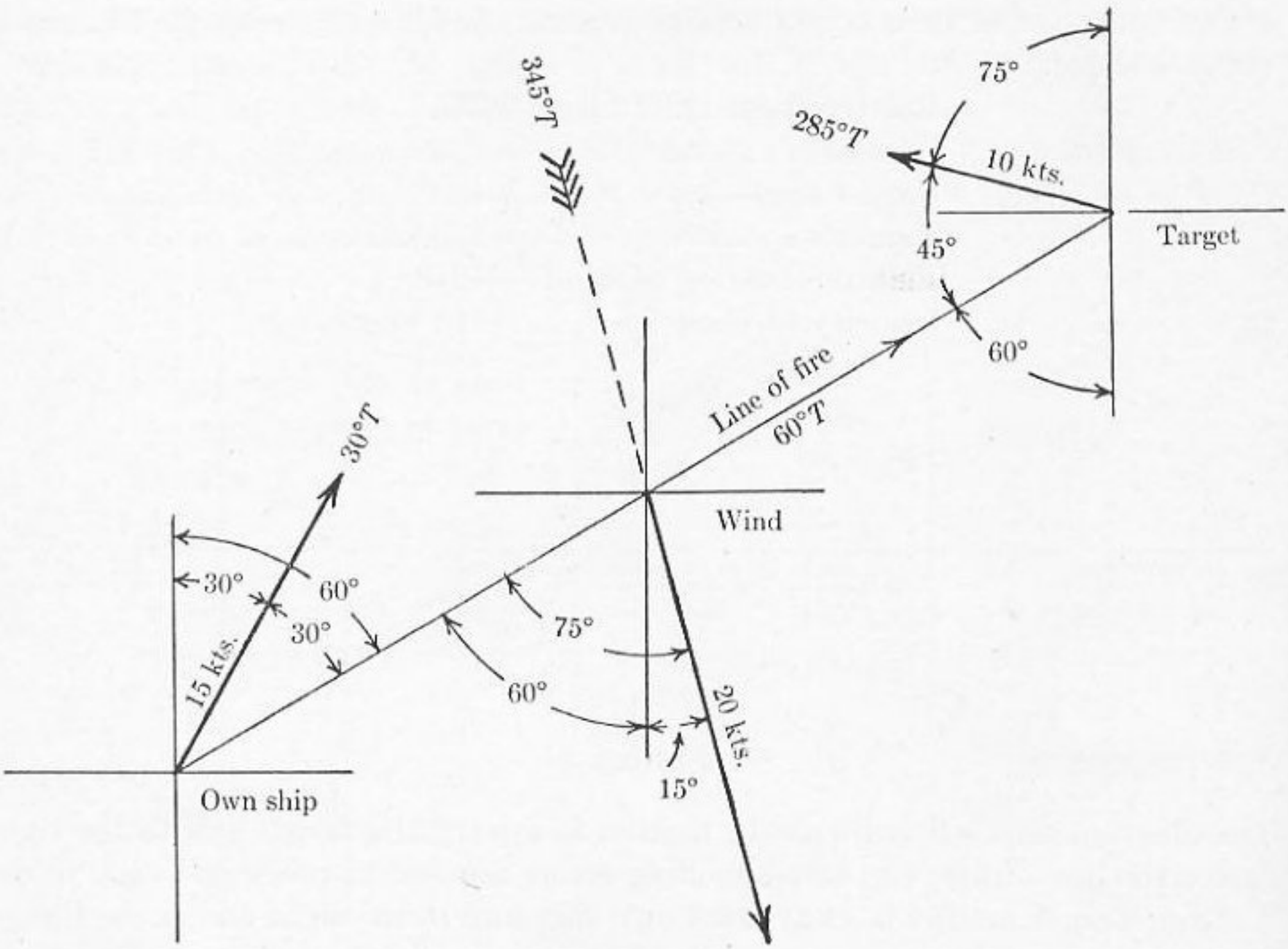


FIGURE 35

* The angles determined from a diagram of this type can, of course, be transferred without difficulty to the type of diagram which is provided for at the top of Sheet 10. The problem in article 1231, being based on the same set-up as stated for the present problem, affords a comparison between the two types of diagram.

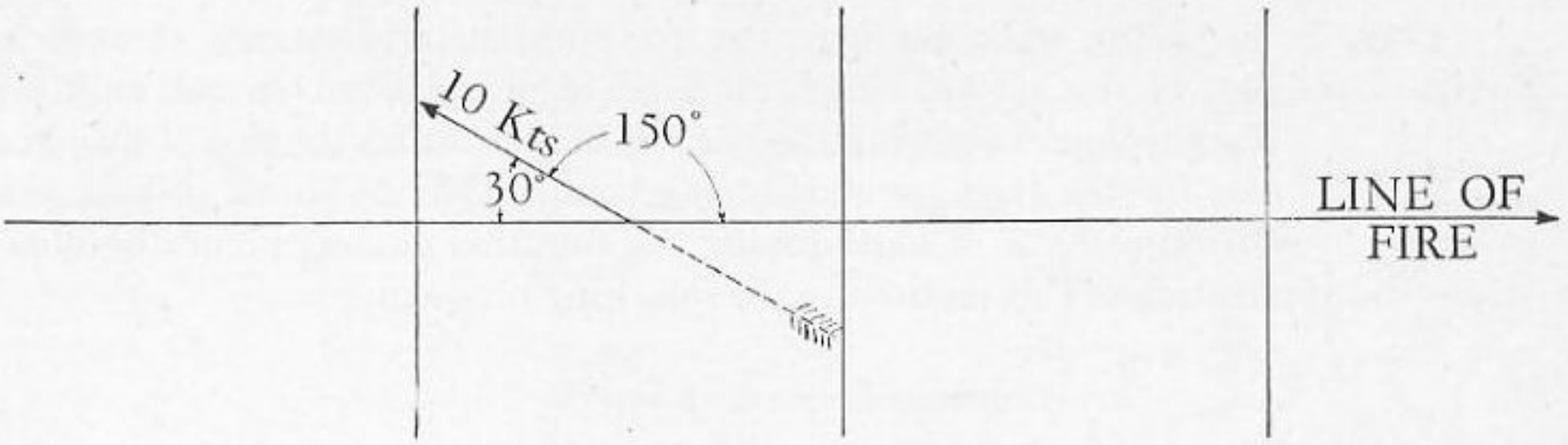


FIGURE 33

1229. The complete diagram is a graphic representation of the fire-control set-up that enters into the determination of the ballistic corrections. By drawing the vectors to scale, the components in and perpendicular to the line of fire can be measured graphically, and their computed values thus checked approximately. More important, however, is the aid that the diagram lends in determining the directions of the several components and of their corresponding errors. The following example illustrates these features.

Construction and use of the complete diagram

Relative bearing of target = 315°
Own ship's speed = 18 knots
Target angle = 120°
Target's speed = 20 knots
Relative bearing of wind = 165°
Velocity of wind = 15 knots

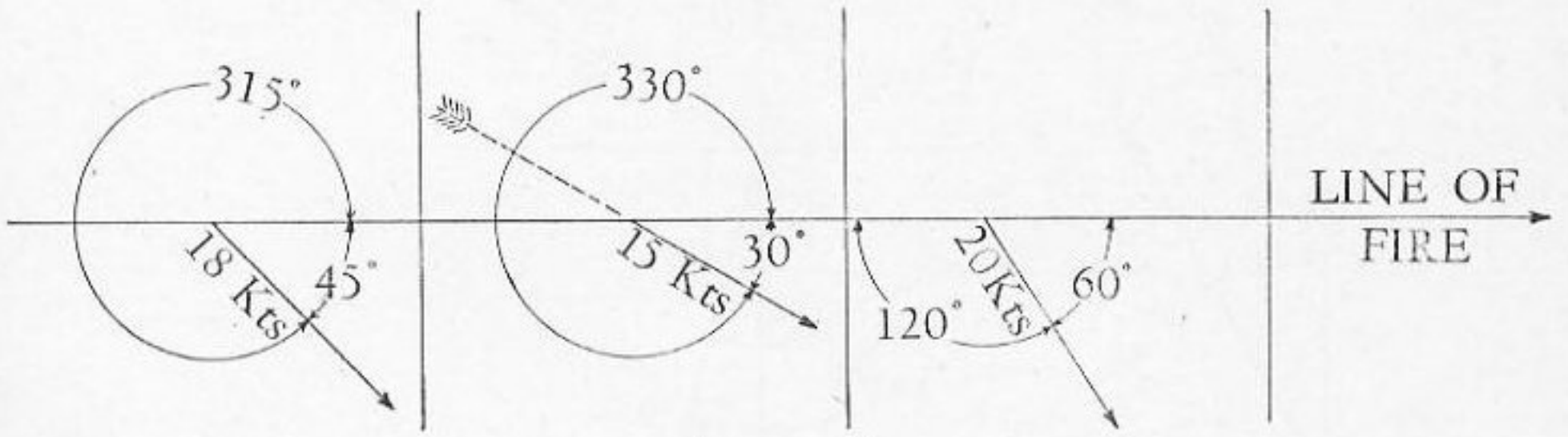


FIGURE 34

The diagram shows that own-ship motion is toward the target and to the right across the line of fire; the corresponding errors are *over* in range and *right* in deflection. Target motion is away from own ship and to the right across the line of fire; the corresponding errors are *short* in range and *left* in deflection. The wind is blowing toward the target and to the right across the line of fire; the corresponding errors are *over* in range and *right* in deflection. It is to be noted that although all three vectors in the above example have range components pointing down the line of fire and deflection components pointing to the right of the line of fire, the errors due to own ship and wind are *over* and *right*, while those due to the target are *short* and *left;* the reason for this has already been given in article 1214.

It will be observed, in the examples given above, that in all cases where the whole angle used in plotting the vector lies in the second, third, or fourth quadrant, the diagram shows not only the whole angle but also the corresponding first-quadrant angle, for the latter can be used more conveniently for finding the required trigonometric functions. In actual practice it is customary to label on the diagram only these first-quadrant angles.

1231. The following problems illustrate the determination of the sight-bar range and scale, taking into account all of the variations from standard conditions that have thus far been considered.

Examples of solution for the gun ballistic

Given: The 16″2600 f.s. gun and its range table; target distance 30,000 yards; powder temperature 82°F.; velocity loss due to erosion 28 f.s.; surface air temperature 48°F.; surface barometer 30″.55; true course of own ship 30°, speed 15 knots; true bearing of target 60°; true course of target 285°, speed 10 knots; true bearing of ballistic *true* wind 345°, velocity 20 knots; average weight of projectiles 2100 lbs.

Find: The ballistic corrections in range and in deflection, and the sight-bar range and scale.

For the purpose of plotting the vectors, the necessary angles are determined as follows:

(a) (See example (b), art. 1226.)

True bearing of target	= 60°
Own ship's true course	= 30°
Relative bearing of target	= 30°

(b) (See example (b), art. 1227.)

True bearing of target	= 60°
Reverse of true bearing of target	= 240°
True course of target	= 285°
difference	= (−) 45°
	(+)360°
Target angle	= 315°

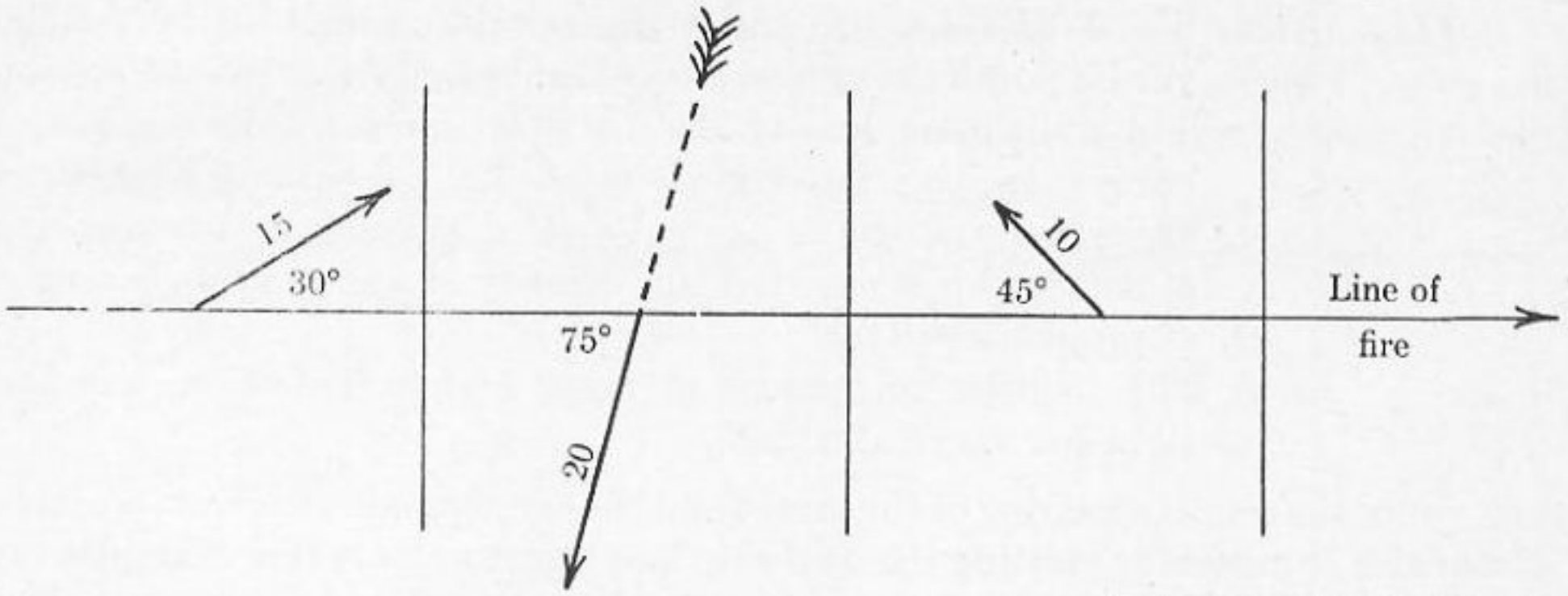
(c) (See example (b), art. 1228.)

True bearing of wind	= 345°
True bearing of target	= 60°
Reverse of true bearing of wind	= 165°
difference	= (−)105°
	= (+)360°
Wind angle	= 255°

The diagram is shown above the form on the opposite page (note that Fig. 35 is a navigational plot based on the same set-up as given in the present example). The diagram shows that the errors due to own-ship motion are *over* in range and *left* in deflection, the errors due to target motion are *over* in range and *right* in deflection, and the errors due to wind are *short* in range and *right* in deflection. Having made this analysis from the diagram, it is a good plan to put crosses in the spaces which are *not* to be used, as shown in the form on the opposite page (this should be done also for the other items), in order to ensure that the values which are to be computed later on will not be recorded in the wrong columns.

Since only surface observations are given in connection with the atmospheric density, the multiplier for Column 12 is obtained, in this case, according to rule number I as stated in article 1212, i.e., by entering Table V with the surface density factor (δ) as obtained from Table III, and with the maximum ordinate corresponding to the given range..

Since *true* wind is to be used, rule number I as stated in article 1219 applies in this case, i.e., errors due to wind are to be found from Columns 13 and 16, errors due to own-ship motion from Columns 14 and 17, and errors due to target motion from Columns 15 and 18.



16″ 2600 f.s. Gun

Target Distance 30,000 yds.

				Range Errors		Deflection Errors	
				Over	Short	Left	Right
Col. 6 656						×	656
Col. 10 173	Erosion Powder temp 82°F., (−)8×2= Tot. vel. var.	− 28 f.s. − 16 f.s. (−)44 f.s.	$\frac{44}{10}\times 173=761$	×	761		
Col. 11 15	Var. from standard weight of projectile	0	$\frac{0}{10}\times 15=0$	×	×		
Col. 12 1160	Air temp. Barometer δ (T. III) Max. ord. Multiplier (T.V)	48°F. 30.″55 1.059 12,001 ft. (−) .41	.41×1160=476	×	476		
Col. 13 145	Wind velocity (true) ∢ with line of fire 20 cos 75°=5.2	20 75°	$\frac{5.2}{10}\times 145=75$	×	75		
Col. 14 160	Own ship's speed ∢ with line of fire 15 cos 30°=13.0	15 30°	$\frac{13.0}{10}\times 160=208$	208	×		
Col. 15 307	Target's speed ∢ with line of fire 10 cos 45°=7.1	10 45°	$\frac{7.1}{10}\times 307=218$	218	×		
Col. 16 94	Wind velocity (true) ∢ with line of fire 20 sin 75°=19.3	20 75°	$\frac{19.3}{10}\times 94=181$			×	181
Col. 17 212	Own ship's speed ∢ with line of fire 15 sin 30°=7.5	15 30°	$\frac{7.5}{10}\times 212=159$			159	×
Col. 18 307	Target's speed ∢ with line of fire 10 sin 45°=7.1	10 45°	$\frac{7.1}{10}\times 307=218$			×	218
				426	1312 426 886	159	1055 159 896

Total range error..... 886 yds. Short
Total deflection error..... 896 yds. Right

$$1\text{ mil}=\frac{30{,}000}{1000}=30\text{ yds.},\quad \frac{896}{30}=30$$

Ballistic Correction in Range.....Up 886 yds.
Ballistic Correction in Deflection...Left 30 mils
Sight-bar Range 30,900 Scale 70

1232. *Given:* The 5″/3150 f.s. gun and its range table; target distance 8,500 yards; powder temperature 79°F.; velocity loss due to erosion 44 f.s.; ballistic density .933 (as determined by the aerological party); relative bearing of target 322°; own ship's speed 18 knots; target angle 22°; target's speed 33 knots; relative bearing of ballistic *apparent* wind 10°, velocity 32 knots; average weight of projectiles 48 lbs.

Find: The ballistic corrections in range and in deflection, and the sight-bar range and scale.

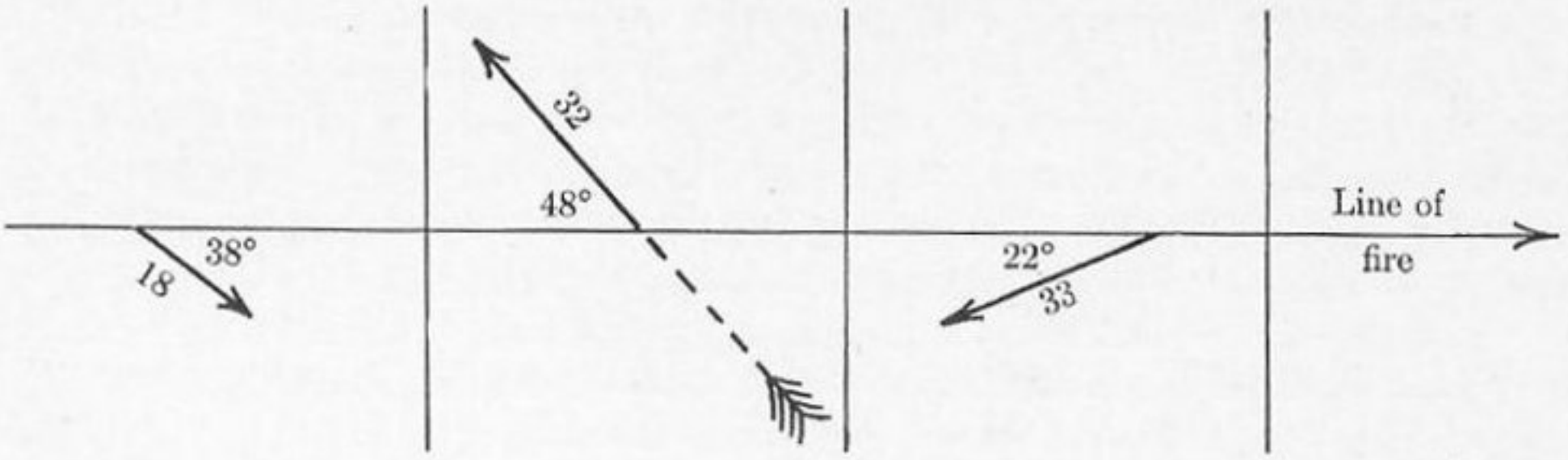
Since the relative bearing of the target and the target angle are given, we have the angles required for plotting the own-ship and target vectors (see examples (a) under articles 1226 and 1227). For the purpose of plotting the wind vector we find the wind angle as follows (see example (a) under article 1228).

Relative bearing of wind	=	10°
Relative bearing of target	=	322°
Reverse of relative bearing of wind	=	190°
Wind angle	=	132°

The diagram is shown above the form on the opposite page, and it shows that the errors due to own-ship motion are *over* in range and *right* in deflection, the errors due to target motion are *over* in range and *left* in deflection, and the errors due to wind are *short* in range and *left* in deflection.

Since the ballistic density is given, the multiplier for Column 12 is obtained, in this case, according to rule number II as stated in article 1212, i.e., by entering Table V with the given ballistic density and zero maximum ordinate.

Since apparent wind is to be used, rule number II as stated in article 1219 applies in this case, i.e., errors due to wind are to be found from Columns 13 and 16, and errors due to own-ship motion and target motion are in both cases to be found from Columns 15 and 18.



5″/3150 f.s. Gun

Target Distance 8500 yds.

				Range Errors		Deflection Errors	
				Over	Short	Left	Right
Col. 6 42.6						×	43
Col. 10 32	Erosion Powder temp. 79°F. (−) 11×2 = Total vel. var.	(−)41 f.s. (−)22 f.s. (−)66 f.s.	$\frac{66}{10}\times 32=211$	×	211		
Col. 11 (−)5	Var. from standard weight of projectile	(−) 2 lbs.	2× 5=10	×	10		
Col. 12 390	Ballistic density Multiplier (T.V)	.933 (+).67	.67×390=261	261	×		
Col. 13 46	Wind velocity (apparent) ∢ with line of fire 32 cos 48° =21.4	32 48°	$\frac{21.4}{10}\times 46=98$	×	98		
Col. 15 74	Own ship's speed ∢ with line of fire 18 cos 38° =14.2	18 38°	$\frac{14.2}{10}\times 74=105$	105	×		
Col. 15 74	Target's speed ∢ with line of fire 33 cos 22° =30.6	33 22°	$\frac{30.6}{10}\times 74=226$	226	×		
Col. 16 27	Wind velocity (apparent) ∢ with line of fire 32 sin 48° =23.8	32 48°	$\frac{23.8}{10}\times 27=64$			64	×
Col. 18 74	Own ship's speed ∢ with line of fire 18 sin 38° =11.1	18 38°	$\frac{11.1}{10}\times 74=82$			×	82
Col. 18 74	Target's speed ∢ with line of fire 33 sin 22° =12.4	33 22°	$\frac{12.4}{10}\times 74=92$			92	×
				592 319 273	319	156 125 31	125

Total range error..... 273 yds. Over
Total deflection error..... 31 yds. Left

$$1\text{ mil}=\frac{8500}{1000}=8.5\text{ yds.},\quad \frac{31}{8.5}=3.6$$

Ballistic Correction in Range..... Down 273 yds.
Ballistic Correction in Deflection..... Right 4 mils.
Sight-bar Range 8,250 Scale 104

1233. Certain other corrections, in addition to those which have been considered in connection with the problems just solved, sometimes enter into the final value of the ballistic corrections which determine the sight-bar range and scale. The final value of the ballistic corrections is called the *total ballistic*, and it includes the *gun ballistic*, *control ballistic*, *arbitrary ballistic*, and *cold-gun correction*. The purpose and composition of each of these portions of the total ballistic is explained below. (See also Sheet 10.)

The total ballistic

1234. The *gun ballistic* includes the corrections for drift, for variations from standard initial velocity, standard air density, and standard weight of projectile, and for wind, motion of gun, and motion of target, that is, all of the elements which have been included in the problems of articles 1231 and 1232, whence the ballistic corrections found in these problems are, in fact, the gun ballistic in range and in deflection.

The gun ballistic

1235. The control ballistic is designed to account for errors due to certain artificialities of control, such as the use of a point of aim which does not agree with the desired point of impact, or of a range scale which does not agree with the latest range table in effect. It includes the *point of aim correction*, and the *sight-scale-range-table correction.** The point of aim correction has already been explained sufficiently in article 1221.

The control ballistic

It sometimes happens that a gun is fitted with an obsolete range scale, although an up to date range table is available. The *sight-scale-range-table correction* is designed to bring the sight-bar range, as set on an obsolete range scale, into accord with the latest range table in effect, and if such a correction is required it is included in the control ballistic. An example will serve to illustrate how this correction is determined. Let us suppose that we have a gun for which a new range table has been issued, but that corresponding sight scales have not been provided. The ballistic corrections should then be computed as usual, using the new range table. Let us suppose that for a target distance of 20,000 yards we have found, according to a new range table, a gun ballistic in range of Up 500 yards, whence the sight-bar range would be 20,500 yards if the range scale were graduated according to the new range table. If the range scale is in fact graduated according to an obsolete range table, we must then adjust the sight-bar range as follows. Let us suppose now that in the new table the range 20,500 yards corresponds to the angle of elevation 12° 41.′6, and that in the *old table* the angle of elevation 12°41.′6 corresponds to the range 20,800 yards. Then in order to give the gun the angle of elevation that is actually required, we must set on its range scale (which is graduated according to the old table) the sight-bar range 20,800 yards. The latter evidently can be considered to be the sight-bar range 20,500 yards, as determined according to the new range table, plus an additional correction of Up 300 yards. This correction of Up 300 yards is the sight-scale-range-table correction in this case, and it is added to the control ballistic. The total ballistic (assuming that no further corrections enter into this case) then becomes 500+300=800 yards, whence the sight-bar range becomes 20,000+800=20,800 yards, as required. In director fire a similar correction is

The sight-scale-range-table correction

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* On Sheet 10, vertical and horizontal parallax corrections are also included in the control ballistic. These corrections apply only in director fire, and in all modern director systems they are taken care of mechanically by the director system itself and do not enter into the ballistic corrections at all.

required if the range converter is not graduated according to the latest range table in effect.

1236. The *arbitrary ballistic* is designed to account for errors which have been observed at previous firings but to which no specific cause can be assigned. It is determined by deducing from the actual results of a firing, the ballistic corrections that would have given the correct range and deflection, and by comparing these with the corrections that can be accounted for according to the best data available after the firing. In making such an analysis the proper procedure is to re-compute the ballistic corrections after the firing, and to compare them with the corrections indicated by actual results to have been necessary. The computations made *before* the firing may have been made on the basis of predictions that did not actually materialize. The arbitrary ballistic should not include errors in the pre-firing determination of the ballistic corrections that can be accounted for by post-firing analysis, such as erroneous estimates of target course or speed, wind, target distance, etc.

The arbitrary ballistic

The best of judgment must be exercised in determining an arbitrary ballistic, and great care must be taken to exclude from it corrections for mistakes that occurred on a previous firing, for evidently the inclusion of such corrections in the arbitrary ballistic would serve merely to subject a subsequent firing to the effects of these mistakes. An arbitrary ballistic should be based on the records of as many previous firings as are available, but it must be borne in mind that re-lining of the guns, or the use of a new range table, will very likely invalidate any previously determined arbitrary ballistic. It is also worthy of note that the use of a new lot of powder may change the arbitrary ballistic materially, for unknown differences among various lots of powder designed for the same gun probably contribute as much as any other cause to the arbitrary ballistic.*

1237. The *cold-gun correction* is designed to account for the reduction in range that usually results from the firing of cold guns; it is to be noted, in this connection, that the range table is based on the performance of the gun after a warming round has been fired (art. 803 (e)), and that the results obtained with a cold round are consequently very likely to differ from those indicated in the range table. Although cold-gun corrections are commonly applied only to the range, there is ample evidence that such corrections are required in deflection also. Cold-gun corrections can be determined only from careful post-firing analysis; they may change materially during the life of the gun, or upon re-lining. The causes and characteristics of cold-gun errors will be discussed further in Chapter 13.

The cold-gun correction

1238. In actual practice, the manner in which the ballistic corrections are determined depends upon the equipment available. Most batteries are now provided with range-keepers which automatically supply some portions of the ballistic corrections. Those portions of the ballistic corrections which are not handled automatically by the range-keeper must be determined separately, and applied manually to the range-keeper. It is of great importance to understand clearly not only what corrections are handled automatically by the range-keeper, but also the manner in which the latter handles them. For example, many range-keepers generate the corrections for own-ship motion on the basis of Columns 15 and 18, and hence render it

Mechanical devices for handling ballistic corrections

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* See also arts. 1906–1908.

necessary to use apparent wind in connection with the wind corrections (art. 1219). Director systems include *erosion correctors* which, when set up with the range and the rounds fired (or the reduced velocity corresponding to the latter), automatically adjust the gun elevation for erosion losses. If the erosion losses are thus accounted for, they must, of course, not be included in the ballistic corrections which are to be applied elsewhere. If, on the other hand, the erosion losses have been included in the ballistic corrections which are to be applied to the range-keeper, the erosion correctors must be set for zero rounds fired, or for zero range. In short, it is necessary to make a complete survey of the fire-control equipment in order to determine how the various elements of the ballistic corrections are to be handled.

1239. *Ballistic-computation diagrams* are provided for the determination of some portions of the ballistic corrections which are not handled automatically by the range-keeper or other mechanical devices. These diagrams are usually constructed in the form of *alinement charts*, i.e., combinations of several graphs in an arrangement which provides for the solution, by a purely graphical process, of quantities which depend on several arguments. For example, the range correction corresponding to a given set of surface atmospheric conditions depends upon the surface temperature and barometric pressure, and upon the maximum ordinate and value of Column 12 corresponding to the given range. The ballistic-computation diagram for this range correction is arranged so that when entered with temperature, barometer, and range, the operations of determining the surface density from Table III, the maximum ordinate from Column 8, the ballistic density and corresponding multiplier from Table V, and the range correction from Column 12, are all accomplished entirely graphically and in a few steps. The diagrams for wind corrections include the operations both of resolving the wind into its components in and across the line of fire and of applying the components to Columns 13 and 16. Detailed instructions concerning their purpose and use are printed on the ballistic-computation diagrams themselves.*

Graphical devices for handling ballistic corrections

1240. Reduced-charge range tables (art. 1006) are provided for use in connection with reduced-charge firing, but the corresponding range scales are not always provided. It is a simple matter, however, to construct a reduced-charge range scale when both the full-charge and the reduced-charge range tables are available. An example will make the process clear. Let us suppose that a 16″ gun is to be fired at target practice, at 1600 yards, with reduced charges which are designed to give an initial velocity of 2000 f.s., and that the gun is provided only with a full-charge range scale which is graduated for the gun's service velocity of 2600 f.s. We find that in the reduced-charge range table (16″2000 f.s.) the range 1600 yards corresponds to the angle of elevation 1°08.′5, and that in the full-charge range table (16″2600 f.s.) the angle of elevation 1°08.′5 corresponds to the range 2656 yards. Hence if we use the full-charge range scale and set it at 2656, we will have the correct setting for 1600 yards with the reduced charge. Or we can accomplish the same result by pasting

Reduced-charge range scales

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* For explanation to the construction of alinement charts, see pp. 179–186, Marks' *Mechanical Engineers' Handbook* (Second Edition).

over the full-charge scale a temporary reduced-charge scale so laid off that the graduation 1600 on the temporary scale corresponds to the graduation 2656 on the full-charge scale. Other graduations for the temporary reduced-charge scale can be determined similarly. Example:

Given: The 16″ gun and its full-charge range table (16″2600 f.s.) and reduced-charge range table (16″2000 f.s.).

Find: The range graduations on the full-charge range scale to which the range graduations for 1500, 1600, 1700, and 1800 yards on the reduced-charge range scale must correspond.

By entering the 16″2000 f.s. range table with each of the given ranges and finding the corresponding angles of elevation, and then entering the 16″2600 f.s. range table with these angles of elevation and finding the corresponding ranges, we obtain the following relation between the graduations of the two scales:

Graduations of reduced-charge scale	Angle of elevation		Graduations of full-charge scale
yds.	°	′	yds.
1500	1	04.1	2493
1600	1	08.5	2656
1700	1	13.0	2822
1800	1	17.5	2989

1241. A problem very much similar to the above arises in the use of sub-caliber guns. A sub-caliber gun is a small gun which is mounted rigidly upon a larger gun, and which is used for training purposes to simulate the firing of the larger gun. The latter is elevated and trained by means of its regular operating gear and sights. If the sub-caliber gun requires an angle of elevation of 5° for a given range, the sights of the large gun must be set at the range which corresponds to the angle of elevation 5° according to the large gun's range table. The 1-pounder gun is usually used for sub-caliber practices, and a 1-pounder range table is available for controlling its fire. For example:

Control of sub-caliber firing

Given: A 1-pounder gun is mounted upon the chase of a 16″2600 f.s. gun, and the bores of the two guns are parallel.

Find: The settings on the 16″ gun's full-charge range scale that are required to give the ranges 500, 600, 700, 800, 900, and 1000 yards with the sub-caliber gun.

The required angles of elevation are found from the 1-pounder range table, and the ranges corresponding to these angles in the 16″2600 f.s. range table are the required settings for the 16″ gun's full-charge range scale.

1-pdr. range	Angle of elevation		16″ full-charge range scale setting
yds.	°	′	yds.
500	0	27.5	1100
600	0	35.1	1392
700	0	43.6	1719
800	0	53.0	2081
900	1	03.3	2463
1000	1	14.7	2885

1242. If the sub-caliber gun's range is controlled by means of the large gun's range scale, the spotter's corrections evidently must be converted to the same scale before being applied to it. It can be seen from the tabulation given above that

in order to increase the 1-pounder gun's range from 500 yards to 600 yards, the 16″ full-charge range scale setting must be increased from 1100 yards to 1392 yards; that is, an increase of only 100 yards in the actual range requires, in this case, an increase of 292 yards in the sight-bar range. Similarly, a 100-yard increase in the actual range from 900 yards to 1000 yards requires a 422-yard increase in the sight-bar range (i.e., from 2463 yards to 2885 yards). This means, then, that for actual ranges around 500 yards, the spotter's corrections must be multiplied by about 3 before being applied, while for actual ranges around 1000 yards they must be multiplied by about 4 before being applied. The necessity for such a highly artificial procedure can be avoided by preparing a sub-caliber range scale and pasting it over the large gun's range scale.

Conversion scale for plotting board

1243. Another method that may be employed very conveniently in connection with main battery guns, is the use of a conversion scale on the range ruler of the Mark II plotting board. This method serves equally well for the reduced-charge problem and for the sub-caliber problem. It is identical in principle with the method of substituting for the regular full-charge scale a reduced-charge or sub-caliber scale, but has the advantage of requiring the preparation of but one scale, instead of one for each gun, and of eliminating the undesirable feature of smearing the full-charge scales.

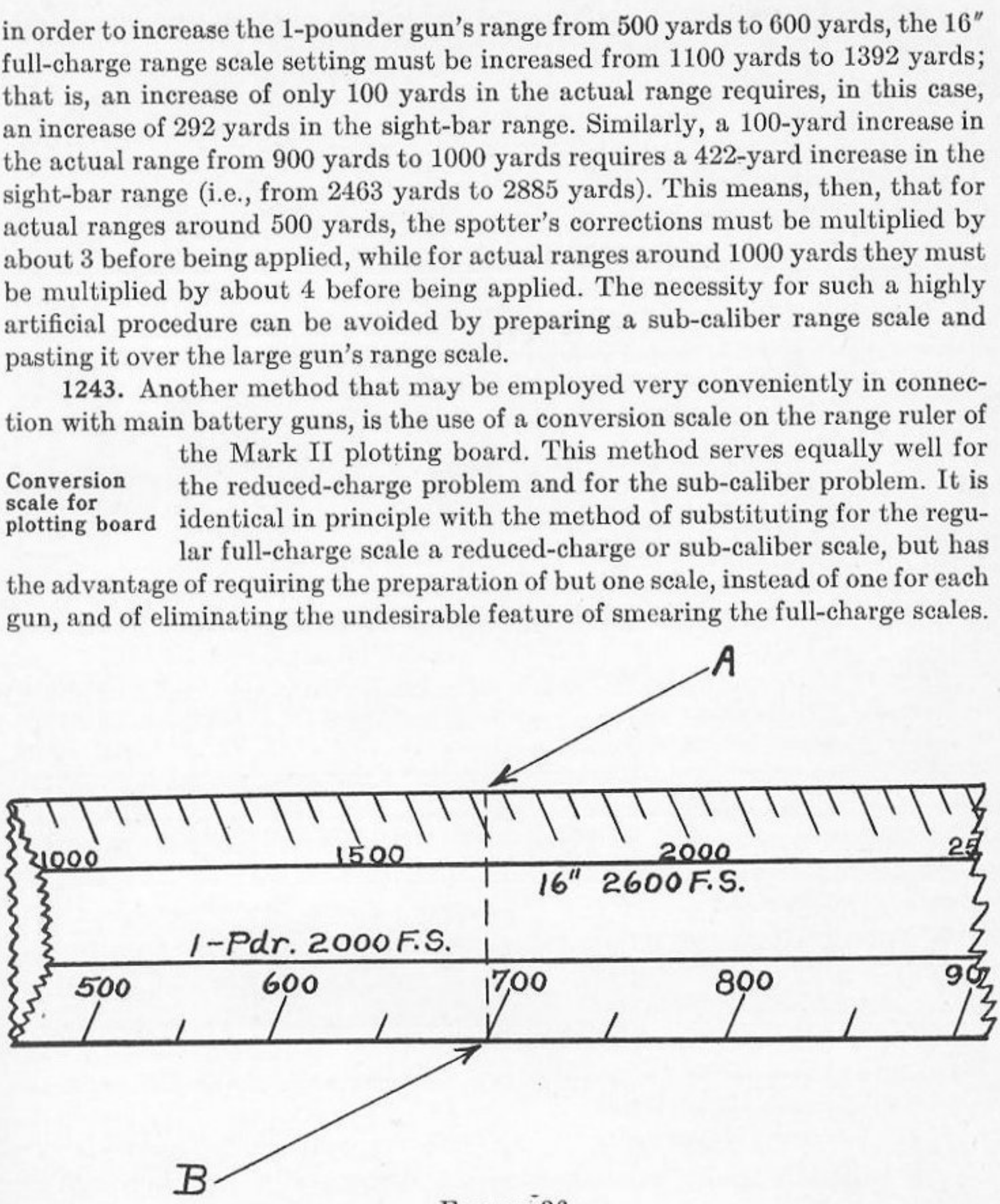


FIGURE 36

Figure 36 represents a section of the range ruler of the Mark II board. Scale A is the usual scale engraved on the upper edge of the ruler; it is labeled, for the purpose of this illustration, 16″2600 f.s. Scale B represents a temporary scale, marked on paper or some other suitable material and pasted along the bottom edge of the ruler; it is labeled, for the purpose of this illustration, 1-Pdr. 2000 f.s.

The graduations of scale B have been made to scale in accordance with the figures of the problem solved in article 1241. For instance, the 700-yard graduation of scale B was located by placing it vertically beneath the point representing 1719 yards on scale A (for the example shows that the 700-yard sub-caliber range corresponds to the 1719-yard range on the 16″ 2600 f.s. scale). Other points were located similarly.

Having graduated the ruler in this manner, range-finder ranges, ballistic corrections, and spots can now be plotted against scale B, and sight-bar ranges

read from scale A. Various other arrangements based on the principle illustrated in Figure 36 may also be devised.

EXERCISES

1. *Given:* The target distance and the total ballistic.
   Find: The sight-bar range and scale. (Nearest 50 yds. and whole mil.)

	Given			Answers	
	Target distance (yds.)	Total ballistic		Sight-bar range (yds.)	Scale (mils)
		Range (yds.)	Deflection (yds.)		
A	2000	Up 163	Right 12	2150	106
B	8500	Down 348	Right 22	8150	103
C	14,000	Down 412	Left 40	13,600	97
D	19,300	Up 580	Right 152	19,900	108
E	32,000	Up 874	Left 620	32,850	81

2. *Given:* The type of gun, the powder temperature, and the velocity loss due to erosion.
   Find: From the range table, at the given range, the range error due to the variations from the standard initial velocity.

	Given				Answers
	Gun	Range (yds.)	Powder temp. (°F.)	Erosion loss (f.s.)	Range error (yds.)
A	3″2700 f.s.	3000	85	6	(−) 22
B	4″2900 f.s.	1600	82	16	(−) 32
C	5″3150 f.s.	12,000	78	14	(−) 148
D	12″2900 f.s.	20,000	80	32	(−) 472
E	16″2600 f.s.	32,000	76	50	(−) 1443

3. *Given:* The type of gun, the surface air temperature, and the surface barometric pressure.
   Find: From the range table, at the given range, the range error for the ballistic density which is assumed to correspond to the given surface conditions.

	Given				Answers	
	Gun	Range (yds.)	Surface conditions		Multiplier, Table V	Range error (yds.)
			Air temp. (°F.)	Barometer (inches)		
A	3″2700 f.s.	3000	52	30.25	(−).40	(−) 49
B	4″2900 f.s.	1600	62	30.30	(−).20	(−) 4
C	5″3150 f.s.	12,000	91	29.20	(+).69	(+)444
D	12″2900 f.s.	20,000	77	29.80	(+).20	(+)148
E	16″2600 f.s.	32,000	88	29.12	(+).28	(+)358

4. *Given:* The type of gun, and the ballistic density as determined for the given range by the aerological party.

Find: From the range table, at the given range, the range error corresponding to the given ballistic density.

	Given			Answers	
	Gun	Range (yds.)	Ballistic density	Multiplier	Range error (yds.)
A	3″2700 f.s.	3000	1.040	(−).40	(−) 49
B	4″2900 f.s.	1600	1.020	(−).20	(−) 4
C	5″3150 f.s.	12,000	.940	(+).60	(+)386
D	12″2900 f.s.	20,000	.982	(+).18	(+)133
E	16″2600 f.s.	32,000	.963	(+).37	(+)474

5. *Given:*

16″2600 f.s. gun
Target distance = 33,100 yards
Surface air temperature = 84°F.
Surface barometer = 29″.30
Magazine temperature = 78°F.
Erosion loss = 58 f.s.
Relative bearing of target = 355°
Own ship's speed = 20 knots
Target angle = 345°
Target's speed = 18 knots
Velocity of ballistic apparent wind = 35 knots
Relative bearing of ballistic apparent wind = 0°

Find:

Gun ballistic in range and deflection.
Sight-bar range and scale.

Answers:

Up 615 yards, Left 33 mils
Range, 33,700, Scale 67

6. *Given:*

14″2600 f.s. gun
Target distance = 14,100 yards
Ballistic density = .974
Magazine temperature = 80°F.
Erosion loss = 42 f.s.
Own ship's true course = 150°
Own ship's speed = 16 knots
True bearing of target = 65°
Target's true course = 190°
Target's speed = 14 knots
Velocity of ballistic apparent wind = 24 knots
Relative bearing of ballistic apparent wind = 330°

Find:

Gun ballistic in range and deflection.
Sight-bar range and scale.

Answers:

Up 336 yards, Left 6 mils
Range 14,450, Scale 94

7. *Given:*

16″2600 f.s. gun
Target distance = 22,700 yards
Surface air temperature = 64°F.
Surface barometer = 29″.60
Magazine temperature = 78°F.
Erosion loss = 28 f.s.
Own ship's true course = 70°
Own ship's speed = 20 knots
True bearing of target = 355°
Target's true course = 90°
Target's speed = 22 knots
Velocity of ballistic true wind = 16 knots
True bearing of ballistic true wind = 45°

Find:

Gun ballistic in range and deflection.
Sight-bar range and scale.

Answers:

Up 657 yards, Left 3 mils
Range 23,350, Scale 97

8. *Given:*

14″2600 f.s. gun
Target distance = 14,000 yards
Ballistic density = .990
Magazine temperature = 82°F.
Erosion loss = 38 f.s.
Relative bearing of target = 265°
Own ship's speed = 16 knots
Target angle = 55°
Target's speed = 12 knots
Velocity of ballistic apparent wind = 26 knots
Relative bearing of ballistic apparent wind = 340°

Find:

Gun ballistic in range and deflection.
Sight-bar range and scale.

Answers:

Up 346 yards, Left 6 mils
Range 14,350, Scale 94

9. *Given:*

16″2600 f.s. gun
Target distance = 24,700 yards
Surface air temperature = 64°F.
Surface barometer = 30″.27
Magazine temperature = 74°F.
Erosion loss = 20 f.s.
Own ship's true course = 50°
Own ship's speed = 16 knots
Target's true bearing = 330°
Target's true course = 80°
Target's speed = 18 knots
Velocity of ballistic true wind = 14 knots
True bearing of ballistic true wind = 30°

Find:

Gun ballistic in range and deflection.
Sight-bar range and scale.

Answers:

Up 773 yards, Left 7 mils
Range 25,450, Scale 93

10. *Given*: The type of gun and a sight-bar range for that gun.
Find: The corresponding range of the 1-pdr. gun, assuming that the latter is rigidly mounted upon the large gun and that its bore is parallel with that of the large gun.

	Given		Answers
	Gun	Large Gun S. B. Range	1-Pdr. Range
A	3″2700 f.s.	3000	1356
B	4″2900 f.s.	1100	443
C	5″3150 f.s.	1700	559
D	12″2900 f.s.	20400	3498
E	16″2600 f.s.	2600	932

CHAPTER 13

THE EFFECTS OF THE EARTH'S ROTATION ON THE TRAJECTORY. ERRORS DUE TO TRUNNION TILT. COLD-GUN ERRORS.

New Symbols Introduced.

ΔX_Ω.....Change in range due to earth's rotation.
D_Ω.....Lateral deviation due to earth's rotation.
A, B, D....Rotation coefficients (see art. 1305); the rotation coefficient D is not to be confused with the drift, which is represented by the same symbol.
a.....Azimuth of the line of fire, measured clockwise from north.
l.....Geographical latitude.
t.....The angle of trunnion tilt, i.e., the angle at which the axis of the gun trunnions is inclined with respect to the horizontal.
ϕ_t'.....The effective angle of elevation as modified from ϕ' by trunnion tilt.
t_D.....The angular deflection resulting from trunnion tilt.
D_t.....Lateral deviation due to trunnion tilt, i.e., linear deflection corresponding to the angular deflection t_D.

1301. The effects of the earth's rotation on the trajectory vary according to the latitude of the gun's location and the azimuth of the line of fire. The assumption of a motionless earth for range-table values (art. 801 (g)) therefore belongs in the same category as the assumption of zero values for other forces which may occur in various combinations of amount and direction. That is to say, just as range-table values may be adjusted for the effects of wind, motion of gun, and motion of target, according to the amount and direction of each of these forces, they may be adjusted also for the effects of the earth's rotation according to the latitude of the gun's location and the azimuth of the line of fire. The errors in range and deflection due to the earth's rotation are not listed in the range table, but they can be determined from formulas which are given in this chapter.*

For the purposes of logical explanation, the effects of the earth's rotation may be resolved into three components which we shall call, for convenience in referring to them, the X-component, the Y-component, and the Z-component. The X-component affects only the range, the Y-component both the range and the deflection, and the Z-component only the deflection. We shall now examine these three components in turn.

1302. Let us suppose that a gun is located at the equator and fired due east. As a result of the earth's rotation, points on the equator are carried through space to the eastward, along a circular path, at a linear velocity of about 1500 f.s.† The projectile, however, after it has left the gun, retains this velocity of 1500 f.s. in a fixed direction with respect to space,

The X-component

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* There appears to be no good reason for not listing these errors in the range tables of the U. S. Navy; they have been listed in the firing tables (range tables) of the U. S. Army since about 1920.

† This is found by dividing the earth's circumference at the equator by the number of seconds in a sidereal day.

i.e., in the direction occupied at the instant of projection by the tangent to the equator at the position of the gun. Let us suppose that, under the range-table assumption of a motionless earth, the point of fall would occur at a distance of 30,000 yards from the gun. The situation then is that the projectile (apart from the motion imparted to it by the charge of the gun) and the predicted point of fall are both in motion to the eastward at a linear velocity of about 1500 f.s., but that the projectile's motion is along a straight path, while the motion of the predicted point of fall is along a circular path. This situation is illustrated in Figure 37, in which EE' represents the earth's surface at the equator, OSH a trajectory based on the assumption of a motionless earth, and HH_1 the distance through which the earth's rotation has carried the predicted point of fall (H) during the predicted time of flight. The projectile, as a result of the earth's rotation, has acquired motion in the direction of the tangent OO'. In the predicted time of flight this motion will have caused displacement of the projectile from the predicted position H to the position H', the displacement HH' being equal in amount to HH_1 but being parallel in direction to OO'. At the expiration of the predicted time of flight the projectile will then still be above the earth's surface, and the actual point of fall will occur at H_2, thus causing the range to be increased by the amount H_1H_2. In this case (firing to eastward) there is an apparent falling away of the earth's surface from the projectile, as a consequence of which the latter's flight is prolonged and the range increased. It can be shown similarly that in the case of firing to westward there is an apparent rising of the earth's surface with respect to the projectile, as a consequence of which the latter returns sooner to the earth and the range is decreased.

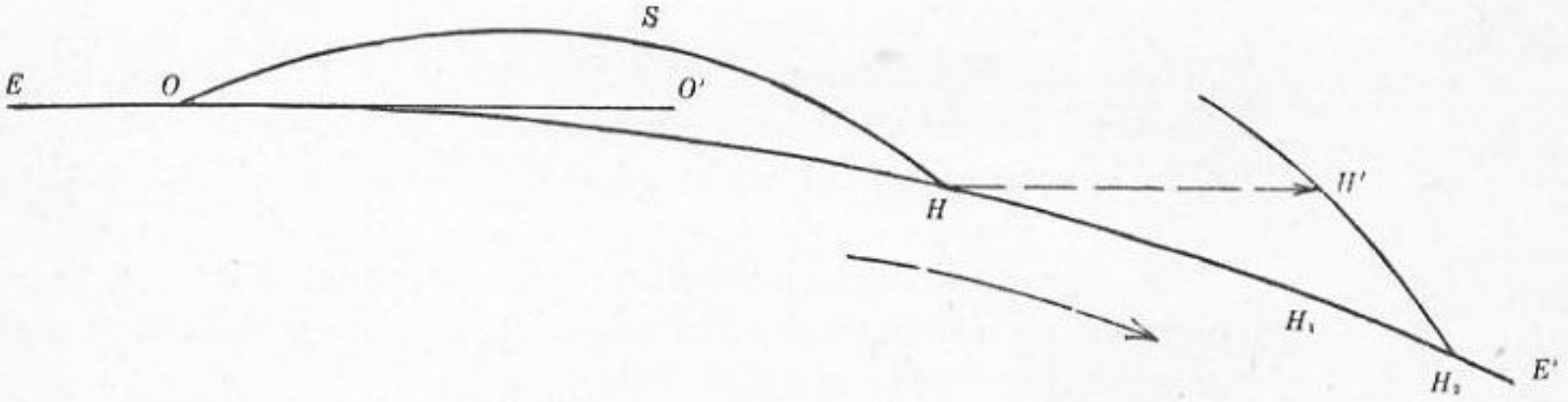


FIGURE 37

The effect of the X-component varies in magnitude according to the linear velocity of the earth's surface with respect to the direction of fire. The linear velocity itself varies as the cosine of the latitude, or as cos l; it is greatest at the equator and vanishes at the poles. At a given latitude, the linear velocity of the earth's surface with respect to the direction of fire varies as the sine of the azimuth of the line of fire, or as sin a (a being measured clockwise from the north); it is greatest for fire to the east or west, and vanishes for fire to the north or south. It is to be noted that the X-component does not directly affect the deflection.

1303. Let us now suppose that a projectile is fired vertically from a position on the equator. In order to remain vertically* above the point on the earth's surface from which it was fired, the projectile, as it rises, evidently would have to increase its velocity to the eastward in proportion to its distance from the center of the earth. Actually, however, the

The *Y*-component

* The term *vertical* as used in this discussion is to be interpreted as defining a direction perpendicular to the earth's surface.

projectile's eastward velocity is limited to that of the point on the earth's surface from which the projectile was fired. As the projectile ascends it therefore lags, by an ever-increasing amount, to the westward of a line extending vertically upward from the gun, and at the end of its ascent the projectile will be vertically above a point on the earth's surface which is to the westward of the gun. At the end of its ascent the projectile will have lost all of the motion imparted to it by the charge of the gun, but will have retained the eastward velocity imparted to it by the earth's rotation. Its motion in descent will be the resultant of this eastward velocity and of the gravitational pull vertically downward. We have seen that at the end of its ascent the projectile is vertically above a point to the westward of the gun; for convenience, let us refer to this point to the westward of the gun as point B. At the beginning of the projectile's descent the gravitational pull is then vertically downward toward point B. But as the projectile, in its descent, lags to the westward of the vertical with respect to point B, the gravitational pull will, accordingly, shift progressively toward points on the earth's surface lying to the westward of point B. The projectile then will eventually return to the earth at a point somewhat to the westward of point B, and hence to the westward of the gun.

For other than vertical fire, points on the actual trajectory will similarly lag to the westward of the corresponding points on the trajectory calculated under the assumption of a motionless earth. As a result of this lag to the westward, the range will be decreased if the direction of fire is to the eastward, and increased if the direction of fire is to the westward. It will be observed that the effects of the Y-component on the range are thus opposite in direction to those of the X-component (art. 1302).

The effect of the Y-component varies in magnitude according to the linear velocity of the earth's surface with respect to the direction of fire. The linear velocity itself varies as the cosine of the latitude, or as cos l; it is greatest at the equator and vanishes at the poles. In the case of fire to the eastward or westward, the westward lag caused by the Y-component operates wholly *in* the line of fire and hence affects only the *range;* in the case of fire to the northward or southward, the lag is still to the westward, and hence it operates wholly *across* the line of fire and affects only the *deflection.* For intermediate directions of fire the westward lag has a component *in* the line of fire which affects the *range*, and a component *across* the line of fire which affects the *deflection.* The range component varies as sin a and the deflection component as cos a (where a is the azimuth of the line of fire, measured clockwise from the north). Since the lag is always to the westward, the deflection errors resulting therefrom are to the *left* when the direction of fire is in the *northerly* quadrants (i.e., between 0° and 90°, or between 270° and 360°), and to the *right* when the direction of fire is in the *southerly* quadrants (i.e., between 90° and 180°, or between 180° and 270°).

1304. The Z-component of the rotational effects results from the fact that the line from the gun to the target changes its direction constantly with respect to space (unless the gun is at the equator), whereas the line of fire moves parallel to itself through space and hence does not change its direction with respect to space. Figure 38 (a) represents the situation that obtains in the case of fire from or to one of the earth's poles. Let us assume first that the gun is exactly at the north pole, P, and the target at A, and that at the instant of fire the gun is trained in the direction PA. At the pole the linear velocity of the earth's surface is, of course, zero, and the projectile therefore acquires no motion from the earth's rotation but travels through space

The *Z*-component

in the direction PA. But the earth's rotation during the time of flight carries the target to A', and the point of fall therefore occurs to the right of the target. The amount of the lateral deviation in this case is approximately equal to the product of the time of flight and the linear velocity of the earth's surface at A; or, expressed in mils, it is approximately equal to the product of the time of flight and the angular velocity (in mils) of the earth's rotation.*

Let us suppose now that the gun is at A and the target at P (Figure 38 (a)), and that at the instant of fire the gun is trained in the direction AP. In this case the earth's rotation imparts no motion to the target, but imparts to the projectile motion in the direction AA'', normal to AP, at the linear velocity of the earth's surface at A. The point of fall will therefore again occur to the *right* of the target by an amount approximately equal to the product of the time of flight and the linear velocity of the earth's surface at A. It is to be noted that the deviation has been to the right in both cases illustrated, although the direction of fire in the first case was to southward and in the second case to northward.

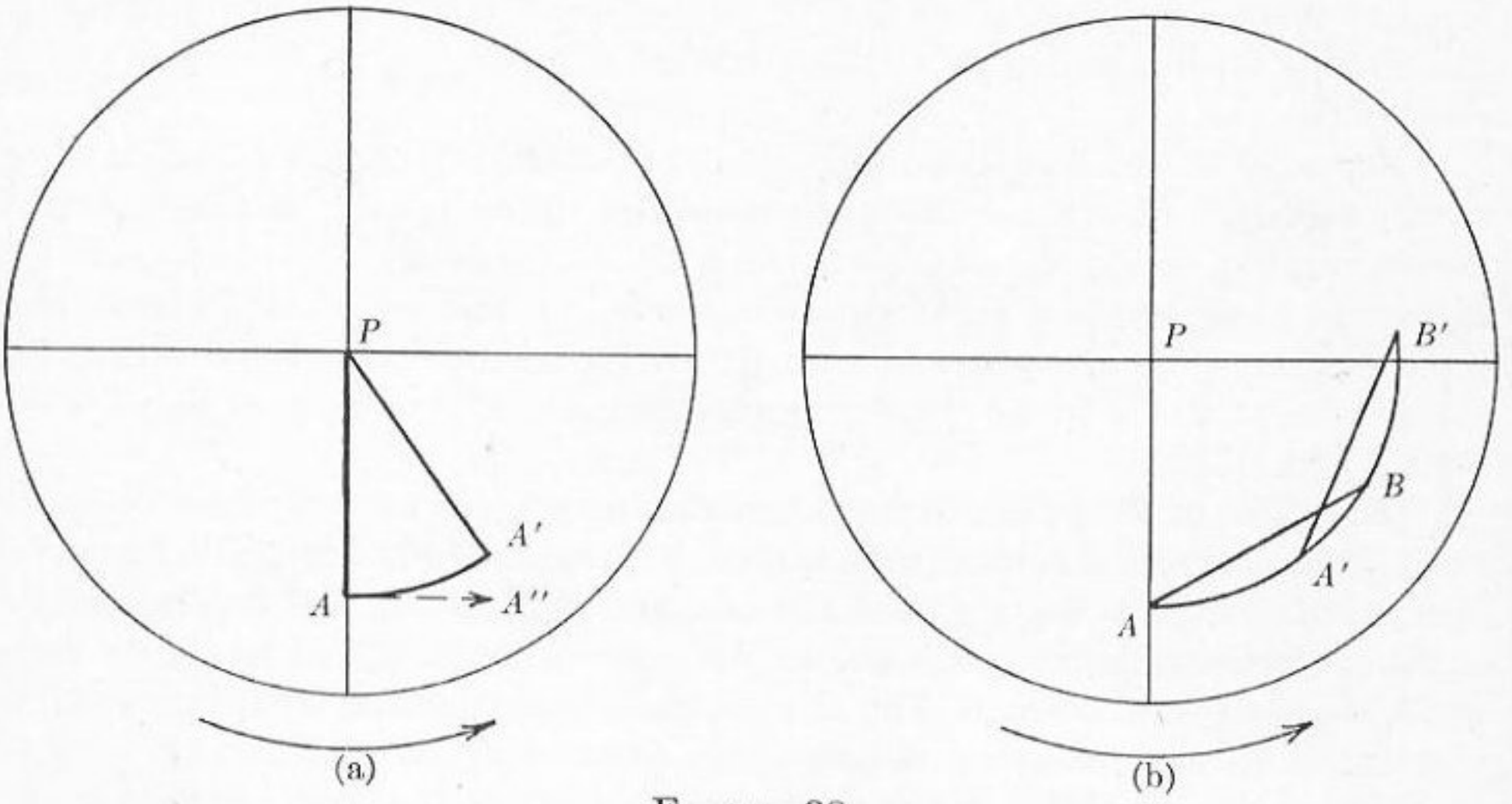


FIGURE 38

Figure 38 (b) represents the situation that obtains in the case of fire to eastward or westward near one of the earth's poles. Let us assume first that the gun is located at A, and the target at B (A and B being equidistant from the north pole, P), and that at the instant of fire the gun is trained in the direction AB. During the time of flight, however, the direction of the line joining the gun and target will have changed from AB to $A'B'$, and the projectile, having been fired in the initial direction AB, will consequently fall to the *right* of the target. Assuming that the gun and target are very near the pole, A, B, and P (Fig. 38 (b)) lie in practically a flat plane, and under this condition the change in direction between AB and $A'B'$ will be practically equal to the angle through which the earth has rotated during the time of flight. The case just considered represents the situation for fire to eastward. In order to examine the situation for fire to westward, let us assume that the gun is at B and the target at A. The line from gun to target changes its direction, in this case, from BA to $B'A'$ during the time of flight, and the initial direction BA which is imparted to the projectile will again cause the latter to fall to the *right* of the target.

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* The angular velocity of the earth's rotation is equal to 2000π mils divided by the number of seconds in a sidereal day, or $6283.2/86{,}164 = .073$ mils per second.

We have now considered the cases of fire to the northward, southward, eastward, and westward (near the *north* pole), and have found in each case that the lateral deviation is to the *right*. We have also found that the change in direction of the line from gun to target, which is responsible for this deviation, depends on the same factors *irrespective of the direction of fire*, being in all cases equal to the angle through which the earth rotates during the time of flight (which applies, however, only to points very near the pole, as here assumed).

Figure 39 illustrates how a change of latitude affects the situation. In this figure the sets of lines AB and $A'B'$, A_1B_1 and $A_1'B_1'$, A_2B_2 and $A_2'B_2'$, etc.,

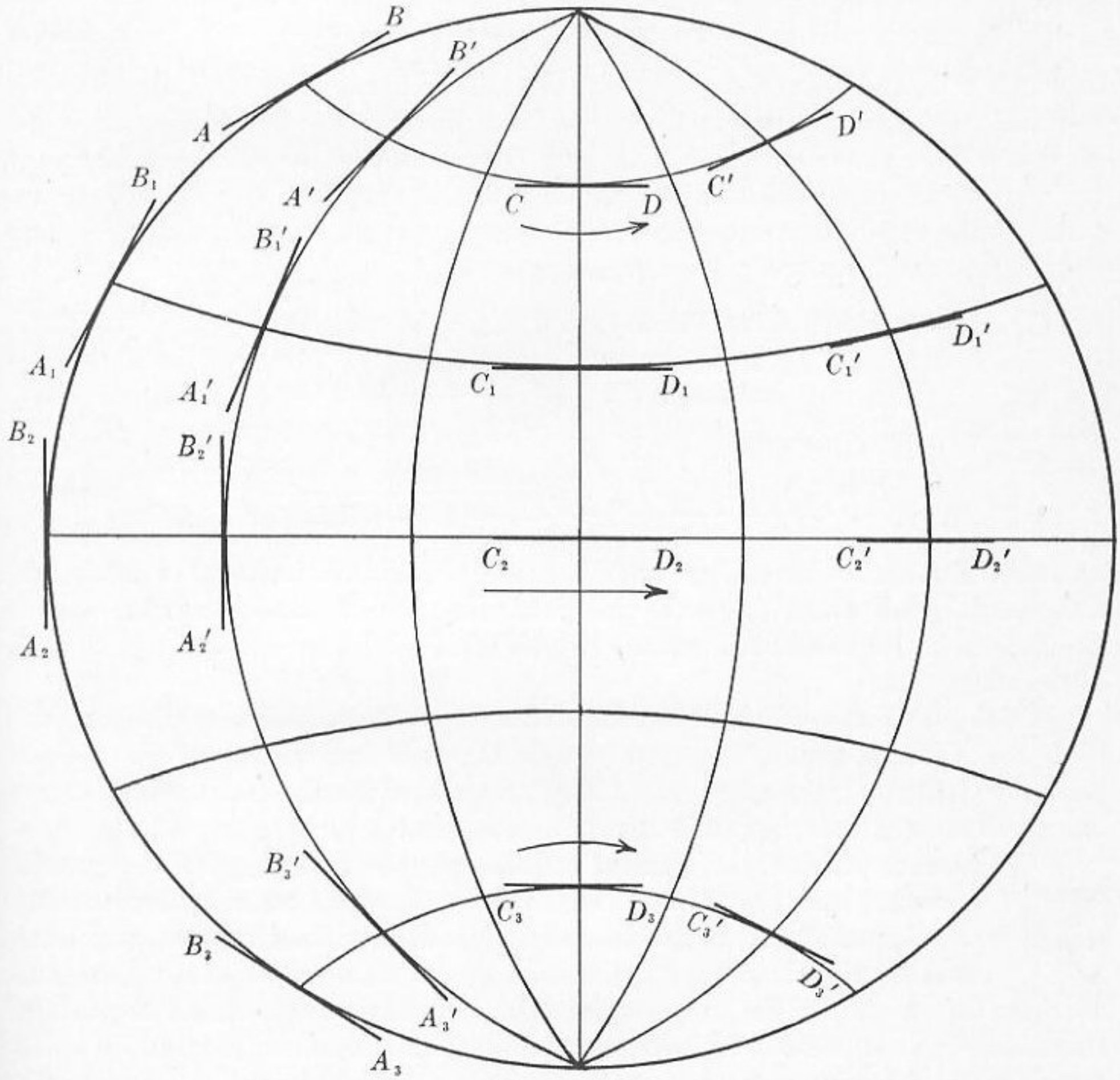


FIGURE 39

illustrate how the directions of north and south lines are changed, with respect to space, by the earth's rotation. It will be observed that the change in direction is greatest near the poles, as illustrated by the lines AB and $A'B'$. At the equator the change in direction due to rotation vanishes, the successive positions of the line A_2B_2 being parallel to each other. In the southern hemisphere we find that the changes in direction are such as to cause deviation to the *left*. (Note that the initial direction A_3B_3 inclines to the *left* of the final direction $A_3'B_3'$, and that the initial direction B_3A_3 also inclines to the *left* of the final direction $B_3'A_3'$; compare with the set A_1B_1 and $A_1'B_1'$.) The effect of the Z-component is therefore greatest at the poles and zero at the equator, and it varies as the sine of the lati-

tude, i.e., as sin l, and is independent of the direction of fire. The sets of lines CD and $C'D'$, C_1D_1 and $C_1'D_1'$, etc., show similarly how the directions of east and west lines are changed, with respect to space, by the earth's rotation.

Formulas for rotation effects

1305. Comparison between the differential equations of the trajectory referred to a set of axes rotating in the direction of and with the angular velocity of the earth's rotation, and the differential equations of the trajectory referred to a set of fixed axes (i.e., a motionless earth), shows that the terms representing the effects of rotation take the forms

$$\Delta X_\Omega = A \cos l \sin a \tag{1301}$$

$$D_\Omega = B \sin l - D \cos l \cos a \tag{1302}$$

in which ΔX_Ω and D_Ω are, respectively, the change in range and the lateral deviation due to rotation, l is the latitude, a is the azimuth of the line of fire (measured clockwise from the north), and A, B, and D are coefficients which depend on the trajectory and whose exact values can be obtained only by integration of certain terms in the differential equations. Sufficiently approximate values of A, B, and D may be found, however, from the expressions.*

Rotation coefficients

$$A = 0.00002431\ T\ (346\ \Delta X_{\phi_{10}} + 1.8\ X \cot \omega) \tag{1303}$$

$$B = 0.00003646\ TX \left(\frac{3 \tan \phi + \tan \omega}{\tan \phi + \tan \omega}\right) \tag{1304}$$

$$D = 0.00001215\ TX \tan \phi \left(\frac{19 \tan \phi + \tan \omega}{7 \tan \phi + 3 \tan \omega}\right) \tag{1305}$$

in which X is the horizontal range, T the time of flight, ϕ the angle of departure, ω the angle of fall, and $\Delta X_{\phi_{10}}$ is the change in range corresponding to an increase of 10 minutes in the angle of departure (which can be determined from Column 2 b of the range table).

1306. The form of (1301) indicates that the coefficient A is associated with both the X-component of rotation (art. 1302), and that portion of the Y-component which affects the range (art. 1303), for we have found that the range effects due to either of these components vary as cos l and sin a. For any angles of departure which occur in naval surface fire, the effect of the X-component is greater than that of the Y-component (in range); ΔX_Ω therefore takes the sign of the X-component's effect, which is positive (i.e., such as to increase the range) for fire to eastward, and negative (i.e., such as to decrease the range) for fire to westward. The term sin a duly takes care of these signs, since sin a is positive for easterly quadrants and negative for westerly quadrants (cos l, of course, remains positive for any value of l in either hemisphere).

Signs of rotation effects

The form of (1302) indicates that the coefficient B is associated with the Z-component of rotation, which varies as sin l (art. 1304), and that the coefficient D is associated with that portion of the Y-component which affects the deflection and which varies as cos l cos a (art. 1303). We have found that the lateral deviation due to the Z-component is positive (i.e., to the right) in the northern hemisphere, and negative (i.e., to the left) in the southern hemisphere; in order

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* These expressions, except for a slight simplification in the expression for A, are in the form given by Dr. T. H. Gronwall in *T.S. 162* (April, 1921, U. S. War Department). For strictly mathematical treatments of the effects of rotation, see arts. 4 and 34, *New Methods in Exterior Ballistics*, F. R. Moulton; also, Chapter XIII, *A Course in Exterior Ballistics*, R. S. Hoar (U. S. War Department Document No. 1051, December, 1920).

that the term $B \sin l$ may have the proper sign, l must therefore be considered as a positive angle for north latitudes and as a negative angle for south latitudes. We have found that the lateral deviation due to the Y-component is positive for directions of fire lying in the quadrants 90° to 180° and 180° to 270°, and negative for directions of fire lying in the quadrants 0° to 90° and 270° to 360°. The minus sign before the term $D \cos l \cos a$, in conjunction with the sign of $\cos a$, duly takes care of the sign of the Y-component's effect. For any angles of departure which occur in naval surface fire, the value of the term $D \cos l \cos a$ remains small, and is exceeded by the value of the term $B \sin l$ for latitudes greater than about 10°. Consequently, except within about 10° of the equator, the sign of D_Ω is positive in the northern hemisphere, and negative in the southern hemisphere, irrespective of the direction of fire. That is to say, the lateral deviation due to the earth's rotation is always to the right in the northern hemisphere and always to the left in the southern hemisphere, except near the equator, and near the equator it is always small.

1307. The calculation of the effects of the earth's rotation is illustrated in the following example.

Calculation of rotation effects

Given: The 16″2600 f.s. gun and its range table; range 30,000 yards; latitude 45° north; direction of fire 240° true.

Find: The change in range and the lateral deviation due to the earth's rotation.

From the 16″2600 f.s. range table we have the data

$$X = 30{,}000 \text{ yards}$$
$$T = 54.49 \text{ seconds (Col. 4)}$$
$$\phi = 23°19.'0 \text{ (Col. 2)}$$
$$\omega = 34°04' \text{ (Col. 3)}$$
$$\Delta X_{\phi_{10}} = \frac{10}{8.2} \times 100 = 122 \text{ yards (Col. 2b)}$$

The calculation of the values of the coefficients A, B, and D follows.

$\phi = 23°19.'0$	$\omega = 34°04'$
$\tan \phi = 0.43101$	$\tan \omega = 0.67620$
$3 \tan \phi = 1.2930$	$3 \tan \omega = 2.0286$
$7 \tan \phi = 3.0171$	
$19 \tan \phi = 8.1892$	

$\Delta X_{\phi_{10}} = 122$	log 2.08636	$X = 30{,}000$	log 4.47712
346	log 2.53908	$\omega = 34°04'$	lcot 0.16992
42,212	log 4.62544	1.8	log 0.25527
79,856			log 4.90231
122,068	log 5.08661		
$T = 54.49$	log 1.73632		
0.00002431	log 5.38578−10		
$A = 161.70$	log 2.20871		

0.00003646	log 5.56182−10
$X = 30{,}000$	log 4.47712
$T = 54.49$	log 1.73632
$(3 \tan \phi + \tan \omega) = 1.9692$	log 0.29429
$(\tan \phi + \tan \omega) = 1.1072$	colog 9.95577−10
$B = 109.50$	log 2.03942

0.00001215	log 5.08458−10
$X=30{,}000$	log 4.47712
$T=54.49$	log 1.73632
$\phi=23°19'.0$	ltan 9.63449−10
$(19\tan\phi+\tan\omega)=8.8654$	log 0.94770
$(7\tan\phi+3\tan\omega)=5.0457$	colog 9.29708−10
$D=15.041$	log 1.17729

The calculation is now completed by the solution of (1301) and (1302).

$A=161.70$	log 2.20871
$l=45°$	lcos 9.84949−10
$a=240°\ (60°)$	(−)lsin 9.93753−10
$\Delta X_\Omega=(-)99$ yds	(−)log 1.99573

$B=109.50$	log 2.03942	$D=15.041$	log 1.17729
$l=45°$	lsin 9.84949−10	$l=45°$	lcos 9.84949−10
77.4	log 1.88891	$a=240°\ (60°)$	(−)lcos 9.69897−10
		(−)5.3	(−) log 0.72575

$D_\Omega=77.4-(-)5.3=83$ yards

The results obtained show that the earth's rotation in this case causes a reduction of 99 yards in the range, and a lateral deviation of 83 yards to the right.

1308. The present practice in the preparation of U. S. Navy range tables is to ignore the effects of the earth's rotation altogether. That is to say, the range and lateral deviation observed at the proving ground, in connection with the ranging of the gun for range-table data (art. 810), are not adjusted for the effects of the earth's rotation.* Consequently the range and drift, as tabulated in our range tables, include the effects of the earth's rotation for the latitude of the proving ground and for the direction of fire at which the ranging was conducted, and corrections to the range-table data must be adjusted accordingly. Practically all U. S. Navy range tables now in service are based on ranging conducted at the U. S. Naval Proving Ground at Dahlgren, Va., in latitude 38°N and in the direction 120° true. In order to determine, for a given latitude and direction of fire, rotation effects that are applicable to these range tables, it is therefore necessary to subtract from the ΔX_Ω and D_Ω for the given l and a, the ΔX_Ω and D_Ω for $l=38°$N and $a=120°$. This operation can be incorporated in the rotation formulas themselves as follows, thus making the formulas directly applicable to the range tables and eliminating the necessity of making double computations.

Formulas for determination of rotation effects in connection with U. S. Navy range tables

$$\Delta X_\Omega = A(\cos l \sin a - 0.68244) \tag{1306}$$

$$D_\Omega = B(\sin l - 0.61566) - D(\cos l \cos a + 0.39400) \tag{1307}$$

The numerical terms that appear in (1306) and (1307) are, of course, merely the

* The practice of ignoring the rotation effects in connection with the proving-ground ranging can hardly be justified, especially in view of the fact that corrections are applied for other causes whose influence is often no greater than that of the earth's rotation. It approaches the absurd, for example, to apply to the observed range a correction for the height of the gun and to ignore the correction for rotation, for at long ranges the height-of-gun correction is generally but a small fraction of the rotation correction. The rotation corrections may, indeed, often be comparable in magnitude to any of the corrections actually applied to the observed range in connection with the proving-ground ranging.

required combinations of the functions for $l=38°$ and $a=120°$. For example, the operation

$$D \cos l \cos a - D \cos 38° \cos 120°$$

results in the expression

$$D \cos l \cos a - D(0.78801)(-0.50000)$$

or

$$D(\cos l \cos a + 0.39400)$$

which appears in (1307), and the other numerical terms were determined similarly.

1309. The following example illustrates the application of formulas (1306) and (1307).

Calculation of rotation effects in connection with U. S. Navy range tables

Given: The 16″2600 f.s. gun and its range table; range 30,000 yards, latitude 45° north; direction of fire 240° true.

Find: The change in range and the lateral deviation due to the earth's rotation, considering that the range table includes these effects for the latitude and direction of fire at which the proving-ground ranging was conducted.

The coefficients A, B, and D are the same as already found in the example given in article 1307. Using these in connection with formulas (1306) and (1307), we then have

$l=45°$	lcos 9.84949−10
$a=240°$ (60°)	(−)lsin 9.93753−10
(−)0.61238	(−)log 9.78702−10
(−)0.68244	
(−)1.29482	(−)log 0.11220
$A=161.70$	log 2.20871
$\Delta X_a=$(−)209 yards	(−)log 2.32091

$B=109.50$ log 2.03942	$l=45°$ lcos 9.84949−10
$l=45°$ sin 0.70711	$a=240°$(60°) (−)lcos 9.69897−10
(−)0.61566	(−)0.35356 (−)log 9.54846−10
0.09145 log 8.96118−10	(+)0.39400
10.0 log 1.00060	0.04044 log 8.60681−10
	$D=15.041$ log 1.17729
	0.6 log 9.78410−10

$D_a=10.0-0.6=9$ yards

The results obtained show that the earth's rotation in this case causes a reduction of 209 yards in the range, and a lateral deviation of 9 yards to the right, *with respect to the range-table values.*

Use of signs in south latitude

1310. The following example illustrates how the signs are to be used in connection with the solution of (1306) and (1307) for a problem involving south latitude. Assuming the same data as already stated for the example in article 1309, except that the latitude is 45° south, the solution is then as follows.

$l=(-)45°$	lcos 9.84949−10
$a=240°\ (60°)$	$(-)$lsin 9.93753−10
$(-)0.61238$	$(-)$log 9.78702−10
$(-)0.68244$	
$(-)1.29482$	$(-)$log 0.11220
$A=161.70$	log 2.20871
$\Delta X_\Omega=(-)209$ yards	$(-)$log 2.32091

$B=109.50$	log 2.03942	$l=(-)45°$	lcos 9.84949−10
$l=(-)45°$	$(-)$sin 0.70711	$a=240°\ (60°)$.	$(-)$lcos 9.69897−10
	$(-)$ 0.61566	$(-)0.35356$..	$(-)$log 9.54846−10
	$(-)$ 1.32277 log 0.12149	$(+)0.39400$	
$(-)144.8$	$(-)$log 2.16091	0.04044	log 8.60681−10
		$D=15.041$	log 1.17729
		0.6	log 9.78410−10

$D_\Omega=(-)144.8-0.6=(-)145$ yards

The results obtained show that the earth's rotation in this case causes a reduction of 209 yards in the range, and a lateral deviation of 145 yards to the left, *with respect to the range-table values*. It will be observed that the change from north to south latitude has affected only the term $B \sin l$; ΔX_Ω therefore has not been affected at all, although D_Ω has changed very materially.

ERRORS DUE TO TRUNNION TILT

1311. It is common knowledge to those familiar with the use of small arms that a rifle must be held so that its sights are vertical in order to shoot correctly, and that if the rifle is canted, the bullet is deflected to the side toward which the sights are inclined. What is commonly termed *cant* in the case of a rifle, is termed *trunnion tilt* in the case of a gun. The trunnions of a gun are said to be tilted when the axis of the trunnions is not horizontal, and the angle of trunnion tilt, which we shall denote by t, is the angle between the horizontal plane and the axis of the trunnions. Trunnion tilt operates to reduce the range and to cause lateral deviation toward the side of the lower trunnion. The reason for this will be apparent if the extreme case is considered. Let us suppose that the range scale of a gun has been set for a range corresponding to the angle of elevation 5°, and that the deflection scale has been set at 100 (i.e., so that the line of sight is parallel to the bore). If the gun is now tilted through 90°, so that its trunnions are vertical, the range setting becomes deflection setting and vice versa, and with the line of sight on the target the gun is actually laid at zero angle of elevation and at 5° in deflection. In any case, trunnion tilt operates to introduce into the deflection a component of the angle of elevation for which the sights are set, and to introduce into the angle of elevation a component of the deflection for which the sights are set. The component of deflection which is introduced into the angle of elevation by trunnion tilt is small enough to be neglected in any practical case. The component of elevation which affects the deflection may, however, cause very serious errors in deflection. The component of elevation which is lost due to trunnion tilt may, under exceptional circumstances, cause noticeable errors in range, but ordinarily these errors are small.

Nature of errors due to trunnion tilt

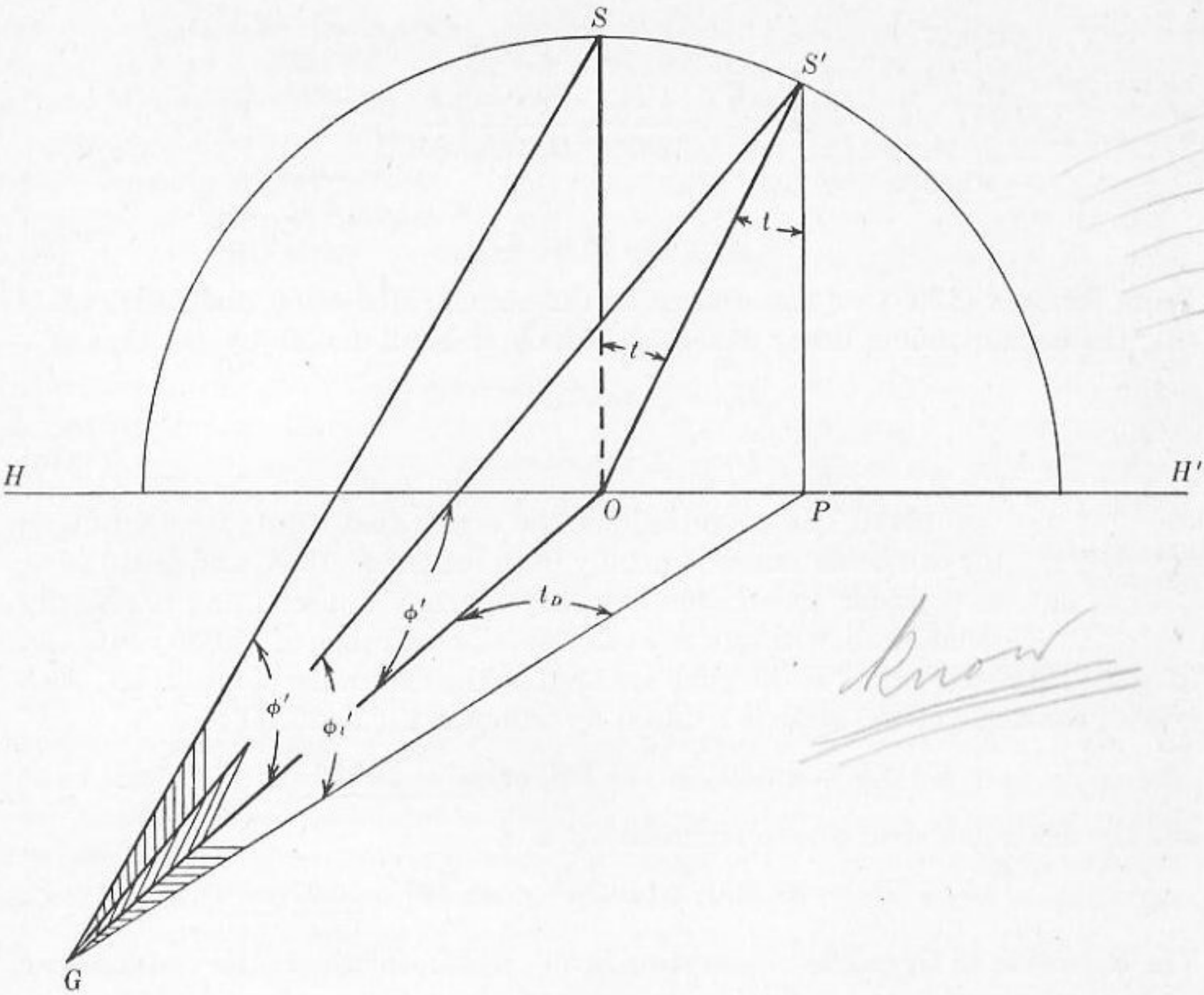


FIGURE 40

1312. In Figure 40, G represents the position of a gun and GO the line of sight; GS represents the axis of the bore with the trunnions of the gun in the horizontal plane $H\,H'$. The angle of elevation is OGS, or ϕ'. GS' represents the axis of the bore with the trunnions tilted to the right by the amount of the angle SOS', or t. (These relations are shown in perspective.) The figure shows that, with the trunnions tilted, the angle of elevation is reduced to $S'GP$, or ϕ_t', and a deflection component equal to the angle OGP, or t_D, is introduced. Noting that the triangles SOG, $S'OG$, $S'PG$, and POG all are right triangles, that $GS=GS'$ and $OS=OS'$, and that the angles OGS and OGS' both represent the angle of elevation ϕ', we find the following relations.*

$$\sin \phi_t' = \frac{S'P}{GS'} = \frac{OS' \cos t}{OS' \csc \phi'}$$

whence

$$\sin \phi_t' = \sin \phi' \cos t. \qquad (1308)$$

* These formulas, of course, do not take into account the angular deflection setting of the sights. Trunnion tilt operates to alter the deflection setting just as it operates to alter the elevation setting; that is, it reduces the effective deflection setting and introduces a component thereof into the angle of elevation. Within practical limits of deflection setting and trunnion tilt, however, the reduction in effective deflection setting is practically insignificant, and the component of deflection setting that is introduced into the angle of elevation is not likely to cause noticeable errors. By far the most important effect of trunnion tilt is the introduction of a component of the angle of elevation into the deflection, i.e., the effect represented by formula (1309) or (1310).

Also

Formulas for effects of trunnion tilt

$$\tan t_D = \frac{OP}{OG} = \frac{OS' \sin t}{OS' \cot \phi'}$$

whence

$$\tan t_D = \tan \phi' \sin t. \tag{1309}$$

From formula (1309) we can determine the angular deflection due to trunnion tilt; the corresponding linear deflection, which we shall denote by D_t, then is

$$D_t = X \tan t_D$$

or

$$D_t = X \tan \phi' \sin t. \tag{1310}$$

Examples of effects of trunnion tilt

1313. The magnitude of the errors that result from trunnion tilt can be determined readily from formulas (1308) and (1310). Let us consider an extreme case, say a heavy cruiser firing practically deadahead, with gun selevated at 30° for a range of 28,000 yards,and rolling 10° to either side. If the guns are fired at the extremity of a roll, i.e., with $t = 10°$, the angle of elevation is reduced by trunnion tilt to 29°30′;

$$\sin \phi_t' = \sin 30° \times \cos 10°, \text{ or } \underline{\phi_t' = 29° 30}$$

and the deflection error due to trunnion tilt is

$$D_t = 28,000 \times \tan 30° \times \sin 10° = \underline{2807 \text{ yards}}$$

The reduction in the angle of elevation is 30′, which, for the battery of a heavy cruiser, corresponds to a reduction in range of about 200 yards at 28,000 yards. Hence it appears that the effect of trunnion tilt on the range is relatively small, even in this extreme case. The significant feature is that trunnion tilt, in this case, can cause deflection errors of any amount up to about 2800 yards (or about 100 mils) either to the right or to the left, depending upon the stage of the roll at the instant of firing.

The following example shows that trunnion tilt can cause very serious errors in deflection even under ordinary circumstances. Let us suppose that a 16″2600 f.s. battery is being fired at a range of 25,000 yards, for which the angle of elevation is practically 17°, and under conditions involving angles of trunnion tilt as great as 2° to either side. From (1308) and (1310) we have

$$\sin \phi_t' = \sin 17° \times \cos 2°, \text{ or } \phi_t' = \underline{16° 59.'4}$$

$$D_t = 25,000 \times \tan 17° \times \sin 2° = \underline{267 \text{ yards}}$$

The maximum reduction in the angle of elevation, due to trunnion tilt, is therefore 0.′6, which corresponds to about 9 yards of range. The deflection error, however, can be as great as 267 yards (or about 11 mils) either to the right or to the left.

1314. Mechanical compensation for errors due to trunnion tilt is provided for in all director systems of recent design. However, there are still in service many director systems of earlier design which do not include this feature. In the absence of mechanical compensation for trunnion tilt errors, firing should be done only when the trunnions are practically horizontal. This applies, of course, to pointer fire, as well as to director fire with systems which are not equipped to

compensate mechanically for trunnion tilt. Although such procedure may entail a material sacrifice in the rate of fire, it should be obvious from the study made above that the alternative procedure of firing without regard for the existing condition of trunnion tilt may result in an extremely erratic deflection performance.*

COLD-GUN ERRORS

1315. It is well known, from practical experience, that a shot from a cold gun usually falls short with respect to succeeding shots which are fired from the same gun under otherwise identical conditions. The abnormal errors which occur on initial shots are naturally associated with the fact that the gun is cold for such shots, and they are accordingly called *cold-gun errors.* It is not a foregone conclusion, however, that the heating of the gun by the initial shot is the only effect which operates to change the range of succeeding shots, for if the heating of the gun were alone responsible for such changes one might reasonably expect the latter to occur with fair regularity at the firings of similar guns, or certainly at different firings of the same gun. That is to say, one might reasonably expect the cold-gun errors of all guns of the same caliber and initial velocity to be approximately the same, and the cold-gun errors of a given gun to remain approximately the same throughout the life of that gun. On the contrary, the performances of different guns of the same caliber and initial velocity, or of the same gun at different stages of its life, are actually found to differ materially in this respect. Although most guns settle down to a steady performance after the first shot, some do not settle down until several shots have been fired. In the case of the 4″2900 f.s. gun, for example, increases of range from shot to shot have been found to persist for as many as six shots. Also, there is strong evidence that some guns actually suffer *reductions* of range after the first shot.

General nature of cold-gun errors

1316. It is probable that the change in the gun's performance after an initial round results from a combination of some or all of the following causes.

(a) A portion of the energy of the charge is expended in heating the gun. This loss of energy is greatest for the first round, and probably diminishes rapidly thereafter. This cause, by itself, serves to explain the increases of range that often occur after the first round, but not the great variation in the amount of such increases for different guns of the same caliber and initial velocity, or for firings of the same gun at different stages of its life.

(b) For the initial round the walls of the gun are at a fairly uniform temperature throughout their thickness; or the outside of the gun, if exposed to the rays of the sun, may be warmer than the bore. After the initial round, the bore is considerably warmer than the outside of the gun. It is not unlikely, therefore, that the structural parts of the gun readjust themselves after the bore becomes heated, and thus cause a change in the

Causes of cold-gun errors

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* For any values of angle of elevation and trunnion tilt that are likely to occur in practice, *the deflection error in mils due to trunnion tilt is approximately equal to one-third of the product of the angle of elevation and angle of trunnion tilt, each of the latter being expressed in degrees*; that is, t_D (in mils) $= \frac{1}{3}\phi' \times t$ (in degrees). Applying this rule to the examples given in article 1313, we find for the first example a deflection error of about $\frac{1}{3} \times 30 \times 10 = 100$ mils, and for the second example a deflection error of about $\frac{1}{3} \times 17 \times 2 = 11$ mils.

droop. This cause can operate either to increase or to decrease the range, according to the manner in which the droop is affected.

(c) The bore of a gun probably remains fairly well lubricated for the initial round, even though it has been washed out carefully prior to firing. The initial round not only removes all traces of lubrication from the bore but also fouls the latter with copper. The bore therefore offers less than the normal resistance to the passage of the projectile on the initial round, and the pressure, muzzle velocity, and range for this round are consequently less than normal (the effect being akin to that of erosion).

(d) The temperature of the powder charge may be increased by exposure to the heat of the chamber, thus causing an increase of range after the initial round. Experiments have indicated, however, that the temperature of the powder charge is increased relatively little by exposure to the heat of the chamber for such periods of exposure as are likely to occur in practice.

1317. Fairly consistent shifts in deflection after the initial round, have been observed in the cases of some guns, and it is probable that such shifts occur for all guns, though they may often be small enough to escape notice.

Cold-gun errors in deflection

It would naturally be assumed that *increases* in range after the initial round would be accompanied by *increases* in drift, but observed results indicate that the drift may actually decrease after the initial round even though the range increases appreciably. In the case of the 5″3150 f.s. gun, for example, an *increase* in range and a shift to the *left* in deflection usually occur after the initial round, and there is evidence to indicate that the amount of the shift to the left in deflection varies approximately in proportion to the amount of the increase in range. Shifts to the left in deflection with increases in range, after the initial round, have also been noted in the case of the 16″2600 f.s. gun. It is to be presumed that a shift to the left after the initial round indicates that the latter suffered excessive drift. The combination of decreased range and increased drift which the initial round suffers in these cases can be accounted for satisfactorily only by irregular projection, as a result of which the drift is increased by a greater amplitude of precession and the range is decreased by a greater average obliquity of the projectile.

1318. Cold-gun errors are usually noticeable only at comparatively long ranges. When reliable information as to the amounts of these errors is available (and this can be gained only from careful analysis of previous firings), *cold-gun corrections* may be included in the ballistic corrections for the first salvo and removed after this salvo has been fired. In some cases the spotter is charged with the responsibility of making an allowance in his spot on the first salvo for the shift that is expected to occur after this salvo. However, the better practice is to handle the cold-gun correction altogether at the range-keeping station and to let the spotter spot for full due on the first salvo.

Application of cold-gun corrections

Typical examples of cold-gun errors

The cold-gun corrections for the 16″2600 f.s. gun, at battle ranges, have been found to vary from about *up* 100 yards in range and *left* 1 mil in deflection when the gun is new, to a maximum of about *up* 700 yards in range and *left* 4 mils in deflection when the gun has fired about 100 rounds. In the case of the 5″3150 f.s. gun at battle ranges, the cold-gun corrections have been found to vary from about *up* 150 yards in range and *left* 1 mil in deflection when the gun is new, to a maximum of about *up* 600 yards in range and *left* 4 mils in deflection when the gun has fired about 250 rounds.

CHAPTER 14

THE ACCIDENTAL ERRORS OF GUNFIRE. THE APPLICATION OF THE LAWS OF PROBABILITY TO THESE ERRORS. DETERMINATION OF THE PROBABILITY OF HITTING A GIVEN TARGET.

New Symbols Introduced

D. . . . True mean dispersion (article 1402 (f)); for range the subscript r is used and for deflection the subscript d.

D'. . . . Apparent mean dispersion (article 1402 (e)); subscripts may be used as with D.

a. . . . . Distance, or error, of any given point from a reference point, such as the mean point of impact or the center of the hitting space (article 1413); subscripts are used as with D.

d. . . . . Error of the M.P.I.; subscripts are used as with D.

S'. . . . Hitting space of a target (article 1402 (h)).

w. . . . . Width of a target across the line of fire.

P. . . . . Probability of hitting a given target; subscripts are used as with D. (The symbol P, however, may also be used to denote probability in connection with any other problem.)

1401. It is common knowledge that even under the best of conditions there is a certain amount of scattering, or dispersion, among the points of impact of shots from a gun. This dispersion obviously may be due to a multitude of causes. Even when a gun is stationary and rigidly fixed in elevation, variations in range and deflection are caused by differences among individual powder charges and projectiles, by variations from shot to shot in the conditions attending the projection and consequently affecting the flight of the projectile, and by variations from shot to shot in the atmosphere. In actual practice, additional variations are caused by errors in sight-setting and in aiming, and by variations from shot to shot in the motion of the gun due to rolling, pitching, and yawing of the ship. We shall, in this chapter, examine the nature and principal causes of dispersion, and determine how dispersion affects the control and effectiveness of gunfire.

Definitions

1402. The following definitions relate to terms which are used in connection with the analysis of fall of shot; these definitions are standard in the U. S. naval service and are used in connection with target-practice reports.

Salvo

(a) A *salvo* consists of two or more shots fired at the same target either simultaneously or within a short interval of time, such as the duration of a firing signal.

Mean point of impact, or M.P.I.

(b) The *mean point of impact* (abbreviated *M.P.I.*) of a salvo is the point which is at the geometrical center of the points of impact of the several shots of the salvo, *excluding wild shots;* it is analogous to such terms as "center of figure," "center of location," etc., when referred to a group of points.

Pattern

(c) The *pattern in range* of a salvo is the distance, measured parallel to the line of fire, between the point of impact of the shot falling farthest beyond the M.P.I. and the point of impact of the shot

falling farthest short of the M.P.I., *excluding wild shots.* The *pattern in deflection* of a salvo is the distance, measured at right angles to the line of fire, between the point of impact of the shot falling farthest to the right of the M.P.I. and the point of impact of the shot falling farthest to the left of the M.P.I., *excluding wild shots.* The term *pattern* has nothing to do with the arrangement of the several points of impact of a salvo, nor with the area covered by them. There is generally more occasion to deal with the pattern in range than with pattern in deflection, and hence the unqualified term *pattern* is often used when referring to the pattern in range.

Dispersion

(d) The dispersion* of a shot is the distance between its point of impact and the M.P.I. of the salvo. The *dispersion in range* is the component of dispersion measured parallel to the line of fire, and the *dispersion in deflection* the component measured at right angles to the line of fire. The dispersion in range of a shot is called positive when the point of impact of the shot is beyond the M.P.I., and negative when the point of impact of the shot is short of the M.P.I. The dispersion in deflection of a shot is called positive when the point of impact of the shot is to the right of the M.P.I., and negative when the point of impact of the shot is to the left of the M.P.I. It is to be noted that the arithmetical sum of the greatest positive and negative dispersions in range is equal to the pattern in range, and that the arithmetical sum of the greatest positive and negative dispersions in deflection is equal to the pattern in deflection; also that the algebraic sum of the dispersions in range of the several shots of a salvo must equal zero, and likewise for the dispersions in deflection.

Apparent mean dispersion

(e) The *apparent mean dispersion in range* (*or deflection*) of a salvo is the arithmetical mean of the dispersions in range (or deflection) of the several shots of that salvo, *excluding wild shots.*

(f) The apparent mean dispersion is not a true measure of the accuracy of fire, since it is based on a limited number of observations. The true measure of accuracy is the mean dispersion of an infinitely great number of shots, all assumed to have been fired under as nearly as possible similar conditions, and it is called the *true mean dispersion.* Although it is impossible to measure the value of the true mean dispersion experimentally, a theoretical value is given by the relation

True mean dispersion

$$D = D'\sqrt{\frac{n}{n-1}} \qquad (1401)\dagger$$

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* The term *dispersion*, as used here, is similar in meaning to the terms *error* and *deviation* as used by other services in the same connection. This usage of the term *dispersion* is standard in the U. S. naval service.

† It is shown in the laws of errors that if a mean error is reckoned from a finite number of observations, the true value (γ) of the mean error bears to the value (γ') of the mean error as reckoned from n observations, the relation

$$\gamma = \gamma'\sqrt{\frac{n}{n-1}}.$$

Proof of this proposition can be found in most treatises on the *Method of Least Squares,*

in which D is the true mean dispersion, D' the apparent mean dispersion, and n the number of shots from which D' has been measured. The following table gives values of the ratio of D to D', i.e., values of the term $\sqrt{\frac{n}{n-1}}$, for salvos of from two to twelve shots. The true mean dispersion is therefore defined as follows.

Ratios for converting apparent mean dispersion to true mean dispersion

No. of shots in salvo	Ratio of true mean dispersion to apparent mean dispersion
12	1.044
11	1.049
10	1.054
9	1.061
8	1.069
7	1.080
6	1.095
5	1.118
4	1.155
3	1.225
2	1.414

The *true mean dispersion in range* (*or deflection*) of a salvo is equal to the apparent mean dispersion in range (or deflection) of that salvo multiplied by a factor which depends on the number of shots in that salvo (excluding wild shots) and which is given in the above table.

Wild shot

(g) A *wild shot* is a shot whose dispersion is abnormal in comparison with the mean dispersion of the salvo in which that shot was fired. A shot may be *wild* either in range or in deflection.

The determination of how great a shot's individual dispersion may be in comparison with the mean dispersion of the salvo, before the shot may be classed as *wild*, is one of the most difficult problems in salvo analysis. Since wild shots are rejected in determining M.P.I.s, mean dispersion, and pattern sizes, it is evident that great care and good judgment must be exercised in determining whether a shot is truly *wild*, or whether the probable frequency of occurrence of a dispersion of the magnitude in question warrants the inclusion of the shot in the analysis. Otherwise the values derived may lead to very erroneous conclusions as to the performance analyzed.

There is considerable divergence among rules stated by various authorities for the rejection of wild shots. The general considerations upon which such rules are based will be treated later (art. 1503). But it may be noted here that in the U. S. naval service the rules to be used for determining wild shots for the purpose of target-practice analysis, are given in *Orders for Gunnery Exercises* and in *Gunnery Instructions*. They will not be stated here as they may vary from time to time.

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or in the following works: art. 20, *Combination of Observations*, Brunt; art. 104, *The Calculus of Observations*, Whitaker and Robinson; art. 61, *Handbook of Ballistics*, Cranz and Becker.

Hitting space

(h) The *hitting space* (S') is the distance, measured parallel to the line of fire, between the point of fall at the surface of the water of a shot that just pierces the top of the target and of one that just pierces the bottom of the target (i.e., the waterline on the engaged side of the target). The hitting space includes the projection of the target's vertical height upon the plane of the water and the target's horizontal dimension in the line of fire (we shall hereafter refer to the latter as the target's *depth* in the line of fire). Sometimes there is also included in the hitting space a distance in front of the target within which impacts are likely to produce either underwater hits or ricochet hits on the target. It is convenient to think of the hitting space as the pattern in range of the shortest and longest shots that can hit the target.

That portion of the hitting space which is due to the height of the target is approximately equal to $h \cot \omega$, where h is the height of the target and ω is the angle of fall. It can be found with sufficient approximation for practical purposes from either Column 7 or Column 19 of the range table, except for very short ranges (see arts. 916 and 1222).*

Error of M.P.I.

(i) The *error of the M.P.I.* is the distance of the M.P.I. from the center of the hitting space, measured parallel to the line of fire for range and at right angles to the line of fire for deflection.

It is to be noted that the *error of the M.P.I.* of a salvo defines the error in location of the entire salvo with respect to the target, while the *mean dispersion* of the salvo relates to the errors of the several shots of the salvo with respect to their own M.P.I. and is entirely independent of the location of the salvo with respect to the target.

1403. Figure 41 is a plan view of a target TT resting on the surface of the water. The line of fire is normal to TT. The area outlined with shading represents the area behind the target within which shots piercing the target must fall. The dimension of this area in the line of fire is equal to the target's hitting space, and the dimension perpendicular to the line of fire is equal to the target's width; the center of the area is at C. The points of impact of a 10-shot salvo are represented by dots. The figure clearly illustrates how the pattern in range and pattern in deflection are measured. Assuming that the figure is plotted to a scale of 10 yards by 10 yards for each small square, we find that the pattern in range is 320 yards and the pattern in deflection 130 yards. (The target dimensions and values of pattern size and dispersion assumed for this illustration are not typical but have been selected for convenience in illustrating.)

The location of the M.P.I. is determined as follows. We shall refer each shot to the center of the target's waterline, denoted by P in the figure, and tabulate the amount it is over (+) or short (−), and right (+) or left (−), with respect to this point. Although any other reference point can be used for this purpose, the center of the target's waterline is usually the most convenient reference point.

We thus find that the M.P.I. is located 80 yards *over* and 40 yards *right* with respect to the center of the target's waterline, and it has been so plotted in Figure 41. The work is shown beneath Figure 41.

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* The determination of the hitting space for very short ranges is treated as a special problem in Appendix B.

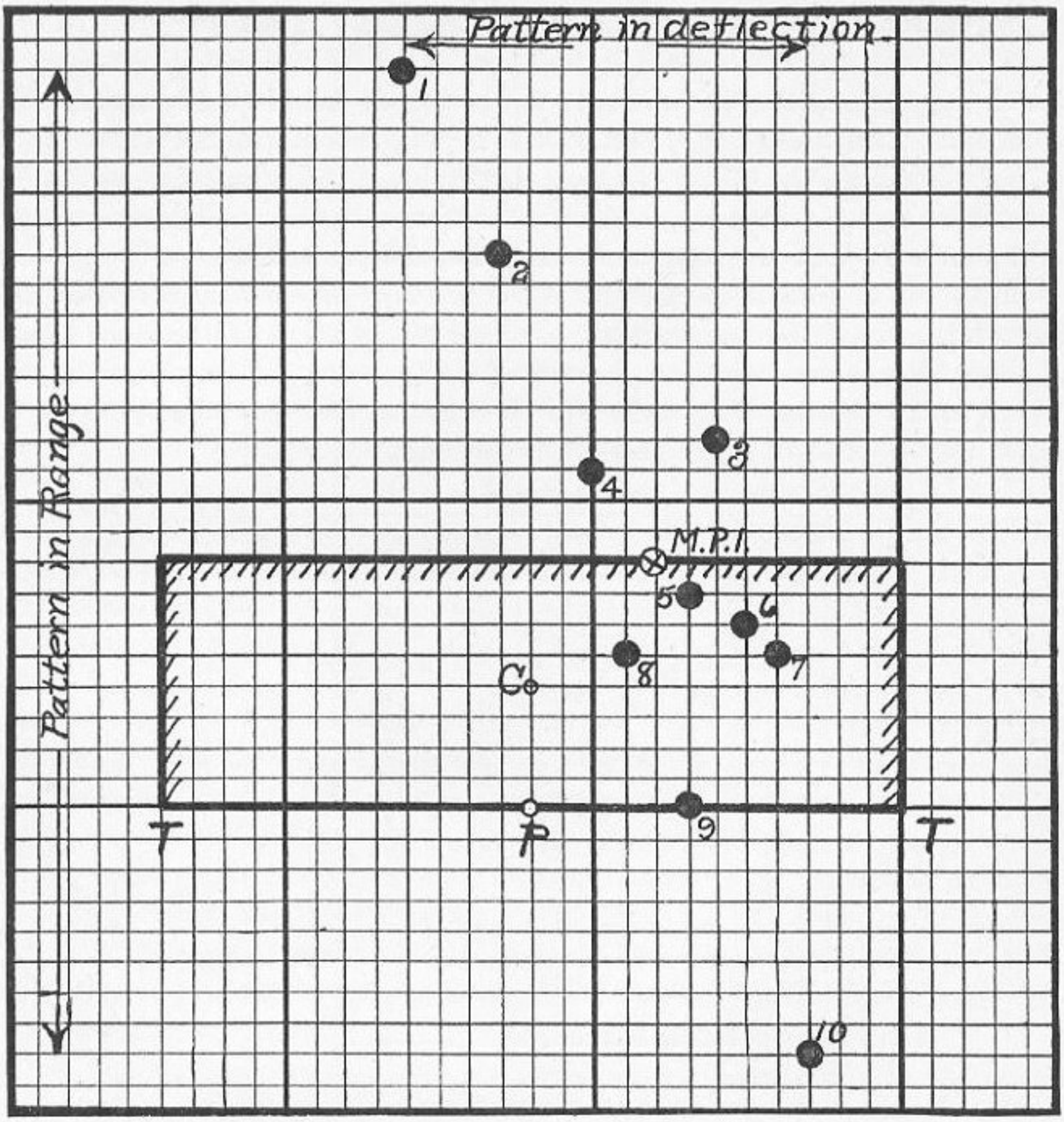


FIGURE 41

Determination of M.P.I., pattern, and error of M.P.I.

Shot No.	Distance from reference point			
	Range		Deflection	
	(+)	(−)	(+)	(−)
1	240			40
2	180			10
3	120		60	
4	110		20	
5	70		50	
6	60		70	
7	50		80	
8	50		30	
9		0	50	
10		80	90	
Sum.....	880	80	450	50
	80		50	
Difference...	800		400	
Mean.....	80		40	

The center of the hitting space, *C*, is located 40 yards *over* with respect to the center of the target's waterline, and the M.P.I. is therefore located 40 yards *over* and 40 yards *right* with respect to the center of the hitting space. According to definition, then, the *error of the M.P.I.* is 40 yards *over* and 40 yards *right.*

In determining the apparent mean dispersion it must be borne in mind that the apparent mean dispersion is the *arithmetical* mean of the dispersions of the several shots. To find the apparent mean dispersion in range we therefore take the sum of the distances the several shots are *over* or *short* with respect to the M.P.I. and divide it by the number of shots in the salvo; to find the apparent mean dispersion in deflection we proceed similarly, using the distances the several shots are to the right or left of the M.P.I. We thus find that the apparent mean dispersion is 66 yards in range and 32 yards in deflection.

Determination of apparent and true mean dispersion

Shot No.	Dispersion	
	In Range	In Deflection
1	160	80
2	100	50
3	40	20
4	30	20
5	10	10
6	20	30
7	30	40
8	30	10
9	80	10
10	160	50
Sum.....	660	320
Mean.....	66 yds.	32 yds.

The true mean dispersion is now found by multiplying the apparent mean dispersion by 1.054, which is the conversion factor for a salvo of 10 shots (as given in the table in article 1402 (f)). Hence the true mean dispersion is $66 \times 1.054 = 70$ yards in range, and $32 \times 1.054 = 34$ yards in deflection.

An erroneous process, often used to shorten the determination of apparent mean dispersion, is to *decrease* the sum of the several distances "over" the reference point by the product of the number of "overs" and the distance the M. P. I. itself is "over" this same point, and to *increase* the sum of the several distances "short" by the product of the number of "shorts" and the distance the M. P. I. is "over" the reference point, and then to add these two quantities and divide by the number of shots, etc. Referring to the example illustrated above, it would appear that this last-described process would be the equivalent of individually decreasing each "over" and increasing each "short" and adding all the distances so found and dividing by the number of shots, which is, indeed, what has been done in finding the apparent mean dispersion above. But applying this supposedly equivalent process to the above example, we get for the apparent mean dispersion in range only 48 yards and in deflection only 26 yards, amounts so far in error as to be utterly worthless. The reason for the discrepancy lies in the fact that when handling the sums in this manner we are not getting the *arithmetical* sum in all cases. For instance, it will be noted that the dispersions in range of shots 5, 6, 7, and 8 of the example are 10, 20, 30, and 30 yards, respectively. If we reduce the sum of the eight "overs" of the example by 8×80, we are, in fact, treating shots 5, 6, 7, and 8 as if their

dispersions were (−)10, (−)20, (−)30, and (−)30, respectively. But the dispersion of shot 5 is 70∼80=10, and not (−)10, and similarly for shots 6, 7, and 8 the dispersions should, as always, have the plus sign, while by using the sum of all the errors we actually would be using these dispersions with the minus sign. The shortened process will fail to give the correct result whenever the salvo contains impacts which are located between the M. P. I. and the reference point from which the individual shots are measured, and as this is almost always the case, the correct method illustrated in this article should be employed habitually.

M.P.I. in the vertical plane

1404. The M.P.I. may also be referred to the vertical plane. The M.P.I. of hits in vertical portions of a target (as for example in a target-screen) can be found readily by the process that has already been illustrated for finding the M.P.I. in the horizontal plane. But if some shots fail to hit in vertical portions of a target, it is then necessary to project their impacts from the horizontal plane to the vertical plane, which can be done by means of Column 19 of the range table. In any event, the M.P.I. in the horizontal plane having been found, it is a simple matter to project it to the vertical plane by means of Column 19. It will be observed that in the example in Figure 41 the M.P.I. in the vertical plane evidently is at the top edge of the target.

Mean dispersion in the vertical plane

By projecting the several impacts of a salvo to the vertical plane we may, by the process illustrated in article 1403, determine the mean dispersion in the vertical plane. However, the mean dispersion in range in the vertical plane, which is called the *mean vertical dispersion*, can also be found readily from the mean dispersion in range in the horizontal plane, by means of Column 19. The mean dispersion in deflection in the vertical plane, which is called the *mean lateral dispersion*, is practically the same as the mean dispersion in deflection in the horizontal plane.

Causes of dispersion

1405. Even at the proving ground, where special facilities are available for ensuring uniformity of powder charges and projectiles and of their loading, for laying the gun accurately in elevation, and for securing uniformity in many other factors upon which the trajectory depends, unaccountable variations in range and deflection occur. In order to understand this it must be accepted, first of all, that it is never possible to measure with absolute certainty, and hence never possible to control completely, all of the factors which influence the flight of a projectile. For example, although great care is exercised to ensure uniformity as to the weight and temperature of the powder charge, the initial velocity nevertheless may vary by small amounts from shot to shot due to unmeasurable and uncontrollable variations in the powder charge. Similarly, although great care may be exercised in laying a gun at exactly the same elevation for a series of shots, the angle of departure nevertheless may vary from shot to shot due to variations in droop and jump. It is likely that irregularities in projection are among the principal sources of unaccountable errors. Such irregularities may result not only from differences in the manner in which the bore of the gun reacts to the pressures, temperatures and shock of firing, but also from differences in the smoothness with which the projectile itself passes through the bore and hence in the position of its axis at the instant of projection, and from differences in the action of the muzzle blast on the projectile. Differences in the action of the rotating band as the projectile passes through the bore, and in the condition of the rotating band during the flight of the projectile, may have a very marked effect on the dispersion. If the band does not engrave properly the pro-

jectile receives insufficient rotation and its flight may be erratic. Erratic flight may result from deformation of the rotating band (such as fringing), or from shearing of the band.

1406. In the service firing of guns dispersion results also from other causes in addition to those which have been outlined above. Differences occur in boresighting and in the lining up of director systems, due both to personal errors and to limitations of material. Powder charges may suffer alteration to varying degrees due to differences in the conditions of storage in different magazines. The temperatures of individual powder charges may differ among one another due to differences in the temperatures of the various magazines in which the charges are stored; sometimes charges are stored in gun compartments or in ready-service lockers on deck for fairly long periods prior to firing, and very material differences in powder temperature may be occasioned thereby. Variations in shell seating cause variations in the density of loading and have a marked effect on dispersion. Then there are differences among the several guns of a battery,—differences in erosion for which only approximate compensation can be made, differences in droop, and differences in mounting (for example, differences in trunnion tilt among the several guns of a battery due to differences among the inclinations of their respective roller-paths).

Additional causes of dispersion in gunfire from ships

Further causes of dispersion are occasioned by the motions of the firing ship and target. Unsteadiness of the gun platform causes dispersion among the shots of a salvo due not only to variable pointing errors, but also to variations in the delay between the closing of the firing key and firing of the gun (such variations may be occasioned by differences in the firing circuits, primers, or ignition pads, or by differences in the location of the powder bag with respect to the primer vent, or even by differences in the condition of the primer vent). Relative motion between the ship and target gives rise to variable control errors, such as errors in range-finding, errors in estimation of target course and speed, errors in sight-setting, etc.

1407. The errors which have been discussed above, as well as many others which contribute to dispersion in gunfire, come under the general classification of *accidental errors*, a classification which applies not merely to errors of gunfire but to occurrences in general. Accidental errors are just as likely to be positive as negative, and they are more likely to be small than large. They follow the laws of probability and their behavior, in the long run can be predicted under the terms of these laws. Knowledge as to the behavior of the accidental errors of gunfire is of importance not only for estimating the effectiveness of gunfire under various conditions, but also for establishing sound doctrines for spotting.

Accidental errors

The laws of probability deal with the prediction of future occurrences on the basis of information gained from past occurrences. Such occurrences may be of various kinds. Thus the laws of probability provide a basis for predictions as to the probable remaining duration of life of a person of a given age, the predictions being based on statistics taken in the past as to the frequency of occurrence of deaths at various ages beyond the given age. Such predictions are the basis of mortality tables and hence the foundation of all life-insurance schemes. Probability laws have an important part in the science of statistics and may, in this connection, apply to almost any kind of event, or occurrence. In gunnery, the occurrences to which the laws of probability are applied are the accidental errors

of gunfire, and the predictions deal with the probable sizes and distribution of such errors.

The Normal Probability Law

Probability laws vary considerably according to the nature of the occurrences to which they pertain.* The law which pertains to symmetrically distributed accidental errors is known as the *normal probability law.* The accidental errors of gunfire generally follow a fairly symmetrical distribution, and the normal probability law is applied to them.

1408. The normal probability law is based upon the following considerations:

1. That positive and negative errors of the same size are equally probable and hence will occur with equal frequencies.
2. That small errors are more probable than large errors and hence will occur with greater frequency than larger errors.
3. That very large errors will not occur (or, more specifically, that very large errors will probably be due to mistakes and not be classifiable as accidental errors).

These considerations all pertain to the probable frequency of occurrence of errors of certain size and sign. If we translate them into a mathematical formula, or equation, the curve representing that equation will have the following characteristics (abscissas measuring the size of error and ordinates the frequency of their occurrence):

1. Since positive and negative errors of the same size occur with equal frequencies, the curve must be symmetrical to the right and left of the point that denotes zero error (i.e., $x=0$), which is the origin.
2. Since small errors are more frequent than large errors, the maximum ordinate must occur at the point that denotes the least error (i.e., $x=0$), which is the origin.
3. Since large errors occur but rarely, and beyond some finite but undefinable limit do not occur at all, the right- and left-hand branches of the curve must each approach the horizontal axis rapidly and meet it at some undefinable point.

These features are illustrated in the curves shown in Figure 42.

The equation representing the probability law must take into account the actual frequency with which each size of error will occur. The derivation of this equation (i.e., for the *normal* probability law) may be found in any treatise on the *Method of Least Squares;*† it will be given here without further proof.

Normal Probability Equation

$$y = \frac{h}{\sqrt{\pi}} \epsilon^{-h^2x^2} \tag{1402}$$

In equation (1402), x and y represent abscissas and ordinates of the curve, the former being a measure of the size of errors and the latter of the frequency of their occurrence; ϵ is the base of the Naperian system of logarithms (approximately 2.7183), and h is a constant determined from the particular set of observations to which a particular curve is to apply.

Index of precision

1409. The constant h is a measure of the precision of the instrument with which the observed results are obtained; we may call it the *index of precision.* As applied to gunfire, h is the index of precision of

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* A thorough and interesting treatment of a wide variety of applications of the laws of probability is given in *Probability and its Engineering Uses,* by Thornton C. Fry.

† Ref. Chapter IV, *The Theory of Errors and Method of Least Squares,* W. W. Johnson; also, Chapter II, *The Combination of Observations,* Brunt. A very complete treatment, with particular reference to applications in gunnery, may be found in §58–60, *Handbook of Ballistics,* Cranz and Becker.

the gun. An index of precision may be defined in several ways, all of which, however, depend upon analysis of results actually given by the instrument to which the index applies.

In the *Theory of Least Squares* it is demonstrated that the most accurate *mean error* of a series of observed quantities from their *arithmetical mean*, is that which results from taking the square root of the sum of the squares of the several observed errors. This mean is usually called the *quadratic mean error*, and it is almost invariably used in connection with problems dealing with precision of measurements. In these cases the quadratic mean error is the index of precision.

In applications to the errors of gunfire it is found more convenient and sufficiently accurate to employ the arithmetical mean, or average, obtained by the simple process of adding together the various errors, without regard to sign, and dividing the sum by the number of errors.* The latter quantity is properly called the *arithmetical mean error* but, since it is so commonly used in gunnery, it is usually referred to simply as the *mean error* or the *mean dispersion*. It is shown in the derivation of the normal probability equation, that if the measure of precision is taken as the arithmetical mean error of the gun, or D, then the index of precision h is related to D by the equation,†

Arithmetical mean error

$$h = \frac{1}{D\sqrt{\pi}}. \tag{1403}$$

Substituting this value of h in (1402) we have the probability equation in the form in which it is commonly used in gunnery,

Probability equation for gunfire

$$y = \frac{1}{\pi D}\epsilon^{-x^2/\pi D^2} \tag{1404}$$

and in this equation D represents the *true mean dispersion*, as defined in article 1402 (f), and the other symbols have meanings as already noted under equation (1402).

1410. The curve representing the normal probability equation is known as the *normal probability curve*. Its general form is illustrated in Figure 42, and it will be noted that the general characteristics of the curve conform to the considerations upon which the normal probability law is based (article 1408), with the exception that the branches of the curve do not actually meet the horizontal axis. For finite considerations, however, this condition is met, for the curve, while not actually meeting the X-axis except at infinity, approaches this axis very closely within relatively small finite distances from the origin.

Normal Probability Curve

While every normal probability curve must have the general characteristics noted in article 1408 and illustrated in Figure 42, the actual form of a particular curve depends upon the value of the index of precision h used in its equation, or, when dealing with our own problem, the mean dispersion D establishes the actual form of the particular curve. This will be understood when we consider that the index of precision is a measure of the mean error of the observations, and that if the mean error is small the several errors must be closely grouped about their mean; thus with a small

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* Ref. §63, *Handbook of Ballistics*, Cranz and Becker, for statements and examples as to the relative accuracy of these "means" Ref. also articles 17–21, *The Combination of Observations*, Brunt.

† Ref. §60, p. 374, *Handbook of Ballistics*, Cranz and Becker, and article 17, *The Combination of Observations*, Brunt.

mean error the frequency of small errors is increased, the maximum ordinate of the curve becomes great, and its branches descend steeply to the horizontal axis. With a large mean error the several errors are more widely dispersed, the central portion of the curve is lower and flatter, and the branches approach the horizontal axis less steeply.

Curve A of Figure 42 illustrates a normal probability curve based upon an index of relatively *great precision*, while curve B is based on an index of relatively *low precision*. Or the former curve might be said to represent the normal probability curve for a gun of relatively *small mean dispersion* and the other to represent relatively *great mean dispersion*. Actually curve A represents a mean dispersion just

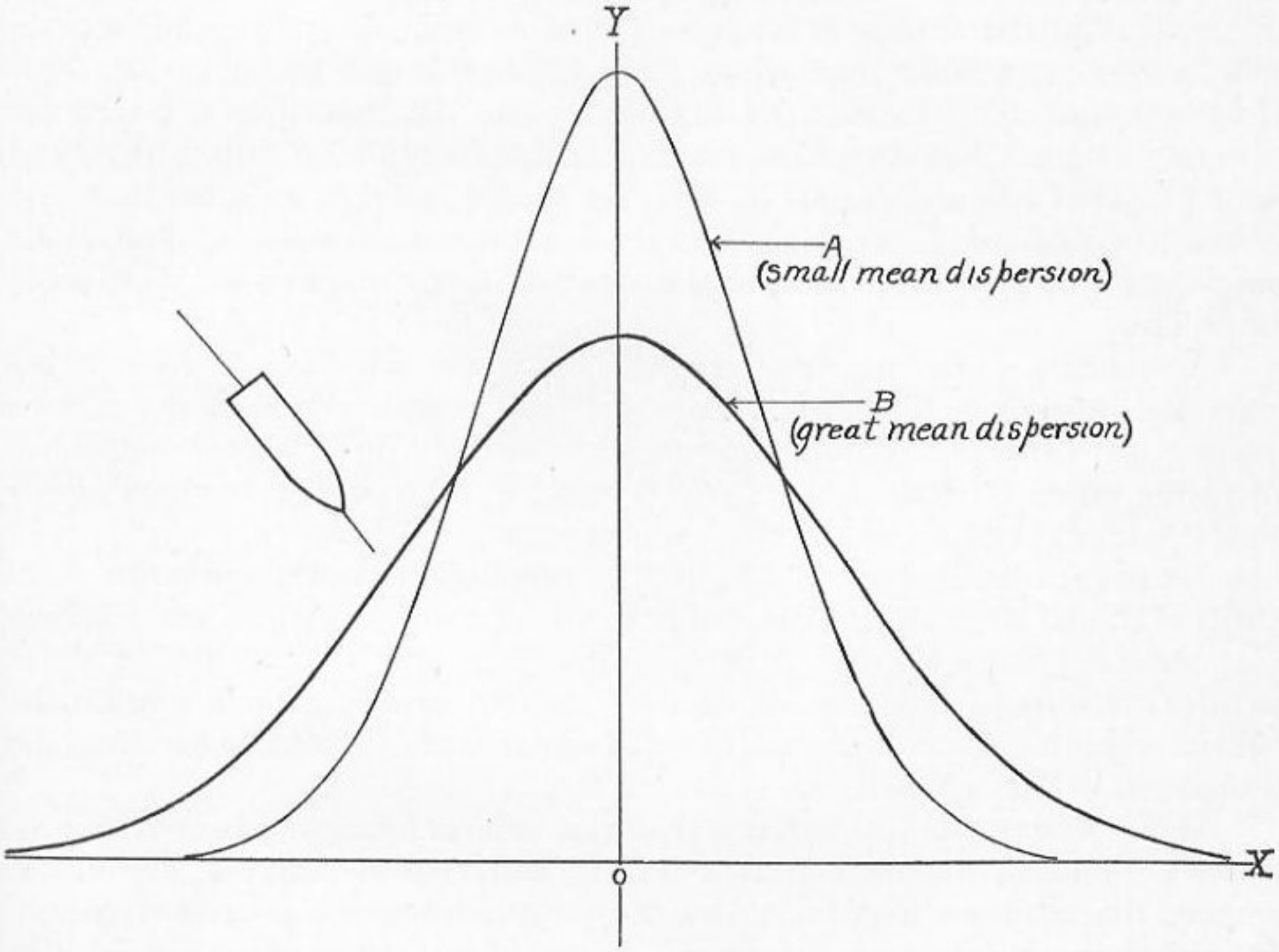


FIGURE 42

two-thirds as great as represented in curve B, both curves being drawn to the same scale.

The general nature of the probability curve may be understood readily if we consider the following illustration. Let us suppose that *O*, in Figure 42, is a point for which we have accurately determined the mean hitting range, and that we fire a great many shots at this point; also let us suppose, for the purpose of this illustration, that each projectile remains exactly where it first strikes. The projectiles would then pile up around the point *O*, which would be their M. P. I. under the conditions we have assumed. If the gun used were a very accurate gun, having a very small mean dispersion, the projectiles would evidently be heaped high around the M. P. I. If the gun were one with large mean dispersion, the pile would be lower and flatter, and the projectiles would straggle out to greater distances from the center of the pile. The contour of a cross section of the pile taken parallel to the line of fire would evidently be the probability curve for that particular gun in range; and the cross section taken normal to the line of fire would represent the probability

curve in deflection. Thus it is seen that the probability curve really gives us a very easily understandable picture of the shot distribution of a gun.

1411. Let us suppose, now, that we desire to estimate our chances of hitting within a given distance of O, in Figure 42, with the same gun with which the experiment described in the previous article was assumed to have been made. Let us deal first with distance in range and assume 10 yards for the amount. If we were to count the number of projectiles that had fallen at ranges not greater than 10 yards beyond O, nor less than 10 yards short of O, and to find the ratio of this number to the total number of projectiles, we would have a numerical measure of our chances of hitting within $\pm$ 10 yards of O in range. If 1000 shots had been fired and 600 were counted in the given strip, we would have 60% of shots with ranges within $\pm$ 10 yards of O. We speak of this percentage as the *probability* of obtaining that result, and the probability of having a range error within $\pm$ 10 yards in this case would be .60. Now if we continued to fire at point O with the same gun, the probability of hitting the given strip being .60, of the next 10 shots 6 would *probably* hit the strip, and so would 60 of the next 100 *probably* hit it, etc. We cannot say that such results would *certainly* be obtained, but on the basis of the best information we have, those results would be the most probable ones and the ones that, in the long run, would probably be fulfilled.

Assuming a similar strip centered at O, but bounded by lines 10 yards to the right and 10 yards to the left of O, and applying the same process as above, let us suppose that a count showed 900 shots to lie within this strip. Then the probability of hitting within 10 yards *right or left* of O would be .90, and what has been said of the ratio in connection with the range applies equally well here.

We have assumed above that 60% of the projectiles fell at ranges within $\pm$ 10 yards of O, and 90% within distances 10 yards right or left of O. Then it follows that of the 60% lying within $\pm$ 10 yards of O in range, only 90% will also lie within $\pm$ 10 yards of O in deflection, whence 90%$\times$60%, or 54%, will lie within the square measuring 20 yards on each side and centered at O. Thus we find that the probability of hitting that square is .54.

1412. The probability equation gives us a formula by means of which we may arrive at probability ratios such as illustrated in the above example. For the example of the previous article it would tell us what percentage of shots would probably fall within any given limits with respect to the M.P.I., or O. In order to give this result for range it must employ as the value of D the true mean dispersion in range, and for deflection the true mean dispersion in deflection.

Since the probability equation expresses the frequency y with which an error of size x will occur, to find the frequency with which all errors from 0 to $\pm a$ will occur we must integrate the equation within the limits $x=+a$ and $x=-a$. Thus the frequency of occurrence of errors within $\pm a$, when the mean dispersion is D, becomes

$$\frac{1}{\pi D}\int_{-a}^{+a} \epsilon^{-x^2/\pi D^2}\, dx\,. \tag{1405}$$

Equation (1404) has been so constructed that its integral between the limits $-\infty$ and $+\infty$ equals unity, and hence (1405) expresses a fraction, or percentage, of unity or, as illustrated in the example above, a probability.* Thus (1405) gives the

* If the integral of the curve between any chosen limits is to express the probability of occurrence of errors within those limits, then by extending the limits to ∞ in both directions we will include all errors, and the probability of any errors falling within these infinite limits becomes certainty, or unity, whence the integral extended to these limits must equal unity.

probability of the occurrence of errors within $+a$ and $-a$ when the true mean dispersion is D.

Referring to Figure 43, this means that the total area under the curve of (1404) is unity, while the area of the strip extending from $+a$ to $-a$ is given by (1405); and that, since the total area equals unity, the area of the strip shown will be found directly as a percentage of the whole. Thus by integrating the probability equation we have merely done by a formal mathematical process what we have done in the example of article 1411; i.e., we have made a *count* of the number of errors within $\pm a$ and found the ratio of this number to the whole.

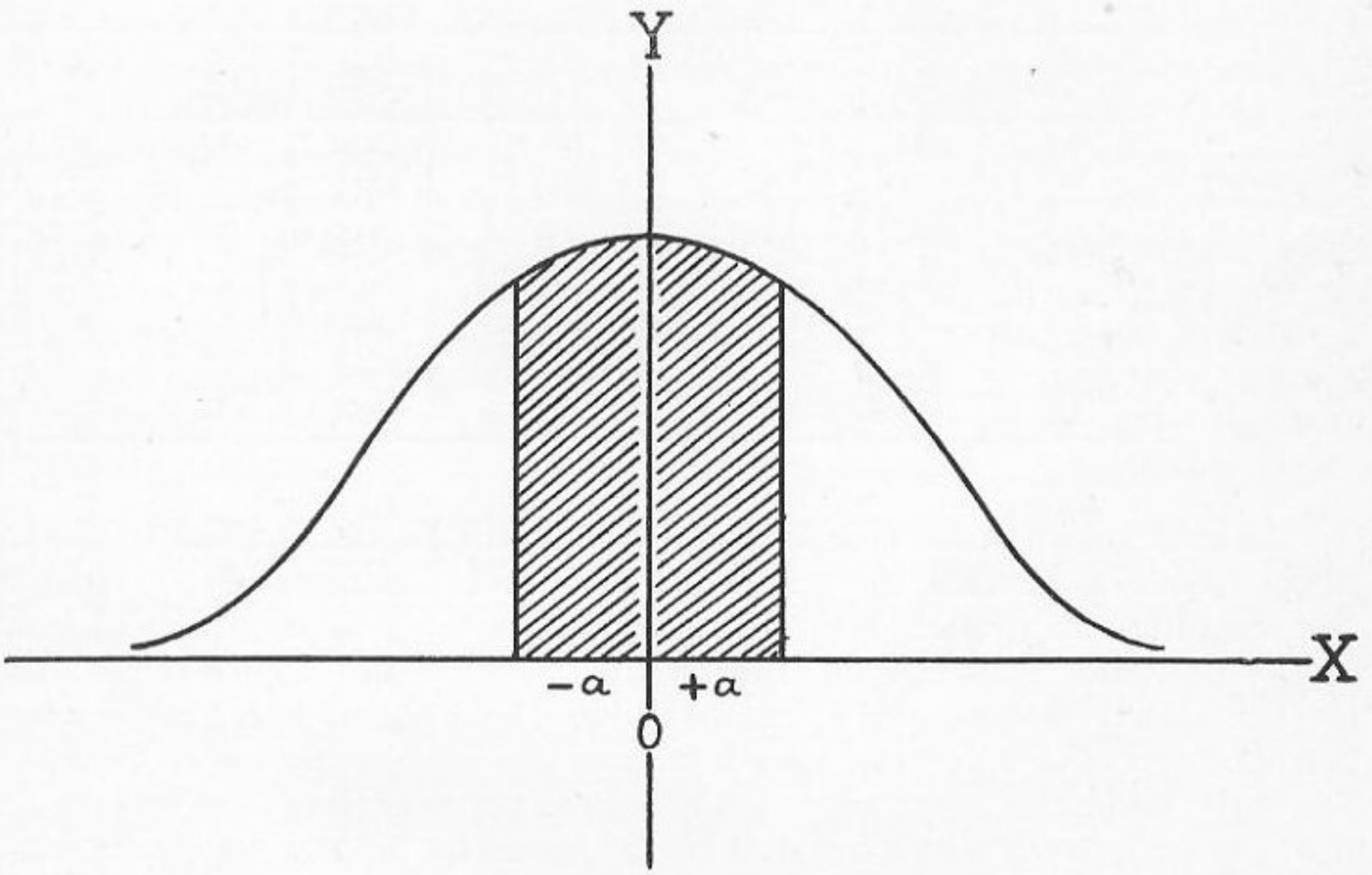


FIGURE 43

1413. In order to eliminate the necessity of integrating (1405) for each individual problem, it is convenient to tabulate the values of the integral for various *ratios* of x to D. If we were to tabulate the values of the integral for various values of x by itself, we would evidently have to provide a different table for each value of D. But if we evaluate the integral for various *ratios* of x to D, we need make but a single table, the entering argument for which will then be the *ratio* x/D. Thus we can tabulate the value of the integral for the value of x which is equal to $1/2D$, or $x/D=1/2$; this value of the integral will apply equally well to a case in which the particular error (x) dealt with is ± 100 yards and the true mean dispersion (D) 200 yards, and to a case in which the particular error is ± 10 yards and the mean dispersion 20 yards, and to any case in which the particular error is half as great as the mean dispersion. And the same applies to any other ratio of x to D for which we evaluate the integral.

When dealing with particular values of the errors, we represent them by the symbol a, and the ratios referred to above then are expressed as a/D. The table of values of the normal probability integral in terms of the ratio a/D is given below; we shall refer to it as the *Probability Table. The entering argument for the probability table is the ratio* a/D, *or the ratio of the particular error* ($\pm a$) *with which we are dealing, to the index of precision of the instrument with which we are dealing, which, in the case of gunfire, is the true mean dispersion* (D). *The quantity* (P) *found from the table is the probability that an error not greater than* $\pm a$ *will occur.*

PROBABILITY OF AN ERROR NOT GREATER THAN $\pm a$
IN TERMS OF THE RATIO $\frac{a}{D}$

Probability Table

$\frac{a}{D}$	P	$\frac{a}{D}$	P	$\frac{a}{D}$	P	$\frac{a}{D}$	P
0.1	.064	1.1	.620	2.1	.906	3.1	.987
0.2	.127	1.2	.662	2.2	.921	3.2	.990
0.3	.189	1.3	.700	2.3	.934	3.3	.992
0.4	.250	1.4	.735	2.4	.945	3.4	.994
0.5	.310	1.5	.768	2.5	.954	3.5	.995
0.6	.368	1.6	.798	2.6	.962	3.6	.996
0.7	.424	1.7	.825	2.7	.969	3.7	.997
0.8	.477	1.8	.849	2.8	.974	3.8	.998
0.9	.527	1.9	.870	2.9	.979	3.9	.998
1.0	.575	2.0	.889	3.0	.983	4.0	.999

An examination of the probability table will show that for the ratio $a/D=4$, the probability is $P=.999$; this means that there is but one chance in a thousand that any future result will be in error by an amount that is greater than about four times the arithmetical mean error of past results. Applying this to our own problem it means that the dispersion of any individual shot from a gun has but one chance in a thousand of being greater than four times the mean dispersion of the gun. Evidently this feature has an important bearing on the determination of wild shots. If we assume that any shot which does not fall within the limits which should contain 99% of all the shots of a series or salvo, is a wild shot, then, according to the table, any shot whose dispersion is greater than 3.2 times the true mean dispersion is a wild shot (for when $P=.99$, $a/D=3.2$). If we extend the limits to include 99.9% of the shots, a dispersion greater than 4 times the true mean dispersion defines a wild shot. The difficulty lies in setting a logical limit when dealing with small numbers of shots, and this matter will be discussed further later on (article 1503). But the point to be noted here is that whatever limits are chosen, the ratio of greatest individual dispersion to true mean dispersion that will conform to these limits may be determined readily from the probability table.

1414. If we enter the probability table with the value $P=.500$ we will find that the corresponding value of a/D is .846. This means that if $\pm a=.846D$, the probability of occurrence of an error within $\pm a$ is just 1/2; i.e., it is just as likely that an error *smaller* than $\pm a$ will occur as that an error *larger* than $\pm a$ will occur. For this reason the error of size $\pm .846D$ is called the *probable error* in many services and this term is found in many textbooks and reference books. The term "probable error" is arbitrarily used to denote what might be called the "50-50 error."

Probable error

If an error of $\pm a=.846D$ will not be exceeded just half the time, then the total width of the band that will be hit just half the time is $2\times.846D$, or $1.692D$. This

The 50% zone

band is called the *50% zone*. In terms of hitting space and mean dispersion, this merely means that in order to receive 50% of hits the hitting space of the target must equal 1.692 times the mean dispersion of the gun. In some services gun errors are commonly spoken of in terms of their 50% zones, and in most others in terms of the "probable error," tables being provided which may be entered directly with the "probable error." In our service the term "probable error" is not used, but the term "50% zone" is used in target practice reports. The latter may be found from the relation noted above, viz.,

$$50\%\ \text{zone} = 1.692D. \tag{1406}$$

1415. An important consideration in any use of the laws of probability is that the value of any prediction based upon them depends largely upon the number of occurrences to which the prediction is applied. For example, let us assume a mean dispersion of 50 yards and find what the probability is that with this mean dispersion shots will fall within ± 100 yards of the M. P. I. The ratio a/D equals 100/50, or 2.0, in this case, whence we find from the table that $P = .889$. Thus the answer is that the probability of obtaining this result is practically .90, or that about 90% of the shots should fall within ± 100 yards of the M. P. I. If we apply this percentage to 3-gun salvos, we get 2.7 hits per salvo within the chosen limits as the probable number of hits per salvo. We can never have 2.7 hits on a salvo, hence with 3-gun salvos we will not get that predicted percentage of hits on any one salvo. If we get 2 hits, the percentage for that salvo will be only 67%, varying very considerably from 90%. Obviously even approximate agreement with the predicted percentage will depend upon firing a considerable number of salvos, and ultimately our total percentage of hits will be about 90%. With 10-gun salvos we could reasonably expect the majority of the several salvos to produce 9 hits each (within the given limits referred to the M. P. I.).

Although these features are obvious enough when we pause to consider the fundamental nature of the laws of probability, they are often lost sight of in practice, and failure to give them due weight often leads to the suspicion that the laws of probability are worthless theories. The value of these laws lies in their application "in the long run." Application of doctrines based on them will produce net profits just as surely as the zero and double zero on a roulette wheel will produce a profit for the bank, "in the long run."

1416. We may define probability as the numerical measure of the expectation that a certain occurrence will take place. We have developed methods for determining this numerical measure for various sets of assumed conditions. Thus, in the preceding article, we found that for the assumed set of conditions $a = \pm 100$ and $D = 50$, the corresponding value of P is .889.

It is an axiom of the laws of probability that if an event is independently controlled by more than one set of conditions, the probability of its occurrence under all

Product of separate probabilities

of these sets of conditions is equal to the product of the several probabilities of its occurrence under the several sets of conditions. This may be illustrated in many ways. For instance, if there is one chance in two that a man A will be at the spot X at the instant Y, and one chance in two that a man B will be at the spot X at the instant Y, then the chance that A will meet B at the spot X at the instant Y is evidently one in four. We can express this also as follows: the probability that A will be at X at the instant Y is $P_1 = \frac{1}{2}$, and that B will be there at the same instant is $P_2 = \frac{1}{2}$, hence the probability that both will be there at the same instant is $P_1 \times P_2 = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$. Again, we know that the probability of drawing the king of

spades from a full deck of cards, in one trial, is $P=\frac{1}{52}$, because there is but one king of spades and there are fifty-two cards. The same result may be obtained in various other ways. The probability of drawing *any* card of the spade suit is $P_1=\frac{1}{4}$, since one-fourth of all of the cards are of the spade suit. The probability that a card of the spade suit, if drawn, will be the *king*, is $P_2=\frac{1}{13}$, since there is only one king among the thirteen cards of the spade suit. Hence the probability that the card drawn will be both a spade and a king is $P=P_1\times P_2=\frac{1}{4}\times\frac{1}{13}=\frac{1}{52}$. Or we have the probability of drawing *any* spade, $P_1=\frac{1}{4}$, of drawing any *face* card of the spade suit, $P_2=\frac{3}{13}$, and the probability that any face card will be the *king*, $P_3=\frac{1}{3}$; whence the probability of drawing the king of spades is $P=P_1\times P_2\times P_3=\frac{1}{4}\times\frac{3}{13}\times\frac{1}{3}=\frac{1}{52}$. The example of article 1411 illustrated how this axiom applies to our problem. It simply means that if the probability of hitting in range is P_r and the probability of hitting in deflection is P_d, then the final probability of hitting is $P_r\times P_d$.

Another axiom of these laws is that if P is the probability that an event *will* occur, then $1-P$ is the probability that it *will not* occur. This is easily understood when we think of probability as a fraction of unity, or a percentage. Thus if we find that the probable percentage of hits is $100P$, the probable percentage of misses evidently must be $100-100P$, or $100(1-P)$.

1417. We are now prepared to deal with some practical applications of the theories we have discussed. Let us investigate the applications to the determination of the probability of hitting a target whose dimensions are known, with a gun whose true mean dispersion is known. Under the assumption that the accidental errors of gunfire follow the distribution indicated by the normal probability equation (and illustrated by the normal probability curve), it is evident that the probability of hitting a given target is greatest when the M.P.I. is located at the center of the target's hitting space. Under this condition the densest portion of the salvo, being symmetrically disposed about the M.P.I., will be centered on the hitting space of the target.* Thus when we speak of the maximum percentage of hits that will probably be made by a certain gun against a certain target, we ordinarily mean the probable percentage of hits that will occur when the M.P.I. is located at the center of the hitting space. The latter condition is, however, by no means one that we may assume as typical in actual service.

Probability of hitting, when M.P.I. is at center of hitting space

If the M.P.I. is located at the center of the hitting space, the allowable error in range (over or short) within which hits will occur, will evidently be one-half the hitting space. We will let S' denote the hitting space (the prime being used to distinguish it from the symbol for danger space). Then under the conditions assumed to govern the maximum percentage of hits, we have $\pm a=\frac{S'}{2}$, and with any given value D for the true mean dispersion, the probability of hitting will be that value of P which will be obtained by entering the probability table with $\frac{a}{D}$, and the percentage of hits will be $100P$. A few problems will illustrate these features.†

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* See article 1425.

† For the sake of simplicity the hitting areas are treated, in these problems, as rectangles. Actually they are irregular, although somewhat resembling rectangles with rounded corners. The deck area itself is approximately elliptical, and equal roughly to 80% of the area of the enclosing rectangle (i.e., length times beam). However, the total hitting area, which includes the areas short of and beyond the actual limits of the ship as well as the deck area itself, equals roughly 90% of the enclosing rectangle (i.e., total hitting space times width across line of fire) at long ranges, and a greater proportion at short ranges. It may be con-

1418. In the problems given here, as well as elsewhere in this chapter, the values of mean dispersion and of pattern size are purely fictitious, this being necessary because real values of such quantities are information of confidential nature. The problems should therefore be viewed merely as illustrations of methods, and not as being designed to illustrate typical results.

1. *Given:* Eight-gun salvos are fired from a ship with its main battery of 16″2600 f.s. guns. The range to the target is 20,000 yards; the freeboard of the target is 28 feet, its beam 90 feet, its length 600 feet, and it is lying perpendicular to the line of fire. It is assumed that shots striking not more than 15 yards in front of the target will make effective under-water hits. The true mean dispersion of the guns is 200 yards in range and 30 yards in deflection.

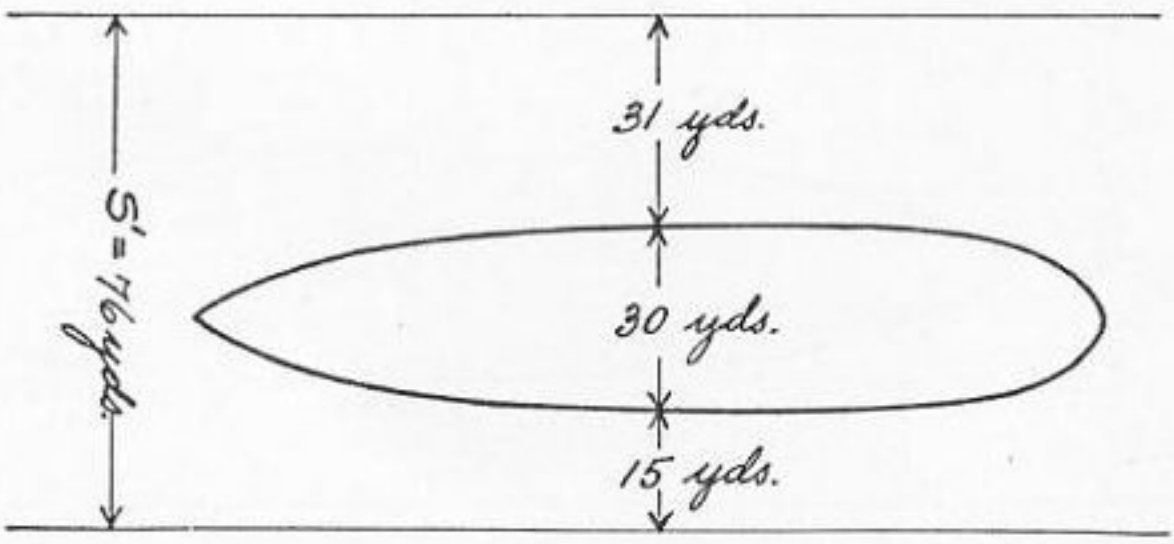
Find: The probable percentage of hits that will be made under the conditions given, and with the M.P.I. at the center of the hitting space.

In accordance with article 1402 (h), the portion of the hitting space due to the height (or freeboard) of the target may be found from Column 7 of the range table, and is,

$$\frac{28}{20} \times 22 = 30.8, \text{ or } 31 \text{ yards.}^*$$

To this must be added the depth of the target in the line of fire (in this case its beam) and the distance in front of the target within which hits will be effective. This gives as the total hitting space,

$$S' = 31 + \frac{90}{3} + 15 = \underline{76 \text{ yards}}$$



With the M.P.I. centered in this hitting space, we will have as the maximum allowable variation in range, $\pm\frac{76}{2} = \pm 38$ yards, whence for range $a_r = \pm 38$ yards; likewise the maximum variation to right and left of the center will be *one-half* the length of target, or $\pm\frac{300}{3}$ yards, whence for deflection $a_d = \pm 100$ yards.

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sidered, therefore, that the probabilities obtained in these problems are of the order of about 10% too great. Considering the general uncertainty of data that enter into problems of this character, it is doubtful whether anything is to be gained by striving for a more exact treatment of the target dimensions. However, methods for dealing with areas of various shapes are outlined in §§ 68–69, *Handbook of Ballistics*, Vol. I, Cranz and Becker.

* It is sufficiently accurate for any practical purpose to take such computations as these to the nearest whole yard or, in the case of the ratio $\frac{a}{D}$, to two decimal places.

For the sake of clarity, we may give subscripts to the symbol for true mean dispersion (as has been done with a above), and thus for range $D_r=200$ and for deflection $D_d=30$. Applying this subscript notation also to the symbol P, we have,

For range, $$\frac{a_r}{D_r}=\frac{38}{200}=.19, \text{ whence } P_r=.121.$$

For deflection, $$\frac{a_d}{D_d}=\frac{100}{30}=3.33, \text{ whence } P_d=.993.$$

Then, in accordance with article 1416, the probability of hitting the target is,

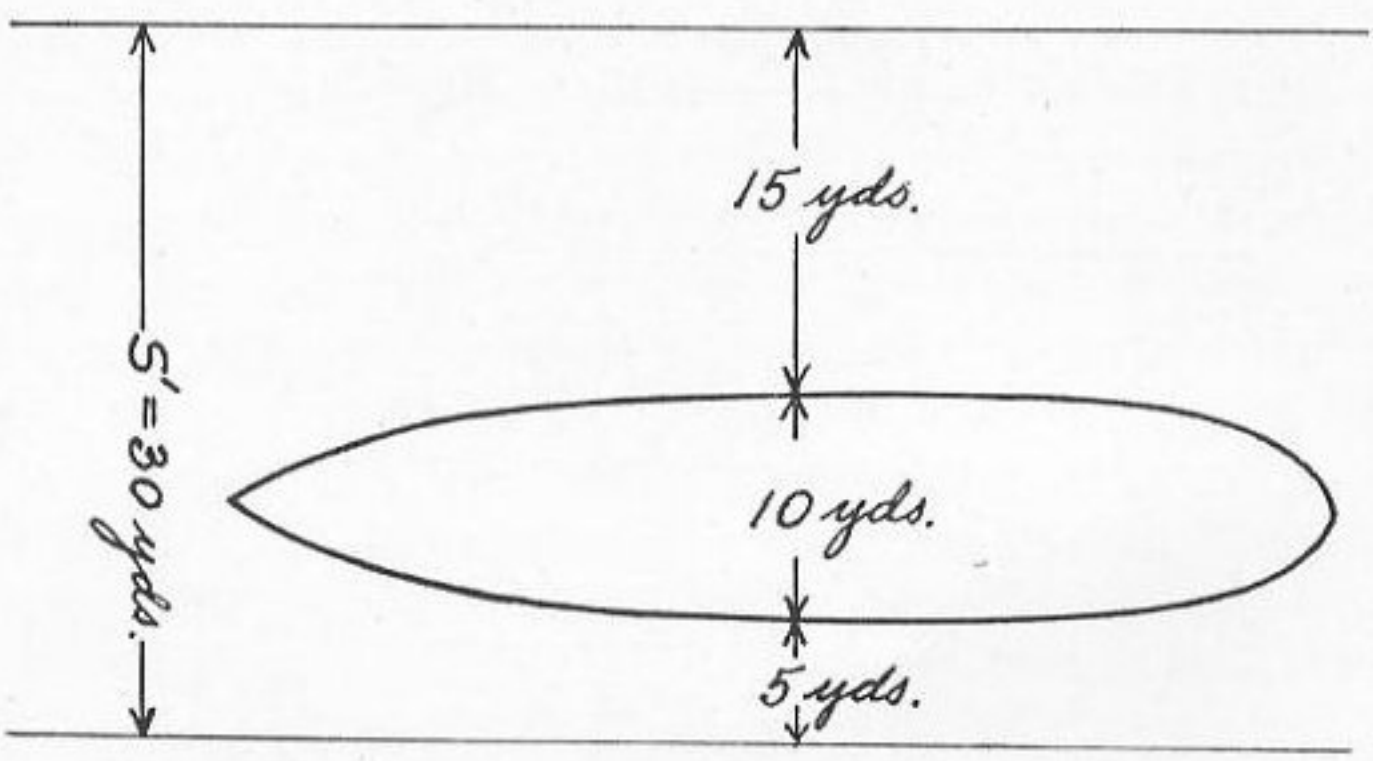
$$P=P_r\times P_d=.121\times .993=\underline{.120}$$

or the probable percentage of hits is 12.0%, giving about one hit per salvo.

2. *Given:* Ten-gun salvos are fired from a ship with its battery of 5″3150 f.s. guns. The range to the target is 12,000 yards; the freeboard of the target is 16 feet, its beam 30 feet, its length 300 feet, and it is lying perpendicular to the line of fire. It is assumed that shots striking not more than 5 yards in front of the target will make effective under-water hits. The true mean dispersion of the guns is 150 yards in range and 25 yards in deflection.

Find: The probable percentage of hits that will be made under the conditions given, and with the M.P.I. at the center of the hitting space.

Proceeding exactly as in the first problem, we have,



$$S'=\left(\frac{16}{20}\times 19\right)+\frac{30}{3}+5=\underline{30 \text{ yards}}$$

whence, $$a_r=\pm\frac{30}{2}=\pm 15 \text{ yards.}$$

Also, $$a_d=\pm\frac{1}{2}\left(\frac{300}{3}\right)=\pm 50 \text{ yards.}$$

$$D_r=150 \text{ yards (as given)}$$
$$D_d=25 \text{ yards (as given)}$$

For range, $$\frac{a_r}{D_r}=\frac{15}{150}=.10, \text{ whence } P_r=.064.$$

For deflection, $$\frac{a_d}{D_d}=\frac{50}{25}=2.00, \text{ whence } P_d=.889.$$

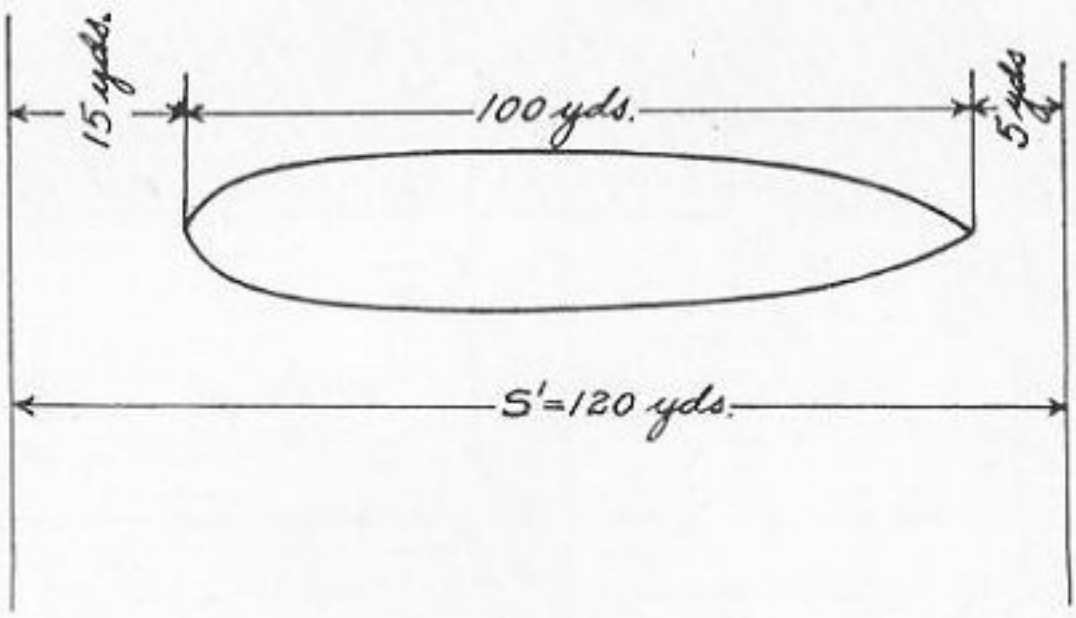
And $$P = P_r \times P_d = .064 \times .889 = .057$$

or the probable percentage of hits is 5.7%, giving about one hit in two salvos.

3. *Given:* The same data as for problem 2 above, except that the target is steaming directly toward the firing ship.

Find: The probable percentage of hits, with the M.P.I. at the center of the hitting space.

Noting that the depth of the target *in* the line of fire is now measured by the target's length, while the target's beam becomes the width *across* the line of fire, we may proceed just as before.



$$S' = \left(\frac{16}{20} \times 19\right) + \frac{300}{3} + 5 = \underline{120 \text{ yards}}$$

whence, $$a_r = \pm \frac{120}{2} = \pm 60 \text{ yards}$$

Also, $$a_d = \pm \frac{1}{2}\left(\frac{30}{3}\right) = \pm 5 \text{ yards}$$

$$D_r = 150 \text{ yards (as given)}$$

$$D_d = 25 \text{ yards (as given)}$$

For range, $$\frac{a_r}{D_r} = \frac{60}{150} = .40, \text{ whence } P_r = .250.$$

For deflection, $$\frac{a_d}{D_d} = \frac{5}{25} = .20, \text{ whence } P_d = .127.$$

And $$P = P_r \times P_d = .250 \times .127 = .032$$

or the probable percentage of hits is 3.2%, giving about one hit in three salvos.

1419. The examples dealt with in the preceding article were all based on the assumption that the M.P.I. coincides with the center of the hitting space. This condition will be met only when control errors have been reduced to zero, which can be considered no more likely than that the gun errors (or dispersion) can be reduced to zero. It is necessary, therefore, to consider methods for determining the probability of hitting when the M.P.I. is *not* at the center of the hitting space; i.e., with an error of M.P.I., and this error may, of course, be an error either in range or in deflection or in both. This problem involves no new principles but merely a further manipulation of the same process already applied to the simpler case.

Probability of hitting, when M.P.I. is not at center of hitting space

It will be noted that the probability table gives us directly the *probability of an error not greater than* $\pm a$ *in terms of the ratio* $\frac{a}{D}$ (article 1413). This means that we get directly the probability that an error will be included within limits which are *symmetrical* with respect to the point from which they are measured. In the examples of the preceding article this direct solution was applicable because the limits within which hits were to be made were also symmetrical with respect to the M.P.I. If, however, we move the M.P.I. from the center of the hitting space, this ceases to be true.

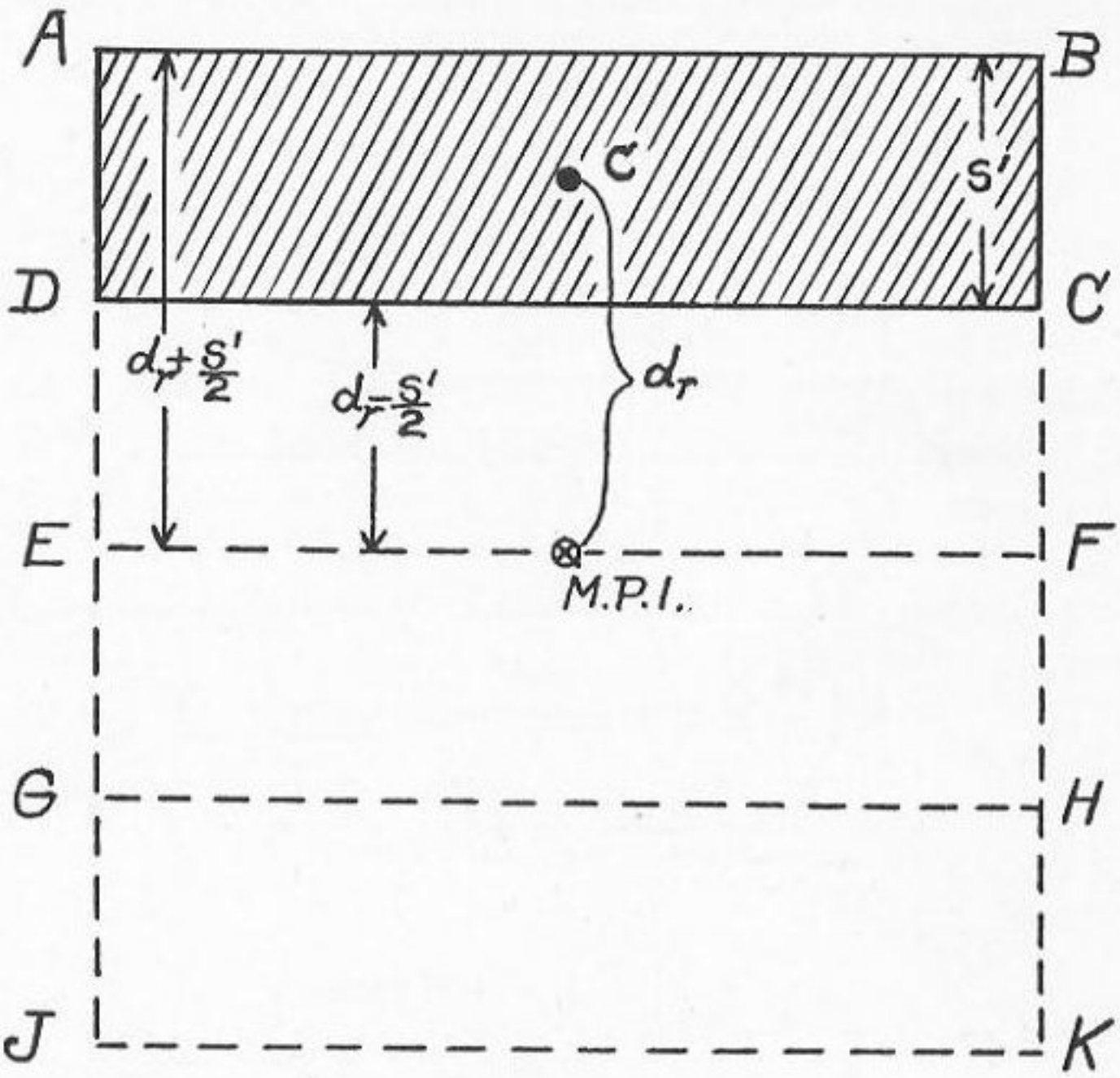


FIGURE 44

Referring to Figure 44, let the rectangle $ABCD$ denote the hitting space, with center at C. Now if the M.P.I. is also at C, then all shots falling within $\pm\frac{S'}{2}$ of C will be hits in range. In the above examples this condition gave us the value $a=\frac{S'}{2}$.

With the same hitting space $ABCD$ (Figure 44), let us now move the M.P.I. to a position away from C. In the figure it is shown at a distance d_r directly in front of the center of the hitting space (C). Thus the error of the M.P.I. is d_r in range, and for the present we will consider it to have no error in deflection. The limits within which shots will be hits in range are now no longer equal distances on either side of the M.P.I. Reference to the figure will show that the limits are now the distances $(d_r+\frac{S'}{2})$ and $(d_r-\frac{S'}{2})$, *both beyond the M.P.I.* In other words, there will fall within the hitting space only those shots whose several dispersions *beyond* the M.P.I. are not more than $(d_r+\frac{S'}{2})$ nor less than $(d_r-\frac{S'}{2})$; no shots falling short of the M.P.I. can hit the area $ABCD$. But if we use the limit $(d_r+\frac{S'}{2})$ for $\pm a$, and with the ratio $\frac{a}{D}$ so obtained enter the probability table, we will find the probability of hitting within distances lying *either* short of or beyond

the M.P.I. the amount of this a; or, as the figure shows, we will find the probability of hitting within the area $ABKJ$ which is symmetrical about the M.P.I. In like manner, if we use the limit $(d_r - \frac{S'}{2})$ for $\pm a$, we will find the probability of hitting within the area $DCHG$ which is symmetrical about the M.P.I. If we subtract the area $DCHG$ from the area $ABKJ$, we will have left the two areas $ABCD$ and $GHKJ$, and from the construction of the figure it is evident that the latter two areas are equal. Thus it follows that if we find the probabilities of hitting the larger area $ABKJ$ and the smaller area $DCHG$, one-half the difference between these probabilities will represent the probability of hitting the area $ABCD$.

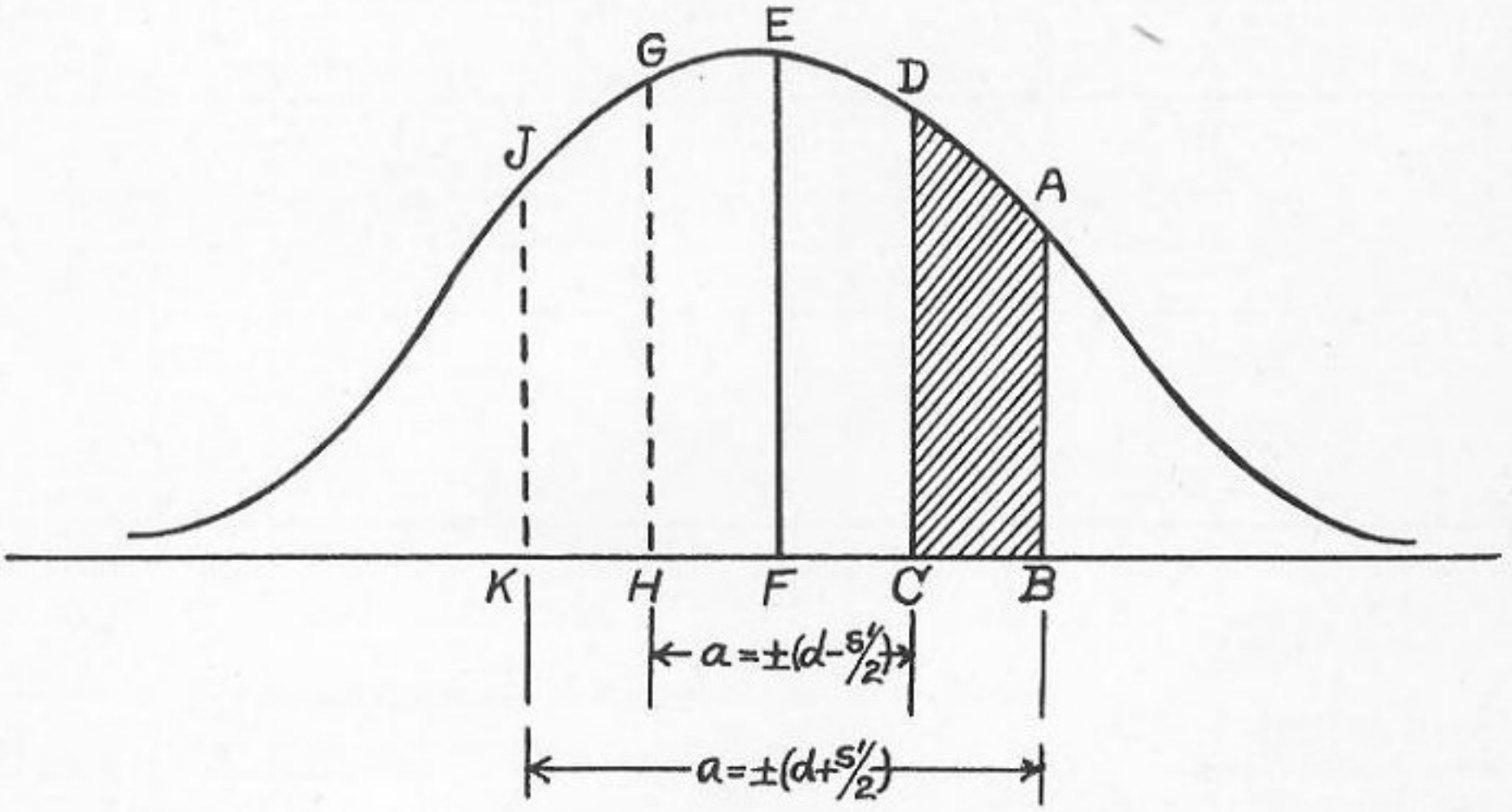


FIGURE 45

In Figure 45 the same features are illustrated in terms of areas under the probability curve. Since the probability table actually consists of a tabulation of areas under this curve between various limits, Figure 45 closely illustrates the actual process employed in this problem.

1420. An illustration entirely similar to the above might be made for the case when the M.P.I. has an error d_d in deflection, and no error in range; it would simply be a repetition of what has been said for the case with an error of M.P.I. only in range, with the substitution of the appropriate symbols for the deflection error. If we turn Figure 44 left through 90°, so that the M.P.I. lies to the right of the shaded area $ABCD$, and if we substitute for S' the symbol w to denote *width* of the target *across* the line of fire, and for d_r the symbol d_d to denote the error of the M.P.I. in deflection, the figure will illustrate the problem of finding the probability of hitting with an error of M.P.I. in deflection.

And, finally, if we consider the M.P.I. to have errors both in range and in deflection, the problem involves a combination of the two elementary cases. Figure 46 illustrates this problem. The M.P.I. is located with an error in range d_r short, and an error in deflection d_d left, of the center C of the hitting area.

Carrying through the process with the range error, we will find the probability of hitting within the uppermost strip (of depth S'), which contains our hitting area. Carrying through the process with the deflection error, we will find the probability of hitting within the farthest right-hand strip (of width w), which also contains our hitting area. The probability of hitting the area itself is then determined by taking the product of the probabilities in range and deflection.

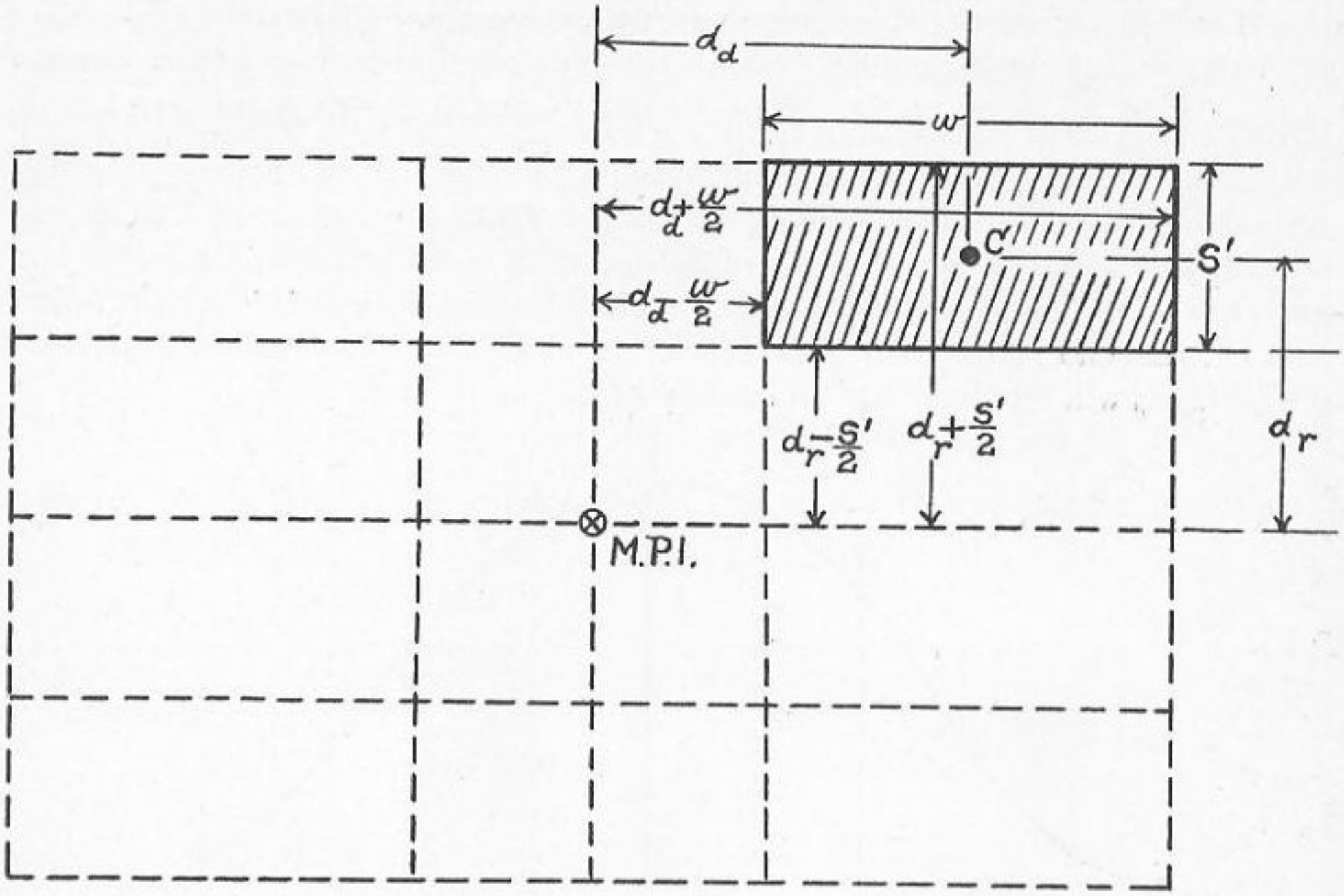


FIGURE 46

Process of determining probability of hitting, when the M.P.I. has an error both in range and in deflection

We may now state the following rules:

If a target's hitting space is S' and its width across the line of fire is w, and if the mean dispersion of the gun is D_r in range and D_d in deflection, and if the error of the M.P.I. is d_r in range and d_d in deflection, then the probability of hitting the target is:

(a) In range.

$$P_r = \tfrac{1}{2}(P_{r_1} - P_{r_2})$$

in which P_{r_1} is found from the probability table with $\pm a_{r_1} = (d_r + \frac{S'}{2})$ and $D = D_r$, and P_{r_2} is found from the probability table with $\pm\, a_{r_2} = (d_r - \frac{S'}{2})$* and $D = D_r$.

(b) In deflection:

$$P_d = \tfrac{1}{2}(P_{d_1} - P_{d_2})$$

in which P_{d_1} is found from the probability table with $\pm a_{d_1} = (d_d + \frac{w}{2})$ and $D = D_d$, and P_{d_2} is found from the probability table with $\pm\, a_{d_2} = (d_d - \frac{w}{2})$* and $D = D_d$.

(c) In both range and deflection:

$$P = P_r \times P_d$$

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* If $\frac{S'}{2} > d_r$ or $\frac{w}{2} > d_d$, which will cause a_{r_2} or a_{d_2} to be negative, respectively, the quantities P_{r_2} or P_{d_2} are additive to P_{r_1} or P_{d_1}, respectively. This case arises whenever the M.P.I. lies within the hitting area, and that we must take one-half the *sum* of the two P's in this case may be verified by drawing the figure.

1421. The following problems illustrate the rules stated above.

1. *Given:* Ten-gun salvos are fired from a ship with its battery of 5″3150 f.s. guns. The range to the target is 12,000 yards; the freeboard of the target is 16 feet, its beam 30 feet, its length 300 feet, and it is lying perpendicular to the line of fire. It is assumed that shots striking not more than 5 yards in front of the target will make effective under-water hits. The true mean dispersion of the guns is 150 yards in range and 25 yards in deflection.

Find: The probable percentage of hits that will be made under the conditions given, and with an error of M.P.I. of 200 yards in range and 40 yards in deflection.

The hitting space is (see sketch for problem 2, article 1418),

$$S' = \left(\frac{16}{20} \times 19\right) + \frac{30}{3} + 5 = \underline{30 \text{ yards}}$$

The width of the target *across* the line of fire is,

$$w = \frac{300}{3} = \underline{100 \text{ yards}}$$

The following values have been given,

$$d_r = 200 \text{ yards} \qquad D_r = 150 \text{ yards}$$
$$d_d = 40 \text{ yards} \qquad D_d = 25 \text{ yards}.$$

Now, following the steps laid down in article 1420, we have:

(a)
$$a_{r_1} = \left(d_r + \frac{S'}{2}\right) = 200 + 15 = \underline{215 \text{ yards}}$$
$$\frac{a_{r_1}}{D_r} = \frac{215}{150} = 1.43, \text{ whence } P_{r_1} = \underline{.745}$$
$$a_{r_2} = \left(d_r - \frac{S'}{2}\right) = 200 - 15 = \underline{185 \text{ yards}}$$
$$\frac{a_{r_2}}{D_r} = \frac{185}{150} = 1.23, \text{ whence } P_{r_2} = \underline{.673}$$
$$P_r = \tfrac{1}{2}(P_{r_1} - P_{r_2}) = \tfrac{1}{2}(.745 - .673) = \underline{.036}$$

(b)
$$a_{d_1} = \left(d_d + \frac{w}{2}\right) = 40 + 50 = \underline{90 \text{ yards}}$$
$$\frac{a_{d_1}}{D_d} = \frac{90}{25} = 3.60, \text{ whence } P_{d_1} = \underline{.996}$$
$$a_{d_2} = \left(d_d - \frac{w}{2}\right) = 40 - 50 = (-)\underline{10 \text{ yards}}$$
$$\frac{a_{d_2}}{D_d} = \frac{10}{25} = (-)0.40, \text{ whence } P_{d_2} = (-)\underline{.250}$$
$$P_d = \tfrac{1}{2}(P_{d_1} - P_{d_2}) = \tfrac{1}{2}(.996 + .250)^* = \underline{.623}$$

(c)
$$P = P_r \times P_d = (.036 \times .623) = \underline{.022}$$

Thus the probable percentage of hits is 2.2%, giving about one hit in five salvos.

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* See note on page 224, which applies in this instance.

2. *Given:* The same data as for problem 1 above, except that the target is steaming directly toward the firing ship.

Find: The probable percentage of hits, the error of M.P.I. being also as in problem 1, above.

We will proceed at once to list our data. (See sketch for problem 3 article 1418.)

$$S' = \left(\frac{16}{20} \times 19\right) + \frac{300}{3} + 5 = \underline{120 \text{ yards}}$$

$$w = \frac{30}{3} = \underline{10 \text{ yards}}$$

$$d_r = 200 \text{ yards} \qquad D_r = 150 \text{ yards}$$

$$d_d = 40 \text{ yards} \qquad D_d = 25 \text{ yards}$$

And the solution follows, according to article 1420.

(a) $$a_{r_1} = \left(d_r + \frac{S'}{2}\right) = 200 + 60 = \underline{260 \text{ yards}}$$

$$\frac{a_{r_1}}{D_r} = \frac{260}{150} = 1.73, \text{ whence } P_{r_1} = \underline{.832}$$

$$a_{r_2} = \left(d_r - \frac{S'}{2}\right) = 200 - 60 = \underline{140 \text{ yards}}$$

$$\frac{a_{r_2}}{D_r} = \frac{140}{150} = 0.93, \text{ whence } P_{r_2} = \underline{.541}$$

$$P_r = \tfrac{1}{2}(P_{r_1} - P_{r_2}) = \tfrac{1}{2}(.832 - .541) = \underline{.146}$$

(b) $$a_{d_1} = \left(d_d + \frac{w}{2}\right) = 40 + 5 = \underline{45 \text{ yards}}$$

$$\frac{a_{d_1}}{D_d} = \frac{45}{25} = 1.80, \text{ whence } P_{d_1} = \underline{.849}$$

$$a_{d_2} = \left(d_d - \frac{w}{2}\right) = 40 - 5 = \underline{35 \text{ yards}}$$

$$\frac{a_{d_2}}{D_d} = \frac{35}{25} = 1.40, \text{ whence } P_{d_2} = \underline{.735}$$

$$P_d = \tfrac{1}{2}(P_{d_1} - P_{d_2}) = \tfrac{1}{2}(.849 - .735) = \underline{.057}$$

(c) $$P = P_r \times P_d = (.146 \times .057) = \underline{.008.}$$

Thus the probable percentage of hits is 0.8%, giving about one hit in twelve salvos. (Compare this result with that of problem 1 above, and with those of problems 2 and 3 under article 1418.)

1422. Having determined the probable percentage of hits for any given set of conditions, it follows directly, from the axiom stated in article 1416, that the percentage of misses will equal one-hundred minus the percentage of hits. It may also be determined what proportions of these misses will be "shorts," "overs," "rights" or "lefts." The determination of the number of "shorts" that are likely

to occur is information useful to the spotter, for at long ranges the spotter on board ship sees little of what goes on beyond the target and must depend to a great extent upon what he sees in front of the target. "Shorts," when referred to in this connection, are considered to be impacts entirely short of the *target*, not merely short of the center of the hitting space.

Determination of number of "shorts" when M.P.I. is at center of hitting space

Let us consider again problem 1 in article 1418. We found there that with the conditions as given, and with the M.P.I. at the center of the hitting space, about one hit per salvo would be probable. Now with the M.P.I. at the center of the hitting space, the misses would be equally distributed short of and beyond the target (i.e., its hitting space). Hence one-half of the misses would be "shorts," and one-half "overs." Under the conditions of the particular problem being considered, we would have $\frac{1}{2}\times(8-1)$ "shorts," or from 3 to 4 "shorts." Thus the spotter would know that salvos which showed from 3 to 4 "shorts" were properly centered over the target.

How many "shorts" will occur, theoretically, with various errors of M.P.I. in range, may also be determined readily enough. It will be of interest to consider how many hits will be probable when the hitting space lies just within the end of the pattern nearest the spotter. The latter condition will evidently result when the M.P.I. lies just one-half pattern beyond the engaged side of the target, or, assuming that the hitting space is bounded by the waterline on the engaged side of the target, when the error of the M.P.I. equals one-half the pattern minus one-half the hitting space. The following two examples will illustrate the method of handling this problem and also furnish the results for further discussion.

1. *Given:* A battery of 12 guns fires in salvo at a target whose total hitting space equals 80 yards (no allowance for distance in front of target for underwater hits is included in this figure). The true mean dispersion of the guns is 115 yards in range.

   Find: How many "shorts" will probably occur when the M.P.I. coincides with the center of the hitting space?

$$S' = 80 \text{ yards (as given)}$$

whence,
$$a_r = 40 \text{ yards.}$$

Also,
$$D_r = 115 \text{ yards (as given)}$$

whence,
$$\frac{a_r}{D_r} = \frac{40}{115} = .35, \text{ and } P_r = \underline{.219}.$$

Therefore the probable percentage of hits is 21.9%, which gives about 2 or 3 hits per salvo and about 9 or 10 misses per salvo. Of the 9 or 10 misses per salvo one-half should be "shorts," hence the probable number of "shorts" when the M.P.I. coincides with the center of the hitting space, is about 5.

2. *Given:* The same data as for problem 1 above, and in addition that the average pattern size of the 12-gun salvos is 500 yards.

   Find: The probable percentage of hits when the M.P.I. is located 250 yards beyond the engaged side of the target.

$$S' = 80 \text{ yards (as given)}$$

and the error of M.P.I., being measured from the center of the hitting space, is,

$$d_r = 250 - \frac{S'}{2} = \underline{210 \text{ yards}}$$

Also,

$$D_r = 115 \text{ yards (as given)}.$$

Applying the rules of article 1420 (and noting that deflection does not enter into the problem), we have,

$$a_{r_1} = \left(d_r + \frac{S'}{2}\right) = 210 + 40 = \underline{250 \text{ yards}}$$

$$\frac{a_{r_1}}{D_r} = \frac{250}{115} = 2.17, \text{ whence } P_{r_1} = \underline{.917}$$

$$a_{r_2} = \left(d_r - \frac{S'}{2}\right) = 210 - 40 = \underline{170 \text{ yards}}$$

$$\frac{a_{r_2}}{D_r} = \frac{170}{115} = 1.48, \text{ whence } P_{r_2} = \underline{.761}$$

$$P_r = \tfrac{1}{2}(P_{r_1} - P_{r_2}) = \tfrac{1}{2}(.917 - .761) = \underline{.078}$$

Therefore, with the M.P.I. just one-half a pattern beyond the waterline on the engaged side of the target, the probable percentage of hits is 7.8%, giving about *one* hit per salvo.

1423. The results found in the second problem of the preceding article are of interest because they represent the best that a spotter can hope for when he sees no "shorts." Probably the most disastrous fault committed by spotters is that of feeling that "shorts" are wasted and should be spotted up to the target. Except at very close ranges, some "shorts" must necessarily occur (on the average) when the M.P.I. is at the center of the hitting space. The greater the range becomes, the more closely the theoretical percentage of "shorts" approaches 50%. Spotters should know, in general terms, how this percentage varies with range for the battery they spot.

Problems such as the above are not offered here as an attempt to establish an absolute spotting gage, but rather to provide a foundation for correct reasoning by spotters. The briefest of attention to the features illustrated above, and the roughest of computations, should make it apparent that the spotter who at battle ranges fails to keep short (on the average) a considerable portion of his shots, is sacrificing hits—certainly not gaining hits. And that the spotter who indulges the temptation to put the salvo all "just 'over' and in the hitting space," is an idle dreamer.

1424. It will be recalled that in article 1407 reference was made to the fact that the normal probability law is applicable to such errors as follow a symmetrical distribution, and it was also noted there that the accidental errors of gunfire are principally of this type. At short and moderate ranges these errors may be considered, for all practical purposes, to occur symmetrically. At long ranges (i.e., ranges resulting from high angles of elevation) the flight characteristics of the projectile operate to cause a measurable dissymmetry in the distribution of errors. The flight characteristics of a projectile are associated with its stability in flight, and are among the causes of dispersion. It appears that for the comparatively short duration of flight at short and moderate ranges, the errors due to variations in flight stability are relatively small and perhaps also practically symmetrical—or, being small themselves, any dissymmetry among them has no appreciable effect upon the resultant of many other elemental errors which are symmetrical. But at

Unsymmetrical errors at long ranges

long ranges the errors due to variations in flight stability become relatively greater. That they should then also be unsymmetrical in range seems plausible when one considers that large *reductions* in range may result from "wobbling," whereas it is less likely that similarly great *increases* due to abnormally good flight will occur. Thus among the many elemental errors contributing to the inherent errors of gunfire, it appears that those due to variations in the projectile's stability in flight are unsymmetrical, and at long ranges, when these errors may become of controlling magnitude, the resultant total errors may also become unsymmetrical to a measurable degree.

In salvo firing at long ranges, the features discussed above sometimes result in a marked straggling of shots at the "short" end of the pattern, and in bunching at the farther end. The M.P.I. of such a salvo would obviously be located *beyond* the center of the pattern, there would be more impacts *beyond* the M.P.I. than short of it, and the average dispersion of the impacts beyond the M.P.I. would be *less* than the average dispersion of those short of it. The normal law, and the normal probability table, would seem not to be applicable to such errors as these, for the normal law is based on symmetrical distribution, which means an equal number of impacts short of and beyond the M.P.I., and average dispersions of equal amount short of and beyond the M.P.I.

The fact that such visible discrepancies are often noted between conditions actually occurring and the theoretical conditions upon which the normal probability law is based, often leads to skepticism as to the worth of any conclusions that may be derived from the applications of this law. Unsymmetrical distributions are common in many other branches of research in which probability laws are used. In the science of statistics unsymmetrical distributions are the rule rather than the exception. A great many special probability laws have been devised to handle such cases, some of which differ very radically from the normal law.* If the deviation of the errors of gunfire from the assumptions governing the normal law were sufficiently great, a special law might be devised for them. But a closer investigation of the situation does *not* indicate that such discrepancies as exist warrant the discarding of the normal law.

1425. In order to illustrate further some of the features just discussed, as well as to provide examples of how theoretical results obtained by application of the normal probability law compare with practical results, two analyses based on recent firings will be presented.†

Illustration of an unsymmetrical range distribution

In Figure 47, curve *A* represents the actual distribution in range of 200 shots fired at a proving ground with various 6″ guns and various types of projectiles. The firings were made in series of from three to ten shots. For each of these series the M.P.I., and the dispersions of the several shots from the M.P.I., were determined. As the mean ranges of the various series differed considerably, the dispersions of the several shots were expressed as percentages of the ranges of their respective M.P.I.s, and these various dispersions were then plotted with reference to a common M.P.I., as shown in the figure. The firings used included a variety of types of

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* Ref. Chapter VIII, *Probability and Its Engineering Uses*, by Thornton C. Fry. Also Chapter IX, *The Combination of Observations*, by Brunt. Also, articles 90 and 160, *The Calculus of Observations*, by Whitaker and Robinson.

† The actual values of the dispersions are not given, but the curves are drawn to scale and the value of *D* to the same scale is shown in order that the distribution may be studied in terms of errors compared to *D*.

projectile and of rifling, and were taken at random from data recorded in a service publication. Altogether the data may be regarded as representing conditions unfavorable, as compared to conditions usually encountered, for testing the applicability of the normal probability law. However, such data have been used here in order that some of the features previously discussed may appear in exaggerated degree for purposes of discussion.

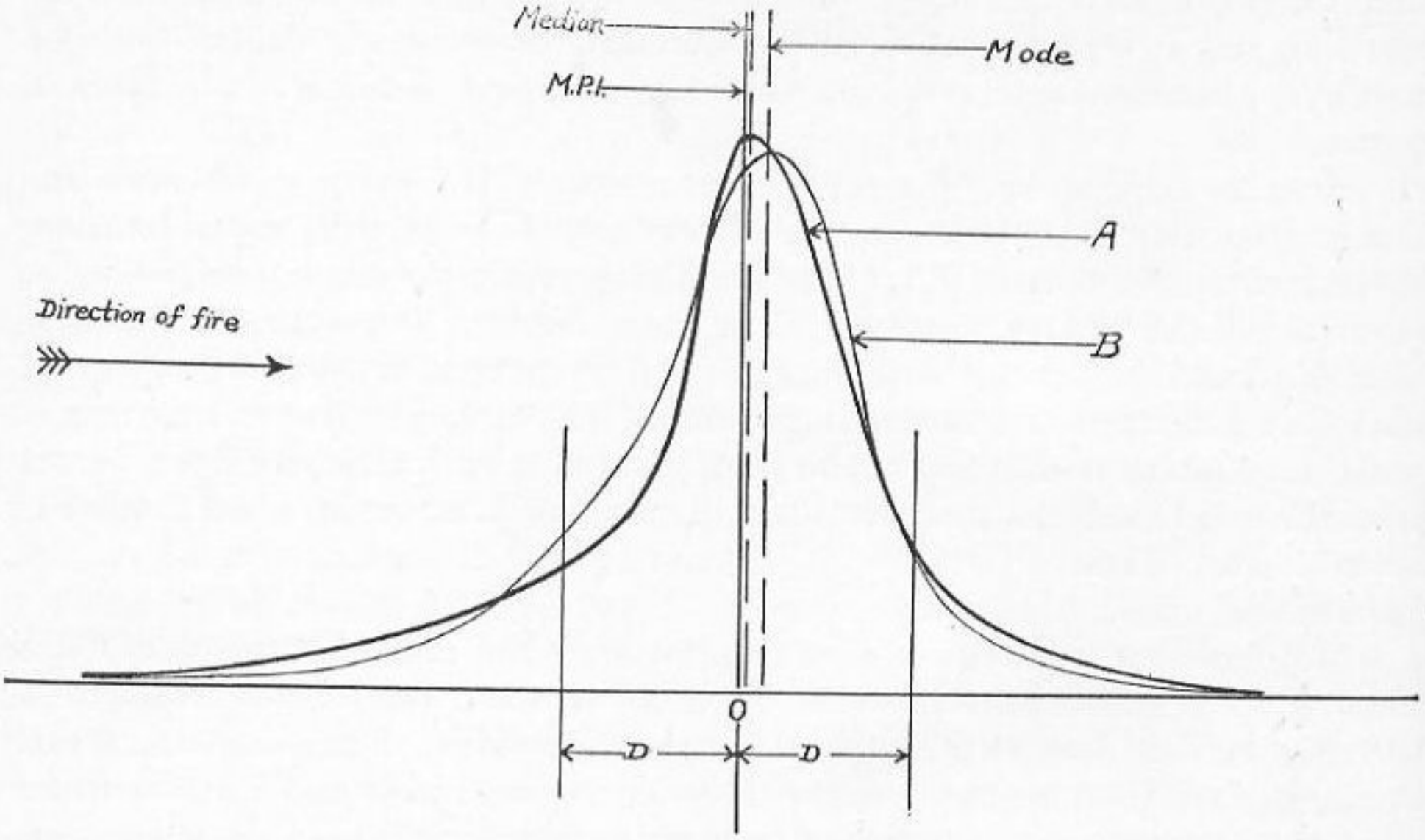


FIGURE 47

It will be noted that curve A, although having some of the general characteristics of the normal probability curve, is markedly lopsided. It falls off from its peak more steeply short of the M.P.I. than beyond it; but short of the M.P.I. it flattens out sooner and ultimately approaches the X-axis less rapidly than beyond the M.P.I. This indicates that shots were bunched more closely beyond the M.P.I. than short of it, and that there were more large errors short of the M.P.I. than beyond it. It indicates that the *density* of impacts decreased less rapidly beyond the M.P.I. than short of it; i.e., that the M.P.I. was not centered in the region where impacts were densest.

If the M.P.I. is not centered in the densest area of impacts, it is evident that the maximum percentage of hits will not occur with the M.P.I. at the center of the hitting space, although the latter assumption was made in arriving at the results in the problems of article 1418. Moreover, the normal probability table would not serve in this case, for it gives results only on the basis of a symmetrical error ($\pm a$).

The "mode" in frequency distributions

To secure the greatest percentage of hits it is evidently necessary to place the salvo so that the point which represents the center of greatest density of impacts coincides with the center of the hitting space. With reference to frequency distributions in general, such a point is called the *mode*,* and the *mode* may be defined, for our case, as that point within the salvo pattern on either side of which the density of impacts

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* The terms *mode* and *median* are used principally in statistical work. Ref: article 61, *The Combination of Observations*, Brunt.

The "median" in frequency distributions

systematically decreases. Another reference point in such distributions is known as the *median*, which is defined for our case as that point which has an equal *number of impacts* on either side of it. It is found that, on the average, the *median* of a frequency distribution lies between the M.P.I. and the *mode.**

Returning now to Figure 47, we find illustrated the *mode* and the *median;* the *mode* was determined by finding the axis about which the peak of the distribution curve becomes symmetrical, and this new curve (B) has also been drawn. Curve B is seen to be much more nearly symmetrical at the peak than curve A, and it is, on the whole, a better approximation of the normal probability curve, although the left branch still shows markedly the straggling short. Thus it appears that the situation can be improved by placing the *mode* at the center of the hitting space and reckoning mean dispersion and probability of hitting from the *mode.* But when we consider the magnitude of the variation between these two axes of reference, it becomes apparent that such a refinement is unnecessary for any practical purpose. In Figure 47 the size of the true mean dispersion of the shots represented there, is indicated to scale. The distance between the M.P.I. and *mode* is about $\frac{1}{7}D$. The mean dispersion from the *mode* is about 2.5% less than the mean dispersion from the M.P.I.

For the conditions represented in the above data, the straightforward application of the normal probability table as outlined in the problems of article 1418, would then cause discrepancies due to an error of about 2.5% in the value of D used, and to an error of about $\frac{1}{7}D$ in the location of the M.P.I.—i.e., we would find a result based on a mean dispersion slightly too great and on an M.P.I. a short distance from the center of the hitting space instead of exactly at the center. And as this applies to a none too favorable case, it seems that the normal law works perhaps even better than we would naturally have expected it to work.

1426. It follows from the above discussion that in cases of unsymmetrical fall of shot the *mode*, rather than the M.P.I., should be spotted to the target.

Spotting the "mode"

Spotting the *mode* is not only the better but also the simpler procedure, since it amounts simply to spotting the densest portion of the salvo to the target. It is often difficult, in unsymmetrical distributions, to locate the M.P.I. quickly by eye.

Another interesting feature brought out by this discussion is that the *median*, for unsymmetrical distributions, lies closer to the *mode* than does the M.P.I. Thus

Spotting the "median"

a spotter who simply spots the 50-50 point of the salvo is, on the average, doing the logical thing—he is actually doing better than he would do if he tried to spot the M.P.I. For aircraft spotters, who can readily count the impacts of a salvo, the spotting of the *median* is easily the simplest method and at the same time probably the most certain. For symmetrical distributions the M.P.I., *median* and *mode* are, of course, identical, hence these observations apply equally well to either case.†

But the discussion in article 1415 again becomes pertinent here. The rules derived here may readily be discredited by applying them to individual instances; any rules that might be devised could likewise be discredited in such a manner. It should be borne in mind that rules based on the laws of probability are designed to serve best "in the long run"; they will prove best in *most* instances occurring, not necessarily in all.

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* Ref: p. 121, *An Introduction to the Theory of Statistics*, Yule (1919).

† Ref. also article 102, *The Calculus of Observations*, by Whitaker and Robinson.

1427. The second analysis is based on the fall of shot that occurred in the long-range battle practices of a division of three ships; the range may be considered as representative of modern battle ranges. Altogether, 134 shots are represented in this analysis, including all shots fired by the three ships on their respective practices, with the exception, in each case, of the first salvo. The ships were equipped with identical batteries and their individual mean ranges were approximately equal, so that the fall of shot could readily be referred to a common M.P.I. by finding the M.P.I. of each salvo and the several dispersions of the shots from their respective M.P.I.s, as was done in the first illustration. The ships selected for this example were not selected because there was any reason to believe

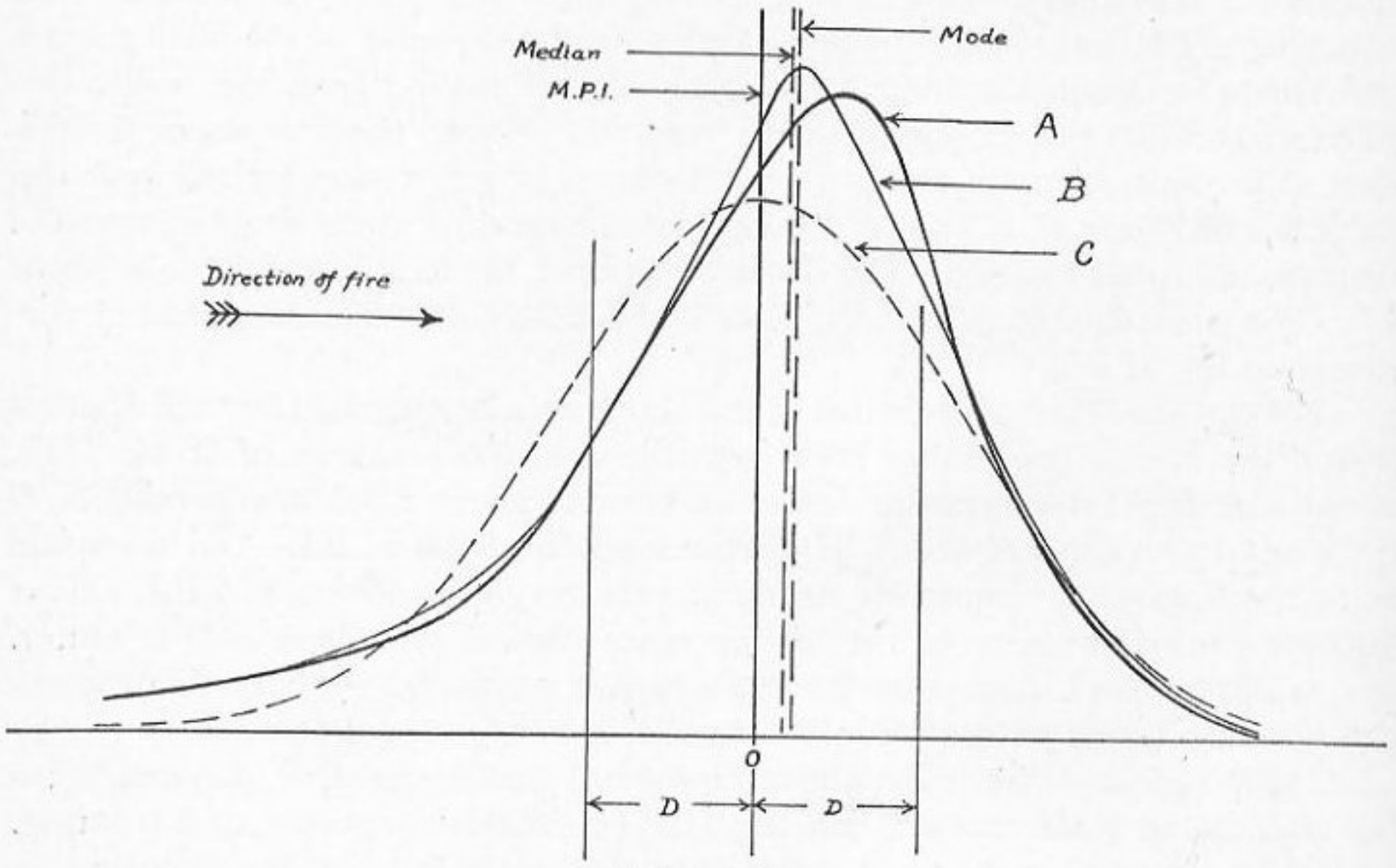


FIGURE 48

that their performances would better than others substantiate theory. On the contrary, the guns of these ships belong to a group whose patterns have been characterized by more than average dissymmetry. The analysis therefore represents conditions leaning toward the unfavorable for a test of the normal probability law.

In Figure 48, curve *A* represents the distribution of all 134 shots, considered as one large salvo with M.P.I. at *O*; wild shots were *not* excluded. We see that this curve is again markedly lopsided and has the same general characteristics as curve *A* of Figure 47. But in this case it is illustrated more clearly how the greatest density of impacts occurs beyond the M.P.I. The *mode* for this shot distribution was determined as described in the previous illustration, and curve *B* is a plot of the shot distribution with respect to the *mode* as axis. The *median* is also shown. Curve *B* is practically symmetrical about its axis (the *mode*) except at the ends of its branches; while the right branch meets the *X*-axis at a distance of about $3.5D$ from the M.P.I., the left branch continues to remain above the *X*-axis well beyond the distance $4D$ from the M.P.I. (see last paragraph of article 1413). This indicates that in the main part of the pattern the distribution is symmetrical but that there are far more large "shorts" than large "overs." The distance between the M.P.I. and *mode* is about one-fourth of the mean dispersion, and the *median* is near the *mode*.

Illustration of a range distribution from fleet firing

Curve C, Figure 48, shows the normal probability curve corresponding to the same data, i.e., to the same mean dispersion D. The discrepancies between curve C and the other curves are apparent, but let us investigate the essential differences. Curve C is *lower* and *flatter* than the others, therefore it actually represents greater mean dispersion than is indicated in the others (see article 1410). If a smaller value of D were used in plotting curve C, the peak of this curve would be extended upward and it would more nearly agree with the others.

The value of mean dispersion having been based on all the shots, *including wild shots*, would naturally be too great. After 12 of the 134 shots had been rejected as "wild,"* the actual distribution curves of the remaining 122 shots were plotted as shown in Figure 49, curve A being again the distribution with respect to the M.P.I., and curve B the distribution with respect to the *mode*. With the mean dispersion obtained from the remaining 122 shots, the theoretical distribution curve (or normal probability curve) was plotted and is shown as curve C. There is now very marked agreement among all three curves, the only difference worthy of note being the small distance between the axis of symmetry of A and B,

Comparison of the theoretical range distribution and the actual range distribution, corresponding to an actual case of fleet firing, excluding wild shots

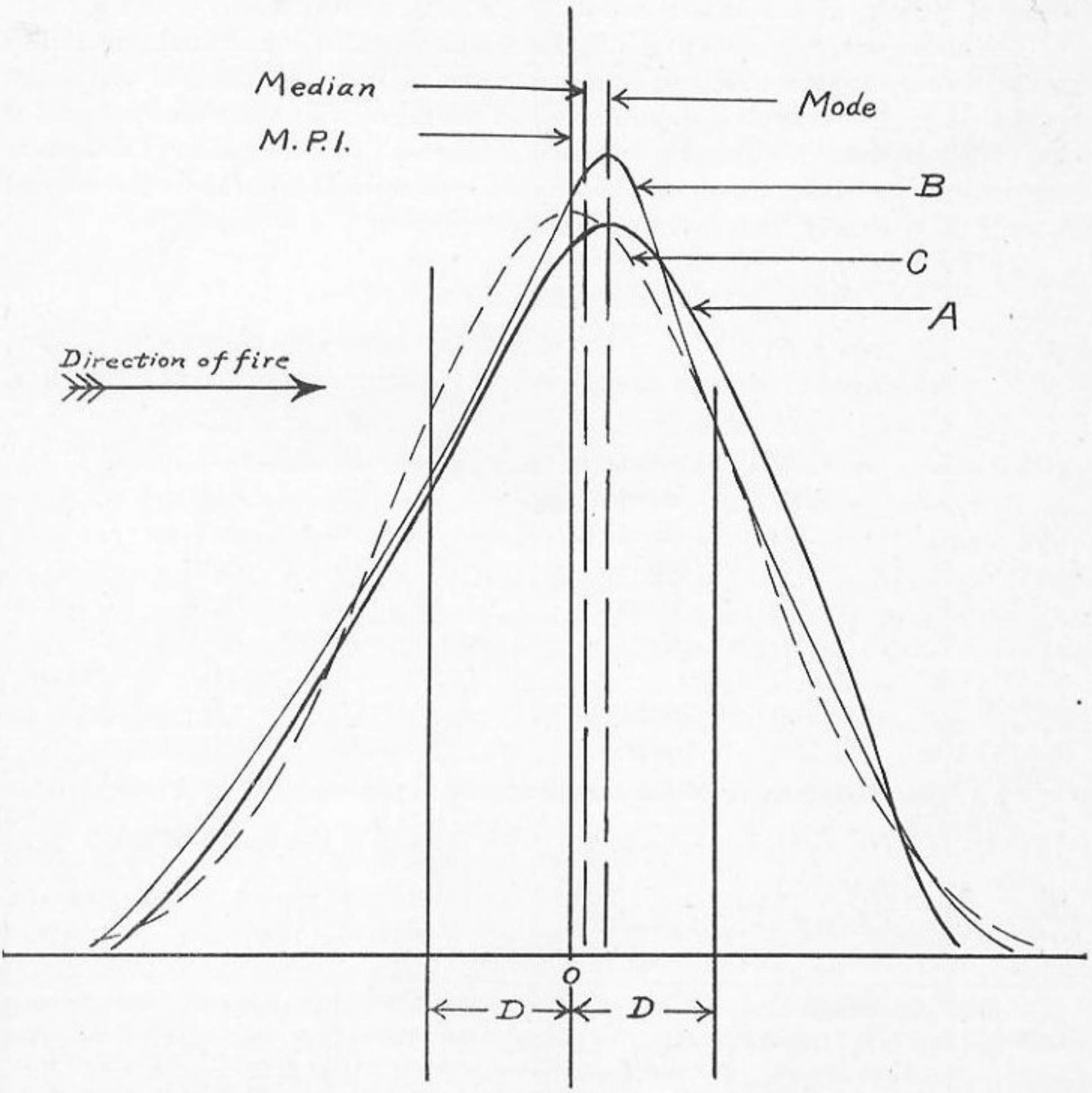


FIGURE 49

* The rejection of these 12 shots was made in accordance with a rule that will be discussed later (article 1503).

and that of C. This merely indicates that the densest portion of the impacts occurred a little beyond their M.P.I. (about $\frac{1}{4}D$), while the impacts taken altogether were distributed practically symmetrically.

The above analysis indicates very close agreement indeed between theory and practice, for somewhat more than 91% of the firing of three different ships throughout an entire battle practice. The ships selected were such as to indicate, if anything, less than the average chance of agreeing with theory. The conditions were actual service conditions and the errors included all errors that one may expect to find, both in the guns themselves and in their control. Of particular importance is the feature of wild shots. The above analysis indicates very strongly that with proper identification and rejection of wild shots, the fall of shot even at the longest of ranges follows the normal probability law. From inspection of Figure 49, which represents 122 out of 134 shots actually fired (or over 91%), it is apparent that percentage of hits by practice and by theory are very nearly the same, whence it appears that after rejecting wild shots we may expect the probability table to give us pretty good results "in the long run." In this case the regular procedure would have to be modified only by applying the probable percentage of hits to the probable percentage of good shots, e.g., if for a certain target we had a 10% chance of hitting, we would now call it 10%×91%, getting finally about 9%.

The above analyses seem to justify the conclusion that the normal probability law applies only approximately to gunfire when we make no rejections of wild shots, but that it applies very well to gunfire when we reject wild shots on the basis of some judicious rule. No mention has been made in all this discussion of deflection distributions, for these are almost invariably symmetrical and follow the normal law even more closely than do range distributions.*

EXERCISES

1. *Given:* The points of fall of the shots of a 10-gun salvo were observed as follows, distances having been measured from the center of the target's waterline.

1.	375 yards short	30 yards right
2.	300 " short	10 " left
3.	250 " short	20 " left
4.	200 " short	0
5.	150 " short	10 " right
6.	50 " short	15 " right
7.	25 " short	40 " left
8.	0	5 " left
9.	100 " over	25 " left
10.	200 " over	35 " left

The hitting space of the target was 84 yards.

Find: (a) The location of the M.P.I. with respect to the center of the target's waterline.
(b) The error of the M.P.I.
(c) The pattern in range and in deflection.

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* Some comparisons between theoretical and practical deflection distributions are given in articles 132 and 148, *Exterior Ballistics, 1906*, Alger. One of these is an analysis of 500 shots and shows theoretical and actual deflection distributions to agree within about 2% for errors from $1/4D$ to $3D$. Comparisons between theory and actual observations, and pertaining to observations of widely varying natures, may be found in articles 19 and 24, *The Combination of Observations*, Brunt.

(d) The apparent mean dispersion in range and in deflection.
(e) The true mean dispersion in range and in deflection.

Answers: (a) 105 yards short, 8 yards left
(b) 147 yards short, 8 yards left
(c) 575 yards in range, 70 yards in deflection
(d) 150 yards in range, 18 yards in deflection
(e) 158 yards in range, 19 yards in deflection

2. *Given:* The points of fall of an 8-gun salvo from a ship carrying 16″2600 f.s guns, were observed as follows, distances having been measured from the center of the target's waterline.

1.	325	yards	over	18	yards	right
2.	300	"	"	14	"	"
3.	225	"	"	12	"	"
4.	175	"	"	6	"	"
5.	125	"	"	9	"	left
6	50	"	"	10	"	"
7.	0			12	"	"
8.	75	"	short	13	"	"

The target was 36 feet high, 130 feet long, and had a depth‡ of 80 feet in the line of fire. The range was 20,000 yards.

Find: (a) The location of the M.P.I. with respect to center of the target's waterline.
(b) The error of the M.P.I.
(c) The pattern in range and in deflection.
(d) The apparent mean dispersion in range and deflection.
(e) The true mean dispersion in range and deflection.

Answers: (a) 141 yards over, and 1 yard right.
(b) 107 yards over, and 1 yard right.
(c) 400 yards in range, 31 yards in deflection.
(d) 116 yards in range, and 12 yards in deflection.
(e) 124 yards in range, and 13 yards in deflection.

3. *Given:* The type of gun, the horizontal range, the true mean dispersions* in range and in deflection, and the height of the target and its depth in and width across the line of fire (feet).

Find: The percentage of hits to be expected under the conditions given, and with M.P.I. at center of hitting space.

	Given							Answers
	Gun	Range (yds.)	True mean dispersion* (yds.)		Target (feet)			% Hits
			Range	Deflection	†Height	‡Depth	Width	
A	3″2700 f.s.	3500	25	3	6	30	60	41.0%
B	4″2900 f.s.	4000	16	1	15	0	18	98.0%
C	5″3150 f.s.	13,000	70	10	30	210	42	17.3%
D	12″2900 f.s.	20,000	130	15	40	90	600	17.7%
E	16″2600 f.s.	38,000	210	25	40	90	600	6.4%

* See art. 1418, par. 1. † Use Col. 7 of the range table. ‡ See art. 1402 (h).

4. *Given:* The type of gun, the horizontal range, the true mean dispersions* in range and in deflection, the height of the target and its depth in and width across the line of fire (feet), and error of M.P.I. as indicated.

Find: The percentage of hits to be expected under the conditions given. (Compare with Exercise 3.)

	Given									Answers
	Gun	Range (yds.)	True mean dispersion* (yds.)		Target (feet)			Error of M.P.I. (yds.)		% Hits
			Range	Deflection	†Height	‡Depth	Width	Range	Deflection	
A	3″2700 f.s.	3500	25	3	6	30	60	50	10	6.5%
B	4″2900 f.s.	4000	16	1	15	0	18	18	2	77.2%
C	5″3150 f.s.	13,000	70	10	30	210	42	200	10	1.2%
D	12″2900 f.s.	20,000	130	15	40	90	600	300	50	3.3%
E	16″2600 f.s.	38,000	210	25	40	90	600	500	100	0.5%

5. *Given:* The type of gun, the horizontal range, the true mean dispersion* in range, and the height of the target and its depth in the line of fire, and the number of guns fired in salvo.

Find: The number of *shorts* that should be seen when the salvo pattern is centered on the target's hitting space.

	Given						Answers
	Gun	Range (yds.)	No. shots in salvo	True mean dispersion*(yds.)	Target (feet)		No. Shorts
					†Height	‡Depth	
A	3″2700 f.s.	3500	4	25	6	30	1
B	4″2900 f.s.	4000	3	16	15	0	0
C	5″3150 f.s.	13,000	7	70	30	210	2
D	12″2900 f.s.	20,000	12	130	40	90	5
E	16″2600 f.s.	38,000	8	210	40	90	4

6. *Given:* A destroyer is 300 feet long and has a 30-foot beam. At battle ranges the hitting space due to the freeboard of the destroyer averages about 30 yards, for secondary battery guns. It is assumed that the true mean dispersion of secondary battery guns averages about 150 yards in range and about 25 yards in deflection at battle ranges, and that at these ranges the average error of M.P.I. for the same batteries is about 300 yards in range and about 50 yards in deflection (see article 1418, par. 1).

Find: How do the destroyer's chances of being hit compare under the following two conditions, (a) when she is steaming directly in the line of fire (target angle 0° or 180°), (b) when she is steaming directly across the line of fire (target angle 90° or 270°)?

Answers: (a) 0.3%.
(b) 1.2%.

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* See art. 1418, par. 1. † Use Col. 7 of the range table. ‡ See art. 1402 (h).

CHAPTER 15

FURTHER APPLICATIONS OF THE LAWS OF PROBABILITY IN CONNECTION WITH SALVO ANALYSIS. DETERMINATION OF PROBABLE PATTERN SIZE, WILD SHOTS, AVERAGE SHIFT OF M.P.I., CONTROL ERRORS.

1501. The determination of the probable pattern size of a salvo is a problem which is beset with great difficulties, for it involves the determination of probabilities pertaining to a definite, small number of occurrences, whereas the laws of probability depend for their accuracy upon great numbers. Thus with the information, from the probability table, that there is about one chance in a thousand that an individual error of size $\pm 4D$ will occur, we can be reasonably certain that in a salvo of 1000 shots an error of this size *will* occur, and that the pattern size of such a salvo will approach $8D$. In a salvo of 10 shots there is still that one chance in a thousand that the error $\pm 4D$ will occur, but evidently it is likely to occur only about once in 100 such salvos. It is likely that the greatest errors occurring in *most* of the 10-gun salvos will be considerably less than $\pm 4D$, that the patterns of most of these salvos will be correspondingly smaller, and hence that the *average* pattern of these salvos will be smaller than $8D$.

Determination of probable pattern size

A number of rules applicable to this problem have been devised by various authorities. One that shows the closest agreement with observed results in actual firings is given here. This rule is due to Mazzuoli and is based on the following considerations.* If $\pm a$ denotes the limiting dispersion of a salvo, and if there are 1000 shots in the salvo, the probability that just *one* of the 1000 shots will *fail* to be included within these limits, is evidently $P = .001$. Likewise the probability that *all but one* will be included, is $P = .999$. If $P = .999$, then $a/D = 4.0$, whence $\pm a = 4.0D$, and the pattern is $8D$. Now if n shots compose the salvo, the probability that all but one of them will be included within the limits $\pm a$, is evidently,

$$P = \frac{n-1}{n} \tag{1501}$$

as, for instance, with 100 shots the probability of all but one falling within the limits $\pm a$ is 99/100, and for 12 shots it is 11/12, etc. Having determined P as above, the value of a/D corresponding to this value of P will then define the limiting error, plus or minus, beyond which *probably* not more than *one* shot will fall. Since the error so found is always plus or minus, the corresponding pattern size will be equal to twice this error.

For a 12-gun salvo, (1501) gives $P = \frac{12-1}{12} = .917$; entering the probability table with this value of P, we find the corresponding value $a/D = 2.17$, whence $\pm a = 2.17D$, and the pattern size is $2 \times 2.17D = 4.34D$. Applying the same process

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* Ref: Page 393, *Handbook of Ballistics*, Cranz and Becker.

to salvos of other sizes, we get the following ratios of pattern size to true mean dispersion. These ratios are slightly greater than those found in practice, but provide very good approximations.

Ratio of pattern size to mean dispersion

Number of shots in salvo	Ratio of pattern size to true mean dispersion
3	2.43
4	2.89
5	3.21
6	3.47
7	3.67
8	3.85
9	4.00
10	4.13
11	4.24
12	4.34

1502. Knowledge of the ratio of pattern size to mean dispersion affords a convenient method for determining approximately the probable percentage of hits. The average pattern size of a series of firings may be determined very readily from rake observations, it being necessary to measure only the distance between the shot farthest over and the shot farthest short in each salvo, and to take the average of these distances for the several salvos observed. Applying to this average pattern the ratio corresponding to the average number of shots of the salvos, we get the approximate mean dispersion.

An example will illustrate the application of the above features.

Percentage of hits, based on average pattern size

Given: A ship carrying a main battery of 12 guns has on several target practices had an average pattern of 1000 yards at battle ranges.

Find: The probable percentage of hits in range for this battery at battle ranges, on targets whose total hitting space averages 60 yards, with the M.P.I. at the center of the hitting space. What would it be if the pattern size were decreased to 600 yards?

For the 1000-yard pattern, $D_r = \dfrac{1000}{4.34} = 230$ yards.

For the 600-yard pattern, $D_r = \dfrac{600}{4.34} = 138$ yards.

In either case, $a_r = \dfrac{S'}{2} = \dfrac{60}{2} = 30$ yards.

For the 1000-yard pattern, $\dfrac{a_r}{D_r} = \dfrac{30}{230} = .13$, whence $P_r = .083$.

For the 600-yard pattern, $\dfrac{a_r}{D_r} = \dfrac{30}{138} = .22$, whence $P_r = .139$.

Thus with 1000-yard patterns the ship would expect a maximum of about 8.3% hits in range, and with 600-yard patterns about 13.9% hits in range. (These percentages are, of course, based on M.P.I. at center of hitting space and will be considerably reduced if the ship has large errors in control. They indicate, however, approximately the *best* the ship can hope to do under the stated conditions.)

1503. A problem similar in some aspects to that of determining the ratio of pattern size to mean dispersion, is that of determining how great the dispersion of a single shot must be before it may be classed as *wild*. The difficulty in establishing a sound criterion for determining what constitutes a *wild* result is by no means unique to the problem of gunfire. It exists throughout the entire field to which the laws of probability are applied and has received close attention by many of the world's foremost mathematicians.

The similarity between the problems of determining pattern size and of determining wild shots will be seen if we return to the discussion of how Mazzuoli's rule was derived (art. 1501, par. 2). We dealt there with the limiting error within which the shots of the salvo would *probably* be included; twice this limiting error gave us the pattern size. Now if this pattern size *probably* includes all shots, then any shots which actually fall outside of such a pattern may be thought of as *improbable* results, i.e., *wild shots*.

Mazzuoli's rule actually accounts for only $(n-1)$ shots of a salvo of n shots, and may therefore appear to be insufficiently rigid. The fact that it actually proves, on the contrary, to be somewhat too rigid when applied to the determination of pattern size (as evidenced by practical results), may indicate that our natural intuition in thinking of it as not rigid enough, is at fault. Mazzuoli himself stated that the rule serves well for determining wild shots, and its application to our own problems only serves to strengthen that conviction. The rejection of wild shots in connection with the analysis made in article 1427 and resulting finally in the curves shown in Figure 49, was made according to Mazzuoli's criterion.

From the discussion in the preceding paragraph it appears that a wild shot, according to Mazzuoli's criterion, is one which falls beyond the limits of the probable pattern size defined by the same criterion. Thus for a 12-gun salvo the probable pattern size is $4.34D$, which includes errors as great as $2.17D$ either short of or beyond the M.P.I., (i.e., $a = \pm 2.17\ D$). This then defines as a wild shot for a 12-gun salvo one whose individual dispersion exceeds 2.17 times the mean dispersion of that salvo. The table in article 1501 can be used conveniently for determining wild shots according to this criterion, since one-half the ratio of pattern size to true mean dispersion defines the size of error which, if exceeded by a shot of that salvo, classifies that shot as wild.

Mazzuoli's criterion for rejection of wild shots

1504. The above rule for determining wild shots is not offered here as a standard rule. Due to wide divergence of opinion on the subject, nothing approaching a "standard" rule exists. In our own service, practice has of late varied widely in this connection. For purposes of standardizing target practice reports, definite rules for determining wild shots are laid down in *Orders for Gunnery Exercises;* these rules must be followed in submitting target practice reports and are, for that purpose, "standard."

Proving grounds favor a criterion somewhat more rigid than Mazzuoli's; it is known as Chauvenet's criterion. The latter criterion is derived by the same reasoning as applied in deriving Mazzuoli's, but Chauvenet excludes but one-half of a shot of the n in a salvo, or series. The probability ratio is thus defined by the relation,

Chauvenet's criterion for rejection of wild shots

$$P = \frac{n - \frac{1}{2}}{n}$$

whence,

$$P = \frac{2n - 1}{2n} \qquad (1502)$$

in place of the relation given by (1501). Applying this to a 12-gun salvo we get, $P=\frac{24}{24}\frac{-1}{}=\frac{23}{24}=.958$, whence $\frac{a}{D}=2.55$, and $\pm a=2.55D$. According to this criterion a *wild* shot, for a 12-gun salvo, is then one whose dispersion exceeds 2.55 times the mean dispersion of that salvo, rather than 2.17 times, as gotten from Mazzuoli's criterion. Whichever of these rules is accepted, it will vary very considerably from the rule of calling *wild* only those shots whose dispersions exceed four times the mean dispersion, which is sometimes employed. The use of the latter rule may cause results to appear to be much more discordant than they actually are.*

1505. If a number of small salvos were fired in succession, each with the same point of aim and range and deflection settings, and if each impact and the M.P.I. of each salvo were plotted, we could consider the several impacts taken altogether as an aggregate salvo, and we could find the M.P.I. of this aggregate salvo in the usual manner. We would now find the M.P.I.s of the several small salvos grouped about the M.P.I. of the aggregate salvo, just as the individual impacts of each salvo would be grouped about their own M.P.I.s. This is explained by the occasional occurrence of large individual dispersions, which carry with them the M.P.I.s of the salvos on which they occur. Another evidence of this movement of the M.P.I.s is given by the fact that the pattern of the aggregate salvo would be considerably larger than that of each small salvo, whence it appears that the small patterns must shift about within the limits comprising the aggregate pattern.

Treating the M.P.I.s of the several small salvos as single impacts and measuring their mean dispersion in the usual manner, we now have what may be termed the *mean dispersion of the M.P.I.* Knowledge of how much this mean dispersion of the M.P.I. may amount to is of importance to the spotter, for it is an error over which he has no possible control. To attempt to correct for this natural movement of the M.P.I. by spots, is as illogical as to attempt to spot the individual shots of a salvo each to the salvo's M.P.I., and would prove as fruitless.

Mean dispersion of M.P.I.

It is shown in the laws of errors that when the arithmetical mean of a number of observations is taken, this arithmetical mean is subject to an error which depends upon the number of observations entering into it; also that if D is the mean error of the individual observations from their arithmetical mean, D_0 the mean error of the arithmetical mean, and n the number of observation, then,

$$D_0=\frac{D}{\sqrt{n}}. \qquad (1503)\dagger$$

This relation is directly applicable to our case if we call D_0 the mean dispersion of the M.P.I., D the mean dispersion of the gun, and n the number of shots in the salvo.

It is to be noted that the value of D_0 derived from formula (1503) represents the mean error of M.P.I. that is likely to occur due solely to the mean dispersion of the guns themselves and entirely independently of errors in control. Let us suppose, for example, that the true mean dispersion in range of a battery of six guns

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* Ref: §65, *Handbook of Ballistics*, Cranz and Becker. In addition to the two mentioned above, three other rules are discussed in this reference, and the limits assigned by these also are very considerably less than $4D$. Ref. also Chapter VIII, *The Combination of Observations*, Brunt.

† Ref: Page 385, *Handbook of Ballistics*, Cranz and Becker. Also, article 101, *The Calculus of Observations*, Whitaker and Robinson. Also, article 21, *The Combination of Observations*, Brunt.

is 100 yards. Now if an unlimited number of salvos were to be fired from this battery with absolutely no differences in control from salvo to salvo, it would nevertheless be likely that the M.P.I. of any one of the 6-gun salvos would be in error, with respect to the M.P.I. of *all* the shots, by an amount equal to $\frac{100}{\sqrt{6}}=41$ yards.

1506. Of more immediate importance to the spotter, however, is knowledge as to the probable *shifts* of the M.P.I. between successive salvos. The amount of the probable shift of M.P.I. from salvo to salvo can be determined by means of the method of successive differences.* In this method the mean dispersion is determined by measuring the differences between successive observations in the order in which they actually occur, rather than by referring each observation to the arithmetical mean of all the observations. Thus in a series of shots the differences between the first and second shots, between the second and third, third and fourth, etc., are determined, and the sum of all these differences is divided by the number of differences. If E_d is the *average difference* between successive shots, as thus determined, the mean dispersion is then given by the relation

Method of determining mean dispersion from successive differences

$$D=\frac{E_d}{\sqrt{2}}. \tag{1504}$$

It will be noted that (1504) gives the relation between the mean dispersion and the *average difference between successive shots of a series.* We may apply this also to successive salvos, thus getting a relation between the mean dispersion of the M.P.I. and the average difference, or *shift,* between successive positions of the M.P.I. Thus, if we let D_{0d} denote the average *shift* of the M.P.I., and D_0 the *mean dispersion* of the M.P.I., we have,

Determination of average shift of M.P.I.

$$D_0=\frac{D_{0d}}{\sqrt{2}}$$

whence,

$$D_{0d}=D_0\sqrt{2}. \tag{1505}$$

For the case cited in the foregoing article, we have $D=100$ yards, whence, from (1503), $D_0=\frac{100}{\sqrt{6}}=41$ yards, and, from (1505), $D_{0_d}=41\sqrt{2}=58$ yards. Thus we find that the *average shift of M.P.I.* is 58 yards, which is $\frac{58}{347}$, or practically $\frac{1}{6}$, of the pattern size.

Applying this process to salvos of from 3 to 12 shots, we get the following values for the *average shift of M.P.I. between successive salvos, due only to the errors of the gun* (i.e., with no errors in control).

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* Ref: §62, *Handbook of Ballistics*, Cranz and Becker.

Average shift of M.P.I., due only to gun errors, in terms of pattern size

Number of shots in salvo	Average shift of M.P.I. between successive salvos, due only to gun errors
3	$^1/_3$ Pattern
4	$^1/_4$ Pattern
5	$^1/_5$ Pattern
6	$^1/_6$ Pattern
7	$^1/_7$ Pattern
8	$^1/_8$ Pattern
9	$^1/_8$ Pattern
10	$^1/_9$ Pattern
11	$^1/_{10}$ Pattern
12	$^1/_{11}$ Pattern

The ratios in the right-hand column, all in terms of pattern size, are given to the nearest simple fraction.

1507. In actual practice it is found that the mean dispersion of the M.P.I. is greater than accounted for by formula (1503), and hence greater than indicated in the above table. The reason for this is the presence of control errors. In speaking of control errors, it is necessary to distinguish between those that influence the mean dispersion of the guns of the battery and those that operate equally upon the battery as a whole. In a sense, control errors may be considered to include the various sight-setting and pointing errors at pointer fire, or gun-laying errors at director fire, etc. These errors, however, can hardly be separated from the errors of the guns themselves; they are reflected in the mean dispersion of the guns and in the pattern of the battery. The shift of M.P.I. accounted for by (1503) will include these errors and we may consider them as part of the gun errors, rather than as control errors.

But errors in range-keeping, transmission of data to the guns, and, in the case of director fire, director-pointing errors, etc., affect the battery as a whole. These errors will not be reflected in increased pattern sizes, i.e., in increased dispersion among the guns, but in increased dispersion of the M.P.I.s of the salvos fired by these guns. The mean dispersion of M.P.I. that is actually observed is therefore composed of that portion which results from the gun errors themselves and is found from (1503), and of a portion due to the control of the battery as a whole. We will consider the term *control error* to refer to the latter portion.

Effect of control errors on the shift of M.P.I.

It follows, now, that the average *shift* of M.P.I. to be expected in practice depends not altogether upon the mean dispersion of M.P.I. due to gun errors, but upon the mean dispersion of M.P.I. due to both gun and control errors. The values given in the above table therefore represent only a minimum. How much these values will be increased depends upon the size of the control errors to be expected. It remains true, however, that the *average shift of M.P.I.* is equal to $\sqrt{2}$ times the *mean dispersion of the M.P.I.*, and hence, by determining from target practice results what mean dispersion of M.P.I. is to be expected, we may determine what average *shift* of M.P.I. is to be expected. An example will illustrate these features.

Given: A ship carrying a 12-gun battery finds from target practice analysis that the average pattern size is 1000 yards and the mean dispersion of the M.P.I. is 100 yards.

Find: What average shift of M.P.I. is to be expected, in terms of pattern size? What would it be if the mean dispersion of the M.P.I. were 200 yards?

We are given, $D_0 = 100$ yards, and $D_0 = 200$ yards. From (1505) we find,

$$D_{0_d} = 100\sqrt{2} = 141 \text{ yards}$$

and,

$$D_{0_d} = 200\sqrt{2} = 282 \text{ yards}.$$

The ratios to pattern size then are,

$$\frac{141}{1000} = .141, \text{ or about } \frac{1}{7}$$

and

$$\frac{282}{1000} = .282, \text{ or about } \frac{1}{4}(+).$$

In the above example the 1000-yard pattern size indicated a mean dispersion of $\frac{1000}{4.34} = 230$ yards (article 1501). This mean dispersion of guns would cause a mean dispersion of M.P.I. of $\frac{230}{\sqrt{12}} = 66$ yards (formula (1503)), and the average shift of M.P.I. due to this would be $66\sqrt{2} = 93$ yards (formula (1505)), or $\frac{93}{1000} = \frac{1}{11}$ pattern size, which agrees with the table in article 1506. The difference between this ratio and those found in the example will illustrate how materially the average shift of M.P.I. may be increased by the presence of control errors. *And it must be borne in mind that the spotter is not justified in applying corrections for control errors of the accidental type, any more than he is justified in applying corrections for the accidental errors of the gun.*

The application of a high degree of mathematical accuracy to the determination of such ratios as dealt with here, is hardly justifiable, in view of the very rough values with which we have to deal. Such figures as have been introduced in this discussion have been used principally to illustrate general conclusions. The table in article 1506 shows that even with perfect control the M.P.I. may shift considerably, and that the average shift for small salvos is a considerably greater proportion of the pattern size than is the case with large salvos. The example in this article shows that the average shift shown in the table may be increased materially by control errors. These considerations underlie the doctrine, followed by experienced spotters, regarding the least size of "centering spot" that they are justified in making; this is usually defined in terms of some simple fraction of the pattern size, as $\frac{1}{4}$ or $\frac{1}{2}$, etc. What this ratio should be for any typical set of conditions may be determined by the methods outlined above.

1508. Experienced spotters generally adhere to the rule that after the hitting range has once been established, a correction for a subsequent error of the M.P.I should not be applied unless the error occurs at least twice in the same direction. It may be considered that this rule is dictated wholly by common sense, but as a matter of fact it is also well founded in the laws of probability. We know that the spotter must not attempt to apply corrections for the accidental errors of gunfire, which manifest themselves not only in the dispersion of the shots of a salvo but also in the dispersion of the M.P.I.s of successive salvos. We also know that it is

a characteristic of accidental errors that they are as likely to occur in one direction as in the opposite direction and that most likely they will alternate in direction. On the first occurrence of an error, i.e., after the hitting range has once been established, the spotter has no means of determining whether the error is one which should be corrected or whether it is an accidental error. If the error is of such size that it can be accounted for by the natural shifts of the M.P.I., the spotter has every reason to assume that it is indeed due to this cause and that a spot should therefore not be applied for it. But even if the error is much larger than can be accounted for by the natural shifts of the M.P.I., the spotter still is not justified in applying a spot for it on its first occurrence, for it may have been due, for example, to an error by the director pointer which will not be repeated. Now if the *large* error (i.e., the one last mentioned) should occur again, in the same direction, on the following salvo, there would be a strong probability that this error is due to a recurring cause; in this case the spotter would be fully justified in applying a correction for the error on its second occurrence. If the *smaller* error (i.e., the one first mentioned above) should occur again in the same direction on the following salvo, the odds would lie somewhat in favor of applying a spot on the second occurrence, but not so strongly as in the case of the larger error, for two successive *accidental* errors in the same direction can hardly be regarded as a highly improbable occurrence. For this reason some spotters prefer to withhold a spot for an error which is within the limits of the natural shifts of the M.P.I. until the error has occurred three times in the same direction.*

All of the above presumes, of course, that the spotter is entirely unaided by information from which he may infer that shifts of M.P.I. in a certain direction are likely to occur. If changes in conditions which he can observe, such as changes in target course, should indicate that a shift of M.P.I. in a certain direction is likely, the spotter should, by all means, at once apply a correction for an observed shift in that direction.

1509. The reduction of gun dispersion and of control dispersion are distinct problems, involving in the first case the design of good guns and the training of good gun crews, and in the other case the design of good range-keeping systems and the training of good fire-control parties It is desirable, therefore, to have a means for separating into its component parts the total error that is observed. It would involve insurmountable difficulties to allocate to all the contributory sources their respective shares, but it is possible to make a reasonably good subdivision among the broader classifications.

One of the fundamental relations given in the theories of errors is that when an aggregate error is composed of separate and independent component errors, the aggregate error is equal to the square root of the sum of the squares of the component errors.† Thus, for our case, if we let the observed mean dispersion of the M.P.I. (D_{0_0}) be composed of a portion D_{0_g} due to the gun, and a portion D_{0_c} due to the control, we have,

Important

$$D_{0_0} = \sqrt{D_{0_g}^{\,2} + D_{0_c}^{\,2}} \tag{1506}$$

whence we get also,

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* The probability that the error will in any one instance occur in one of two opposite directions is $\frac{1}{2}$. The probability that it will occur n successive times in the same direction is $(\frac{1}{2})^n$. Hence there is one chance in four that it will occur twice in the same direction and only one chance in eight that it will occur three times in the same direction. Ref. §25 and §52, *Probability and Its Engineering Uses*, Fry.

† Ref: Articles 15 and 26, *The Combination of Observations*, Brunt. Also, p. 381, *Handbook of Ballistics*, Cranz and Becker.

$$D_{0_c} = \sqrt{D_{0_0}^2 - D_{0_g}^2} \tag{1507}$$

Now if we analyze a target practice and find a certain mean dispersion for the guns and also a certain mean dispersion of M.P.I., formula (1507) enables us to estimate how great the mean dispersion in control was. An example will best illustrate this.

Given: From target practice results it was determined that the mean dispersion in range of the guns of a certain main battery was 200 yards. The mean dispersion in range of the M.P.I. was found to be 150 yards. The average number of shots per salvo was 11.

Find: The mean dispersion in control.

We are given $D = 200$ yards, and from (1503) we get,

$$D_{0_g}{}^* = \frac{200}{\sqrt{11}} = 60 \text{ yards}.$$

We are given, also, $D_{0_0} = 150$ yards. Substituting in (1507) we get

$$D_{0_c} = \sqrt{150^2 - 60^2} = \underline{137 \text{ yards}}.$$

Whence it appears that the mean dispersion in control was 137 yards.

By means of the same principle we may determine the size of the errors entering into the mean dispersion of the guns on board ship and not accounted for by the mean dispersion measured at the proving ground. For instance:

Determination of components of mean dispersion due to shipboard use of guns

Given: The mean dispersion of a certain gun as measured at the proving ground was found to be 100 yards at a certain range. On board ship a battery of these guns gives a mean dispersion of 150 yards at the same range.

Find: What is the size of the errors incident to the use of the guns on board ship, at that range?

We shall let D_x denote the error to be found. The aggregate error being composed of the proving-ground component (100 yards) and the ship component (D_x), the square root of the sum of the squares of these must equal the aggregate error (150 yards). Whence,

$$\sqrt{D_x{}^2 + 100^2} = 150$$

or

$$D_x = \sqrt{150^2 - 100^2} = \underline{112 \text{ yards}}$$

The result shows that a mean error of 112 yards is caused by factors entering into the use of the guns on board ship, which includes variations in ammunition and loading, pointers' errors, etc., as well as differences among the various guns of the battery (see also article 1406).

1510. Probability of hitting, i.e., the expected percentage of hits, is often thought of only in terms of the mean dispersion of the guns. Percentage of hits deduced only on the basis of gun dispersion is not a true measure of the battle efficiency of a ship or of a fleet. The percentage of hits to be expected depends also upon the distance of the M.P.I. from the center of the hitting space, i.e., upon the *mean error of the M.P.I.* The mean error of the M.P.I. might be found by taking the square root of the sum of the squares of all the separate errors entering into it. This would be a highly theoretical value unless we knew accurately the values of all the separate errors. Actually

Mean error of M.P.I.

* If the value of D used in formula (1503) is the mean dispersion of the gun, the resulting value of D_0 will be the mean dispersion of M.P.I. due only to the gun, or D_{0_g}.

it is far more practical to measure the value of the mean error of the M.P.I. from target practice results, just as we measure gun dispersion by actually firing the gun, rather than by trying to deduce it theoretically. For the purpose of studying the individual causes of the error of the M.P.I., we may break it down into such components as we desire to study (for instance as noted in article 1509). But as far as determination of the probability of hitting is concerned, we are interested only in what the mean error actually amounts to. No other measure of this will be better than that obtained from actual practice.

Target practice reports include a record of the mean error of the M.P.I. from the center of the hitting space, for each ship at each long range practice. These quantities may be averaged over a period of years for a particular ship, or by classes of ships, or even for an entire fleet, depending upon the nature of the problem with which we may be concerned. The quantity so obtained will represent, for the particular ship, class of ships, or fleet, respectively, the success which may be expected in placing the M.P.I. on the target. Let us consider, for instance, a problem of the following type. It is desired to compare the probable effects of fire against ships which are steaming broadside to the line of fire (i.e., with target angles of 90° or 270°), and the probable effects of fire against ships which are steaming *in* the line of fire (i.e., with target angles of 0° or 180°). The answers at which we would arrive would vary considerably depending upon whether we considered the M. P.I. to be at the center of the hitting space, or considered it to be at some average distance from that point. Evidently the true answer would be that which would be arrived at by using a typical mean error of M.P.I., as well as a typical mean dispersion.

The methods of solving such problems as these are illustrated in article 1420, and the problems in article 1421 are of the nature just discussed here. In these problems we found 2.2% to be the probable percentage of hits on a certain target steaming *broadside* to the line of fire, and 0.8% the probable percentage of hits on the same target steaming *in* the line of fire, the M.P.I. having in both cases been considered to be a certain distance away from the center of the hitting space. In article 1418 (problems 2 and 3) the same conditions were assumed, except that the M.P.I. was considered to be at the center of the hitting space; the results were respectively, 5.7% and 3.2%. While the values of mean dispersion and error of M.P.I. assumed were not necessarily typical, the results of these solutions show how misleading the results may be if we neglect, in any study of this nature, to consider the error of the M.P.I.

EXERCISES

1. *Given*: The type of gun, the horizontal range, the number of guns usually fired in salvo, the average pattern in range,* and the average hitting space for the given gun at the given range against its usual target.
   Find: The maximum percentage of hits to be expected from the given battery under the given conditions.

	Given					Answers
	Gun	Range (yds.)	Size of Salvo	Average Pattern* (yds.)	Hitting Space (yds.)	% Hits
A	3″2700 f.s.	3000	4	250	180	58.9%
B	4″2900 f.s.	5000	3	250	180	51.2%
C	5″3150 f.s.	8000	7	400	120	33.9%
D	12″2900 f.s.	20,000	12	1000	60	8.3%
E	16″2600 f.s.	25,000	8	700	70	12.1%

* See article 1418, par. 1.

APPENDIX A*

THE PRACTICAL APPLICATION OF SIACCI'S METHOD TO THE SOLUTION OF THE TRAJECTORY. INGALLS' METHOD.

New Symbols Introduced.†

a, b, a', b', t', z ..... The general form of Ingalls' secondary functions, pertaining to elements at any point in the trajectory (art. 1611).

A, B, A', B', T', Z .... The special form of Ingalls' secondary functions, pertaining to terminal elements (art. 1611).

$a_s, b_s, a_s', b_s', t_s', z_s$ ..... The special form of Ingalls' secondary functions, pertaining to elements at the vertex (art. 1611).

H ..... A special secondary function used only for finding the maximum ordinate (art. 1615).

Q ..... A special secondary function used in connection with the solution for the coefficient of form (art. 1621).

1601. It is to be expected that with the completion of Volume III of the War Department Tables (art. 714), Siacci's Method will be abandoned altogether. In the meantime Siacci's Method continues to be the most convenient method for solving the trajectory for angles of departure not exceeding 15°. The following outline covers the principal features entering into the practical application of Siacci's Method; extracts from the tables required therewith are given at the end of this appendix. A more complete treatment of Siacci's Method may be found in Chapters 5–11 of the 1926 and 1930 editions of *Exterior Ballistics*, and the complete tables required therewith in the 1926 and 1930 editions of *Range and Ballistic Tables*.‡

1602. We have already seen in article 510 how Siacci's range formula was simplified to the form

$$x = C_s(S_u - S_V) \qquad (519)\ (1601)$$

in which S_u, which is called the *space function*, is a tabulated function representing the integral

The space function

$$S_u = -\frac{1}{A}\int \frac{du}{u^{(a-1)}} \qquad (518)\ (1602)$$

Proceeding in similar fashion, other formulas are set up as follows.

1603. The solution of (515) can be simplified by tabulating the values of an *inclination function* I_u which represents the integral

The inclination function

$$I_u = -\frac{2g}{A}\int \frac{du}{u^{(a+1)}}. \qquad (1603)$$

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* It is desirable that the study of this appendix be preceded by a review of Chapter 5.

† The functions A and a introduced here are not to be confused with Mayevski's constant A and exponent a (Chapter 4).

‡ The 1926 and 1930 editions of *Exterior Ballistics* and *Range and Ballistic Tables* are carried in most ships' libraries.

Formula (515), after integration, then becomes

$$\tan \phi - \tan \theta = \frac{C_s}{2 \cos^2 \phi} (- I_V + I_u)$$

whence

$$\tan \theta = \tan \phi - \frac{C_s}{2 \cos^2 \phi} (I_u - I_V). \tag{1604}$$

1604. Substituting in (504) the value of v given by (508), we have

$$dt = - \frac{u \cos \phi}{g} \sec^2 \theta \, d\theta$$

and substituting in the above the value of $\sec^2\theta \, d\theta$ given by (514)

$$dt = - \frac{u \cos \phi}{g} \times \frac{C_s g}{A \cos^2 \phi} \times \frac{du}{u^{(a+1)}}$$

which reduces to

$$dt = - \frac{C_s}{A \cos \phi} \times \frac{du}{u^a}.$$

The integration of the above is expressed as follows

$$\int_0^t dt = - \frac{C_s}{A \cos \phi} \int_V^u \frac{du}{u^a}. \tag{1605}$$

The solution of (1605) is simplified by tabulating the values of a *time function* T_u which represents the integral

The time function

$$T_u = - \frac{1}{A} \int \frac{du}{u^a}.$$

Formula (1605), after integration, then becomes

$$t = \frac{C_s}{\cos \phi} (T_u - T_V). \tag{1606}$$

1605. Formula (1604) can be re-written

$$\tan \theta - \tan \phi = - \frac{C_s}{2 \cos^2 \phi} (I_u - I_V).$$

Multiplying the above by formula (516) we have

$$dx \tan \theta - dx \tan \phi = \frac{C_s}{2 \cos^2 \phi} (I_u - I_V) \times \frac{C_s}{A} \times \frac{du}{u^{(a-1)}}.$$

Since $dx \tan \theta = dy$ we may make this substitution in the above, and we may further rearrange it as follows,

$$dy - dx \tan \phi = \frac{C_s^2}{2 \cos^2 \phi} \left[I_u \left(\frac{1}{A} \times \frac{du}{u^{(a-1)}} \right) - I_V \left(\frac{1}{A} \times \frac{du}{u^{(a-1)}} \right) \right].$$

The integration of the above is expressed as follows,

$$\int_{y}^{0} dy - \int_{x}^{0} dx \tan \phi = \frac{C_s^2}{2 \cos^2 \phi}\left[\frac{1}{A}\int_{u}^{V} I_u \times \frac{du}{u^{(a-1)}} - I_V \times \frac{1}{A}\int_{u}^{V} \frac{du}{u^{(a-1)}}\right]. \quad (1607)$$

The solution of (1607) is simplified by tabulating the values of an *altitude function* A_u which represents the integral

The altitude function

$$A_u = -\frac{1}{A}\int I_u \times \frac{du}{u^{(a-1)}}. \quad (1608)$$

It will be observed that A_u is the negative of the quantity which appears in (1607) as the left-hand member within the bracket; also that the right-hand member within the bracket is equal to $I_V \times S_u$. Formula (1607), after integration, then becomes

$$y - x \tan \phi = -\frac{C_s^2}{2 \cos^2 \phi}\left[A_u - A_V - I_V(S_u - S_V)\right]. \quad (1609)$$

It is to be noted that I_V, which is a definite value of I for the definite value $u = V$, is handled as a constant in the integration of the right-hand member within the bracket; I_u, on the other hand, is a variable value of I and hence enters into the integration.

Dividing (1609) by (1601), and transposing $\tan \phi$, we have

$$\frac{y}{x} = \tan \phi - \frac{C_s}{2 \cos^2 \phi}\left[\frac{A_u - A_V}{S_u - S_V} - I_V\right]. \quad (1610)$$

1606. We have now found a series of expressions which give us useful relations between certain elements of the trajectory, and they are grouped here for convenience:

$$x = C_s(S_u - S_V) \quad (1601)$$

$$\tan \theta = \tan \phi - \frac{C_s}{2 \cos^2 \phi}(I_u - I_V) \quad (1604)$$

Siacci's general ballistic formulas

$$t = \frac{C_s}{\cos \phi}(T_u - T_V) \quad (1606)$$

$$\frac{y}{x} = \tan \phi - \frac{C_s}{2 \cos^2 \phi}\left[\frac{A_u - A_V}{S_u - S_V} - I_V\right] \quad (1610)$$

For finding the value of v corresponding to a given value of u we may rewrite (508),

$$v = u \cos \phi \sec \theta \quad (1611)$$

The above equations are *Siacci's ballistic formulas* and they are the fundamental equations upon which the solutions by Siacci's Method are based.

1607. The values of the *inclination*, *time*, and *altitude functions* have been tabulated in the manner already indicated in article 510 for the *space function*.

Ingalls' Table I

Siacci prepared tables of these functions, but the tables used in the United States were prepared by Ingalls and are generally known as *Ingalls' Table I*; the latter table includes values of S_u, A_u, I_u, and T_u for velocities from 3600 f.s. to 100 f.s.*

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* Further details connected with the tabulation of these functions are given on pp. 80-83, *Exterior Ballistics, 1926*, and pp. 75–78, *Exterior Ballistics, 1930*.

1608. The ballistic formulas listed in article 1606 are expressions for finding the various elements at *any point* in the trajectory. In order to apply them to the point of fall we proceed as follows.

For the point of fall we have,

$$x=X, \quad y=0, \quad t=T, \quad v=v_\omega, \quad u=u_\omega, \quad \theta=-\omega$$

Making the substitutions in (1601) we have

$$X=C_s\,(S_{u_\omega}-S_V)$$

whence

$$S_{u_\omega}=S_V+\frac{X}{C_s} \tag{1612}$$

From (1610) we have,

$$\frac{0}{X} = \tan\phi - \frac{C_s}{2\cos^2\phi}\left[\frac{A_{u\omega}-A_V}{S_{u\omega}-S_V} - I_V\right]$$

whence

$$\tan\phi = \frac{C_s}{2\cos^2\phi}\left[\frac{A_{u\omega}-A_V}{S_{u\omega}-S_V} - I_V\right] \tag{1613A}$$

and since

$$2\cos^2\phi\tan\phi = \frac{2\cos^2\phi\sin\phi}{\cos\phi} = \sin 2\phi$$

we have finally,

$$\sin 2\phi = C_s\left[\frac{A_{u\omega}-A_V}{S_{u\omega}-S_V} - I_V\right] \tag{1613}$$

From (1604) we have,

$$\tan(-\omega) = \tan\phi - \frac{C_s}{2\cos^2\phi}\,[I_{u_\omega}-I_V]$$

Putting for $\tan\phi$ its value from (1613A) we have,

Siacci's ballistic formulas for terminal elements

$$\tan(-\omega) = \frac{C_s}{2\cos^2\phi}\left[\frac{A_{u_\omega}-A_V}{S_{u_\omega}-S_V} - I_V\right] - \frac{C_s}{2\cos^2\phi}\,[I_{u_\omega}-I_V]$$

which simplifies to

$$\tan\omega = \frac{C_s}{2\cos^2\phi}\left[I_{u\omega} - \frac{A_{u\omega}-A_V}{S_{u\omega}-S_V}\right] \tag{1614}$$

From (1606) we have,

$$T=C_s\sec\phi(T_{u_\omega}-T_V) \tag{1615}$$

From (1611) we have,

$$v_\omega=u_\omega\cos\phi\sec\omega \tag{1616}$$

1609. We have in formulas (1612) to (1616) a series of expressions from which the desired terminal elements may be found as follows. Let us suppose that it is

desired to determine the angle of departure that is required for a given range and initial velocity, and also the angle of fall, time of flight, and striking velocity corresponding to the given conditions. X and V are then the given quantities; C can be found as usual from δ, w, i, and d, but the value of C_s which is required for the solution of the formulas depends upon the values of f_a and β (art. 509), and the latter depend, respectively, on the maximum ordinate and angle of departure (arts. 507 and 512), neither of which is known. To overcome this difficulty we may proceed as follows.

Solution of the trajectory by means of Siacci's ballistic formulas

Knowing V we find S_V from *Ingalls' Table I*, and using this in (1612), with C in place of C_s, we get a first approximation of S_{u_ω}, and reentering *Ingalls' Table I* with this value of S_{u_ω} we find u_ω. Knowing u_ω and V we now find from *Ingalls' Table I* the values of A, S, and I required for the solution of (1613), and, still using C in place of C_s, we find from (1613) a first approximation of ϕ. Proceeding similarly, we get from (1615) a first approximation of T; with the latter we can find an approximate value of the maximum ordinate from

$$y_s{}^* = \frac{gT^2}{8}. \qquad (313\ \text{C})$$

With the first approximation of ϕ we can now get a first approximation of β from the relation (see art. 512)

$$\beta = \sqrt{\sec \phi}$$

and using $\frac{2}{3}$ of the approximate value of y_s (see art. 507) we can get a first approximation of f_a from a table in which f_a is tabulated against altitude (an extract from this table is given in art. 1617). The first approximations of β and f_a are now used to get a first approximation of C_s by means of formula (513), and the entire procedure outlined above is repeated, and second approximations of β, f_a, and C_s are obtained. Further approximations are made until two successive values of C_s agree within the chosen limits of accuracy.

The solution of formulas (1612) to (1616) is then completed as follows. Entering *Ingalls' Table I* with the given V, the values of S_V, A_V, I_V, and T_V are found. With the given value of X, the value of C_s found by successive approximations, and the value of S_V found from the table, formula (1612) is solved for S_{u_ω}. Entering the table with the latter, u_ω, A_{u_ω}, I_{u_ω}, and T_{u_ω} are found, and formulas (1613), (1614), (1615), and (1616) are solved for ϕ, ω, T, and v_ω, respectively.

1610. The procedure outlined in the preceding article constitutes a comparatively direct solution of the trajectory by means of Siacci's principles, and it was at one time employed in the computation of U. S. Navy range tables.† This procedure is, however, a laborious one, not only because of the many values that must be taken from *Ingalls' Table I* and of the tedious interpolations involved therein, but also because the formulas themselves are not in convenient form for solution by logarithms.

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* Although this formula is based on the trajectory in vacuum, it gives a fairly good value of the maximum ordinate of the trajectory in air when the value of T used in it is the time of flight in air. However, a somewhat more accurate determination of the maximum ordinate is rendered possible by a further development of Siacci's Method, which will be outlined presently.

† Ref. Chapter VII, *Exterior Ballistics, 1906*, and Chapter 8, *Exterior Ballistics, 1915*, by Professor P. R. Alger.

The idea of further simplifying Siacci's Method by tabulating values of the frequently recurring combinations of the functions S_u, A_u, I_u, and T_u, was suggested as early as 1883 by Braccialini. The development of this idea to a point of practical usefulness is due to Ingalls, and the greatly simplified method resulting from Ingalls' further treatment of Siacci's Method is therefore commonly referred to as *Ingalls' Method*, or as the *Ingalls-Siacci Method*.

1611. Ingalls created a number of *secondary functions* which represent combinations of the *primary functions* S_u, A_u, I_u, and T_u. In the form in which they are applicable to elements at *any* point in the trajectory, these secondary functions are defined by the relations

$$a = \frac{A_u - A_V}{S_u - S_V} - I_V \tag{1617}$$

$$b = I_u - \frac{A_u - A_V}{S_u - S_V} \tag{1618}$$

Ingalls' secondary functions for elements at any point in the trajectory

$$a' = a + b = I_u - I_V \tag{1619}$$

$$t' = T_u - T_V \tag{1620}$$

$$b' = \frac{b}{a} \tag{1621}$$

$$z = \frac{x}{C_s} \tag{1622}$$

In the form in which they are applicable to the *terminal elements*, the same secondary functions are defined by the relations

$$A = \frac{A_{u_\omega} - A_V}{S_{u_\omega} - S_V} - I_V \tag{1617A}$$

$$B = I_{u_\omega} - \frac{A_{u_\omega} - A_V}{S_{u_\omega} - S_V} \tag{1618A}$$

Ingalls' secondary functions for terminal elements

$$A' = A + B = I_{u_\omega} - I_V \tag{1619A}$$

$$T' = T_{u_\omega} - T_V \tag{1620A}$$

$$B' = \frac{B}{A} \tag{1621A}$$

$$Z = \frac{X}{C_s} \tag{1622A}$$

Special forms of the same secondary functions may be denoted by special subscripts. For example, at the vertex, where u is denoted by u_s, we have

Ingalls' secondary functions for elements at the vertex

$$b_s = I_{u_s} - \frac{A_{u_s} - A_V}{S_{u_s} - S_V} \tag{1618B}$$

$$a_s' = a_s + b_s = I_u - I_V \tag{1619B}$$

and others similarly.

It is to be noted that the secondary functions do not involve any new factors, but that they merely represent combinations of the primary functions. The Siacci C (i.e., C_s) is to be used with the secondary functions just as with the primary functions.

1612. The secondary functions are introduced into Siacci's ballistic formulas (art. 1606) as follows.

From (1601) we have,

$$S_u = S_V + \frac{x}{C_s}$$

whence

$$S_u = S_V + z \tag{1623}$$

From (1613) we may write directly,

$$\sin 2\phi = AC_s \tag{1624}$$

for it will be noted that the expression in the bracket is identical with that given for A in (1617A).

From (1604) we may write,

$$\tan\theta = \tan\phi - \frac{C_s a}{2\cos^2\phi} \tag{1625}$$

and from (1610)

$$\frac{y}{x} = \tan\phi - \frac{C_s a}{2\cos^2\phi} \tag{1626}$$

From (1625) we may write

$$\tan\theta = \tan\phi\left(1 - \frac{a'C_s}{2\cos^2\phi\tan\phi}\right)$$

or

$$\tan\theta = \tan\phi\left(1 - \frac{a'C_s}{2\cos^2\phi} \times \frac{\cos\phi}{\sin\phi}\right)$$

whence

$$\tan\theta = \tan\phi\left(1 - \frac{a'C_s}{\sin 2\phi}\right) \tag{1627}$$

From (1626), by the same process, we get

$$\frac{y}{x} = \tan\phi\left(1 - \frac{aC_s}{\sin 2\phi}\right) \tag{1628}$$

Putting in (1628) the value of $\sin 2\phi$ from (1624) we have,

$$\frac{y}{x} = \tan\phi\left(1 - \frac{aC_s}{AC_s}\right)$$

or

$$\frac{y}{x} = \tan\phi\left(1 - \frac{a}{A}\right)$$

whence

$$\frac{y}{x} = \frac{\tan\phi}{A}\left(A - a\right)$$

or

$$y = \frac{\tan\phi}{A}(A-a)x \tag{1629}$$

From (1627), by the same process, we get,

$$\tan\theta = \frac{\tan\phi}{A}(A-a') \tag{1630A}$$

and since $a' = a+b$

$$\tan\theta = \frac{\tan\phi}{A}\left[A-(a+b)\right]$$

or

$$\tan\theta = \frac{\tan\phi}{A}\left[A - a\left(1 + \frac{b}{a}\right)\right]$$

and since $b' = b/a$

$$\tan\theta = \frac{\tan\phi}{A}\left[\, A - a(1+b') \,\right] \qquad (1630)^*$$

From (1606) we may write directly,

$$= C_s t' \sec\phi \qquad (1631)$$

1613. We now have the following formulas from which the elements at *any* point in the trajectory can be found.

$$y = \frac{\tan\phi}{A}(A - a)x \qquad (1629)$$

Ingalls' general ballistic formulas

$$\tan\theta = \frac{\tan\phi}{A}\left[A - a(1 + b')\right] \qquad (1630)$$

$$t = C_s t' \sec\phi \qquad (1631)$$

$$v = u\cos\phi\sec\theta \qquad (1611)$$

1614. In order to solve the above formulas we must know the values of the secondary functions. We have already seen in article 1611 that the secondary functions are merely combinations of the primary functions, and it will be apparent that if we have enough information to find the values of the primary functions we also have enough information to find the values of the secondary functions. The tabulation of the secondary functions was performed by Ingalls, and the tables containing them are generally known as *Ingalls' Table II*.

Ingalls' Table II

An important consideration which enters into the tabulation of the secondary functions is the fact that these functions pertain to an integration between two velocities both of which depend upon the immediate problem at hand. *Ingalls' Table II* is therefore actually a series of tables each based on a certain value of V. In the complete *Ingalls' Table II* there are forty-nine separate tables for as many different values of V, ranging from 825 f.s. to 3600 f.s. The initial velocity V is thus one of the arguments on which *Ingalls' Table II* is based. There must be one more argument, for V pertains only to the initial conditions of the problem. The other argument must be an element pertaining to the point to be found. The element chosen for this purpose is the range x, or abscissa, of the point to be found. It is necessary, however, to base the range argument on certain standards, and this is done by applying to it the ballistic coefficient. The second argument on which *Ingalls' Table II* is based is therefore $\frac{x}{C_s}$, and we have seen in article 1611 that this relation is represented by the secondary function z.

We have seen in article 1611 that z pertains to *any* point in the trajectory, Z to the point of fall, and z_s to the vertex. Consequently if we are dealing with elements at *any* point (x, y) the entering arguments for *Ingalls' Table II* are V

* Although formula (1630A) is actually in more convenient form than formula (1630), the latter renders it unnecessary to tabulate the function a'.

and $z=\dfrac{x}{C_s}$; if we are dealing with terminal elements the arguments are V and $Z=\dfrac{X}{C_s}$; if we are dealing with vertex elements the arguments are V and $z_s=\dfrac{x_s}{C_s}$.

Likewise, if we enter the table with z we will find a, b', t', etc.; if we enter with Z we will find A, B', T', etc.; and if we enter with z_s we will find a_s, b_s', t_s', etc.

1615. The solution for the terminal elements and maximum ordinate is further simplified as follows.

From (1613) and (1617A) we get directly,

$$\sin 2\phi = AC_s. \tag{1632}$$

From (1614) and (1618A) we get,

$$\tan \omega = \frac{BC_s}{2\cos^2 \phi}$$

or

$$\tan \omega = \frac{BC_s}{2\cos^2 \phi} \times \frac{\sin \phi}{\sin \phi}$$

whence, since $2 \sin \phi \cos \phi = \sin 2\phi$, and since $\dfrac{\sin \phi}{\cos \phi} = \tan \phi$, we may write,

$$\tan \omega = \frac{BC_s}{\sin 2\phi} \tan \phi.$$

Substituting for $\sin 2\phi$ its value AC_s from (1632) we have,

$$\tan \omega = \frac{BC_s}{AC_s} \tan \phi$$

and remembering from (1621A) that $B/A = B'$ we have, finally

$$\tan \omega = B' \tan \phi \tag{1633}$$

From (1615) and (1620A) we have,

$$T = C_s T' \sec \phi \tag{1634}$$

From (1616) we have,

$$v_\omega = u_\omega \cos \phi \sec \omega. \tag{1635}$$

In article 1612 we have derived the formula

$$\tan \theta = \frac{\tan \phi}{A}(A - a') \tag{1630 A}$$

which pertains to *any* point in the trajectory. If we apply this formula to the vertex, we have $\tan \theta = 0$ and $a' = a_s'$, while $\tan \phi$ and A are not affected, for ϕ depends only on the initial conditions and A depends on X which is the total range. Rewriting (1630 A) for the vertex we have

$$0 = \frac{\tan \phi}{A}(A - a_s').$$

The quantity $\tan \phi / A$ cannot equal zero, for both ϕ and A have finite values. It follows that $(A - a_s')$ must equal zero, and hence we have

$$A - a_s' = 0$$

or

$$A = a_s'.$$

This means, then, that the a'-function for the vertex, which is a_s', is equal to the a-function for the point of fall, which is A. This is a fortunate coincidence, for it means that if A is known a_s' also is known, and hence z_s and all other secondary functions pertaining to the vertex can be found. However, we are interested principally in the maximum ordinate, and in simplifying the procedure for finding this element.

By substituting a_s' for A in (1629), the latter may then be written for the vertex as follows

$$y_s = \frac{\tan \phi}{a_s'} (a_s' - a_s) x_s .$$

Now from (1619 B) we have the relation $a_s' = a_s + b_s$, whence $b_s = a_s' - a_s$. Making this substitution in the above we get

$$y_s = \frac{b_s x_s}{a_s'} \tan \phi . \tag{1636}$$

We have already seen that if we know A we also know a_s' and hence can find z_s and b_s; knowing z_s we can find x_s from the relation $z_s = \frac{x_s}{C_s}$. In order to find A, however, we must know Z, and in order to find Z we must know X; the solution of (1636) therefore must be based on a given value of X. The latter can be introduced into (1636) as follows

$$y_s = \left[\frac{b_s x_s}{a_s' X}\right] X \tan \phi . \tag{1637}$$

The secondary function H

making it possible to compute the quantity in the bracket for any value of X and to tabulate it. The quantity in the bracket thus defines another secondary function, which is denoted by H, whence we have

$$H = \frac{b_s x_s}{a_s' X} . \tag{1638}$$

Substituting H in (1637) we have

$$y_s = HX \tan \phi . \tag{1639}$$

The function H is found from *Ingalls' Table II* in exactly the same manner in which the functions for the terminal elements are found. That is to say, although H pertains to the vertex it is not to be found with z_s as argument as is the case with other vertex functions (art. 1614). H is, in fact, a combination of vertex functions which, for convenience, has been tabulated in terms of the same argument which pertains to the terminal elements, i.e., $Z = \frac{X}{C_s}$.

1616. We now have the following formulas from which the terminal elements and the maximum ordinate can be found.

Ingalls' ballistic formulas for terminal elements and maximum ordinate

$$\sin 2\phi = AC_s \tag{1632}$$

$$\tan \omega = B' \tan \phi \tag{1633}$$

$$T = C_s T' \sec \phi \tag{1634}$$

$$v_\omega = u_\omega \cos \phi \sec \omega \tag{1635}$$

$$y_s = HX \tan \phi . \tag{1639}$$

Let us suppose that our problem is to find the angle of departure that is required for a given range and initial velocity, and also the angle of fall, time of flight, striking velocity, and maximum ordinate corresponding to the given conditions. This is what we have to do in constructing a range table. V and X are known; from δ, w, i, and d we can compute C. With X and C we can get a first approximation of Z from formula (1622A). With the given value of V and with the first approximation of Z we now enter *Ingalls' Table II* and find first approximations of the secondary functions A and H. With the latter we solve (1632) and (1639) and thus obtain the first approximations of ϕ and y_s, and thence the first approximations of β, f_a, and C_s, as already outlined in article 1609. With the first approximation of C_s we find second approximations of Z, A, and H, and thence of ϕ and β and of y_s and f_a, and eventually of C_s. Further successive approximations are made until two successive values of C_s agree within the chosen limits of accuracy. The final value of Z is then computed by means of formula (1622A), the corresponding values of A, B', T', u_ω, and H are found from *Ingalls' Table II*, and formulas 1632–1635, and 1639, are solved for ϕ, ω, T, v_ω, and y_s, respectively.

Solution of the trajectory by means of Ingalls' ballistic formulas

1617. The practical application of Ingalls' formulas to the solution of trajectories is illustrated in the following problems. The values of the secondary functions required in connection with these problems may be found in the extracts from *Ingalls' Table II* appended to this chapter; these extracts represent about one one-hundredth of the complete table, which includes values of the argument Z from 0 to 20,000 for each of forty-nine values of the argument V.*

The values of f_a required in connection with these problems may be found from the following table, which is an extract from the *altitude-factor table* used with Ingalls' Method; although the complete table includes values of the argument h up to 40,000 feet, the maximum value of h that occurs within the limits of angle of departure to which Ingalls' Method is now restricted, is about 4000 feet.*

Altitude-factor table

$h = \frac{2}{3} y_s$

h (ft.)	f_a	Δ	h (ft.)	f_a	Δ	h (ft.)	f_a	Δ
0	1.0000	.0030	1000	1.0300	.0032	2000	1.0620	.0032
100	1.0030	.0030	1100	1.0332	.0032	2100	1.0652	.0032
200	1.0060	.0030	1200	1.0364	.0032	2200	1.0684	.0032
300	1.0090	.0030	1300	1.0396	.0032	2300	1.0716	.0032
400	1.0120	.0030	1400	1.0428	.0032	2400	1.0748	.0032
500	1.0150	.0030	1500	1.0460	.0032	2500	1.0780	.0032
600	1.0180	.0030	1600	1.0492	.0032	2600	1.0812	.0032
700	1.0210	.0030	1700	1.0524	.0032	2700	1.0844	.0032
800	1.0240	.0030	1800	1.0556	.0032	2800	1.0876	.0032
900	1.0270	.0030	1900	1.0588	.0032	2900	1.0908	.0032
1000	1.0300	.0032	2000	1.0620	.0032	3000	1.0940	.0034

It will be recalled (art. 507) that the altitude factor f_a, as used in Siacci's Method, accounts for the variation of atmospheric density with altitude and is based on a mean density for the entire trajectory; and that the practice in our services, in this connection, is to assume for this mean density the density at an altitude equal to two-thirds of the maximum ordinate of the trajectory. The value of f_a is therefore to be found from the above table by entering the latter with a value of h which is equal to two-thirds of the maximum ordinate, i.e., with $h = 2/3 y_s$.

* The complete *Ingalls' Table II* and *altitude-factor table* are given in *Artillery Circular "M"* (U. S. War Department, 1917). Extracts of *Table II* covering the initial velocities of practically all U. S. naval guns, as well as the complete *altitude-factor table*, are given in the 1926 and 1930 editions of *Range and Ballistic Tables*.

1618. The following example illustrates the process of solving for range-table values of ϕ, ω, T, v_ω, and y_s corresponding to a given range, initial velocity, and projectile.

Solution for other terminal elements when X is known

Given: The 16″2600 f.s. gun, weight of projectile 2100 lbs., coefficient of form .61, horizontal range 10,000 yards, atmospheric conditions standard.

Find: The angle of departure, angle of fall, time of flight, striking velocity, and maximum ordinate corresponding to the given conditions.

The first step is to determine C_s by successive approximations (see arts. 1609 and 1616).

First Approximation

$$C = \frac{w}{\delta i d^2} \tag{406}$$

$w=2100$		log 3.32222
$\delta=1.00$		colog 0.00000
$i=.61$	log 9.78533−10	colog 0.21467
$d^2=256$	log 2.40824	colog 7.59176−10
C		log 1.12865

$$Z_1 = \frac{X}{C} \tag{1622A}$$

$X=30{,}000$ feet		log 4.47712
C	log 1.12865	colog 8.87135−10
$Z_1=2230.8$		log 3.34847

From Ingalls' Table II, with $V=2600$ and $Z=2230.8$

$A_1=.01259+.308\times.00069=.012803$

$H_1=.27322+.308\times.00115=.27357$

$\sin 2\phi_1 = A_1 C$ (1632) $\qquad y_{s_1} = H_1 X \tan \phi_1$ (1639)

$A_1=.012803$	log 8.10731−10		
C	log 1.12865	$H_1=.27358$	log 9.43709−10
$2\phi_1=9°\ 54'\ 51''$	lsin 9.23596−10	$X=30{,}000$ feet	log 4.47712
$\phi_1=4°\ 57'\ 26''$			ltan 8.93820−10
$y_{s_1}=711.8$ feet			log 2.85239

From altitude-factor table, entering with $\frac{2}{3}y_s=475$, $f_{a_1}=1.0142$

$\beta_1=\sqrt{\sec\phi_1}$; 4°57′26″, lsec .00162, $\frac{1}{2}$ lsec .00081

Second Approximation

$$C_{s_1} = \frac{f_{a_1}}{\beta_1} C \tag{513}$$

C		log 1.12865
$f_{a_1}=1.0142$		log .00613
β_1	log .00081	colog 9.99919−10
C_{s_1}		log 1.13397

$$Z_2 = \frac{X}{C_{s_1}}$$

X = 30,000 feet	log 4.47712
C_{s_1} ..... log 1.13397	colog 8.86603 − 10
Z_2 = 2203.7	log 3.34315

From Ingalls' Table II, with $V = 2600$ and $Z = 2203.7$

$A_2 = .01259 + .037 \times .00069 = .012616$

$H_2 = .27322 + .037 \times .00115 = .27326$

$\sin 2\phi_2 = A_2 C_{s_1}$ $\qquad$ $y_{s_2} = H_2 X \tan \phi_2$

A_2 = .012616	log 8.10092 − 10		
C_{s_1}	log 1.13397	H_2 = .27326	log 9.43658 − 10
$2\phi_2$ = 9° 53′ 22″	lsin 9.23489 − 10	X = 30,000 feet ..	log 4.47712
ϕ_2 = 4° 56′ 41″			ltan 8.93712 − 10
y_{s_2} = 709.3 feet			log 2.85082

From altitude-factor table, with $\frac{2}{3} y_s = 473$, $f_{a_2} = 1.0142$

$\beta_2 = \sqrt{\sec \phi_2}$; 4°56′41″, lsec .00162, $\frac{1}{2}$ lsec .00081

Third Approximation

$$C_{s_2} = \frac{f_{a_2}}{\beta_2} C$$

C	log 1.12865
f_{a_2} = 1.0142	log .00613
β_2 ..... log .00081	colog 9.99919 − 10
C_{s_2}	log 1.13397

$$Z_3 = \frac{X}{C_{s_2}}$$

X = 30,000 feet	log 4.47712
C_{s_2} ..... log 1.13397	colog 8.86603 − 10
Z_3 = 2203.7	log 3.34315

From Ingalls' Table II, with $V = 2600$ and $Z = 2203.7$

$A_3 = .01259 + .037 \times .00069 = .012616$

$H_3 = .27322 + .037 \times .00115 = .27326$

$\sin 2\phi_3 = A_3 C_{s_2}$ $\qquad$ $y_{s_3} = H_3 X \tan \phi_3$

A_3 = .012616	log 8.10092 − 10		
C_{s_2}	log 1.13397	H_3 = .27326	log 9.43658 − 10
$2\phi_3$ = 9° 53′ 22″ ...	lsin 9.23489 − 10	X = 30,000 feet ..	log 4.47712
ϕ_3 = 4° 56′ 41″			ltan 8.93709 − 10
y_{s_3} = 709.2 feet			log 2.85079

From altitude-factor table, with $\frac{2}{3} y_s = 473$, $f_{a_3} = 1.0142$

$\beta_3 = \sqrt{\sec \phi_3}$; 4°56′41″, lsec .00162, $\frac{1}{2}$ lsec .00081

Fourth Approximation

$$C_{s_3} = \frac{f_{a_2}}{\beta_3} C$$

It will be noted that $f_{a_3} = f_{a_2}$ and $\beta_3 = \beta_2$, whence it follows that $C_{s_3} = C_{s_2}$; it is therefore unnecessary to continue with the fourth approximation. The final values of C_s, Z, ϕ, and y_s are those which have been arrived at in the third approximation, viz.,

$$\log C_s = 1.13397, \quad Z = 2203.7, \quad \phi = 4^\circ\, 56'\, 41'', \quad y_s = 709.2 \text{ feet}.$$

The solution is now completed as follows:

From Ingalls' Table II, with $V = 2600$ and $Z = 2203.7$

$$\text{Log } B' = .07820 + .037 \times .00370 = .07834$$
$$T' = .969 + .037 \times .051 = .97089$$
$$u_\omega = 1984.4 - .037 \times 25.4 = 1983.5$$

	$\tan \omega = B' \tan \phi$, (1633)	$T = C_s T' \sec \phi$, (1634)	$v_\omega = u_\omega \cos \phi \sec \omega$ (1635)
C_s		log 1.13397	
$\phi = 4°56'41''$	ltan 8.93709−10	lsec 0.00162	lcos 9.99838−10
B'	log 0.07834		
$\omega = 5°54'57''$	ltan 9.01543−10		lsec 0.00232
$T' = .97089$		log 9.98717−10	
$T = 13.267$ sec.		log 1.12276	
$u_\omega = 1983.5$			log 3.29743
$v_\omega = 1986.7$ f.s.			log 3.29813

1619. The method of solving for elements at a point other than the point of fall is illustrated by the following example.

Solution for elements at any point x, y.

Given: The terminal elements of the 10,000-yard trajectory of the 16″2600 f.s. gun, under standard atmospheric conditions, as already found in the example of art. 1618.

Find: The ordinate, angle of inclination, time of flight, and remaining velocity at the point whose abscissa is 2000 yards.

From the example of art. 1618 we have

$$\log C_s = 1.13397, \quad Z = 2203.7 \quad \phi = 4^\circ\, 56'\, 41''.$$

From formula (1622) we have $z = \dfrac{x}{C_s}$

$x = 6000$ feet	log 3.77815
C_s	colog 8.86603−10
$z = 440.74$	log 2.64418

From Ingalls' Table II

With $V = 2600$ and $Z = 2203.7$

$A = .01259 + .037 \times .00069 = .012616$

With $V = 2600$ and $z = 440.74$

$a = .00197 + .4074 \times .00051 = .0021778$

$\log b' = .01380 + .4074 \times .00350 = .01523$

$t' = .158 + .4074 \times .041 = .17470$

$u = 2479.3 - .4074 \times 29.5 = 2467.3$

$$y = \frac{\tan \phi}{A}(A - a)x, \tag{1629}$$

$$\tan \theta = \frac{\tan \phi}{A}[\, A - a(1 + b')\,], \tag{1630}$$

$$t = C_s t' \sec \phi, \tag{1631}$$

$$v = u \cos \phi \sec \theta \tag{1611}$$

$A = .012616$, $a = .0021778$, $(A - a) = .010438$
$\log b' = .01523$, $b' = 1.0357$, $(1 + b') = 2.0357$

$(1+b') = 2.0357$		log .30871
$a = .0021778$		log 7.33802−10
$a(1+b') = .0044333$		log 7.64673−10

$a(1+b') = .0044333$, $A = .012616$, $[A - a(1+b')] = .0081827$

$\phi = 4°56'41''$	ltan 8.93709−10	ltan 8.93709−10	lsec .00162	lcos 9.99838−10
$A = .012616$	colog 1.89908	colog 1.89908		
$(A - a) = .010438$	log 8.01862−10			
$x = 6000$ feet	log 3.77815			
$y = 429.5$ feet	log 2.63294			
$[A - a(1+b')] = .0081827$	log 7.91290−10			
$\theta = 3°12'42''$	ltan 8.74907−10		lsec .00068	
C_s		log 1.13397		
$t' = .17470$		log 9.24229−10		
$t = 2.3872$ sec.		log 0.37788		
$u = 2467.3$				log 3.39222
$v = 2462.0$ f.s.				log 3.39128

1620. The following example illustrates the method of solution when ϕ is given instead of X (compare with the example in art. 1618).

Solution for terminal elements when ϕ is known

Given: The 16″2600 f.s. gun, weight of projectile 2100 lbs., coefficient of form .61, angle of departure 4°56′41″, atmospheric conditions standard.

Find: The horizontal range and maximum ordinate.

As in the example of art. 1618, the first step is to determine C_s by successive approximations; in the present case, however, the successive approximations are required only for the purpose of determining f_a, for β can be found directly from the given value of ϕ.

Let us compute C and then apply β to it and call the result C_{s_1}.

First Approximation

$$C = \frac{w}{\delta i d^2} \tag{406}$$

$w = 2100$			log 3.32222
$\delta = 1.00$			colog 0.00000
$i = .61$	log 9.78533−10		colog 0.21467
$d^2 = 256$	log 2.40824		colog 7.59176−10
C			log 1.12865

$$C_{s_1} = \frac{1}{\beta} C$$

$\beta = \sqrt{\sec \phi}$, $\phi = 4° 56' 41''$, lsec. .00162, $\frac{1}{2}$ lsec .00081

C		log 1.12865
β	log .00081	colog 9.99919−10
C_{s_1}		log 1.12784

From (1632) we get,

$$A_1 = \frac{\sin 2\phi}{C_{s_1}}$$

$\phi = 4° 56' 41''$		
$2\phi = 9° 53' 22''$		lsin 9.23490−10
C_{s_1}	log 1.12784	colog 8.87216−10
$A_1 = .012796$		log 8.10706−10

From Ingalls' Table II, with $V = 2600$ and $A = .012796$

$$Z_1 = 2200 + \frac{.000206}{.00069} \times 100 = 2229.9$$

$$H_1 = .27322 + \frac{.000206}{.00069} \times .00115 = .27356$$

$X_1 = Z_1 C_{s_1}$ (1622A) $y_{s_1} = H_1 X_1 \tan \phi$ (1639)

$Z_1 = 2229.9$	log 3.34828
C_{s_1}	log 1.12784
$X_1 = 29{,}931$ feet	log 4.47612
$H_1 = .27356$	log 9.43706−10
$\phi = 4° 56' 41''$	ltan 8.93709−10
$y_{s_1} = 708.4$ feet	log 2.85027

From altitude-factor table, with $\frac{2}{3} y_s = 472$, $f_{a_1} = 1.0142$

Second Approximation

$$C_{s_2} = \frac{f_{a_1}}{\beta} C \qquad (513)$$

C		log 1.12865
$f_{a_1} = 1.0142$		log .00613
β	log .00081	colog 9.99919−10
C_{s_2}		log 1.13397

$$A_2 = \frac{\sin 2\phi}{C_{s_2}}$$

$2\phi = 9° 53' 22''$		lsin 9.23490−10
C_{s_2}	log 1.13397	colog 8.86603−10
$A_2 = .012616$		log 8.10093−10

From Ingalls' Table II, with $V=2600$ and $A=.012616$

$$Z_2 = 2200 + \frac{.000026}{.00069} \times 100 = 2203.8$$

$$H_2 = .27322 + \frac{.000026}{.00069} \times .00115 = .27326$$

$X_2 = Z_2 C_{s_2}$ $y_{s_2} = H_2 X_2 \tan \phi$

$Z_2 = 2203.8$	log 3.34317
C_{s_2}	log 1.13397
$X_2 = 30{,}001$ feet	log 4.47714
$H_2 = .27326$	log 9.43658
$\phi = 4^\circ\ 56'\ 41''$	ltan 8.93709
$y_{s_2} = 709.3$ feet	log 2.85081

From altitude-factor table, with $\frac{2}{3}\ y_s = 473$, $f_{a_2} = 1.0142$

Third Approximation

$$C_{s_3} = \frac{f_{a_2}}{\beta} C$$

C	log 1.12865
$f_{a_2} = 1.0142$	log .00613
β log .00081	colog 9.99919−10
C_{s_3}	log 1.13397

We now find that $C_{s_3} = C_{s_2}$. Since we have already used this final value of C_s in the second approximation and since there would be no changes in any other quantities, X_2 and y_{s_2} represent the final values of X and y_s.

1621. The considerations involved in the determination of the coefficient of form i from the results of experimental ranging have already been discussed in articles 624–626, and the practical features entering into the determination of i by means of numerical-integration ballistic tables have been treated in detail in articles 802–814. It will be recalled that the process used in connection with the numerical-integration tables is the equivalent of a solution backwards from a measured X in order to find the value of i which satisfies this known value of X. The process used with Ingalls' Method is essentially the same, although less direct. It involves, first of all, the use of an additional secondary function which is denoted by Q.

The function Q is actually the ratio between the ranges in vacuum and in air for conditions that are otherwise identical. It enters into the problem as follows. From formula (308 B) we have

$$\sin 2\phi = \frac{g}{V^2} \times X_{(vacuum)} \qquad (1640)$$

and from (1632),

$$\sin 2\phi = AC_s$$

whence, since $C_s = \dfrac{X_{(air)}}{Z}$, (1622A)

$$\sin 2\phi = \frac{A}{Z} \times X_{(air)} \tag{1641}$$

Since we are using the same conditions for both air and vacuum, $\sin 2\phi$ is the same in both (1640) and (1641); equating the right-hand members of (1640) and (1641) we have,

$$\frac{g}{V^2} \times X_{(vacuum)} = \frac{A}{Z} \times X_{(air)}$$

whence,

The function Q

$$\frac{X_{(vacuum)}}{X_{(air)}} = Q = \frac{AV^2}{gZ} \tag{1642}$$

With (1642) we may compute Q for any given values of V and Z, and tabulate it with the other functions on the proper page for V and in the proper line for Z. The quantity tabulated is log Q. For instance, suppose that we have $V = 2600$ and $Z = 5000$; we find the corresponding value of A to be 0.03762. Then Q may be computed from (1642),

$$Q = \frac{.03762 \times (2600)^2}{32.16 \times 5000} = 1.5815$$

$$\log Q = .19907$$

which, we see, is the value tabulated in the 2600 f. s. table for $Z = 5000$.

It will be apparent, now, that *if we know V and Q we may find the corresponding value of Z*, just as we might find Z with V and any other function. In the experimental determination of i we use this feature. Q is the ratio between range in vacuum and range in air for conditions otherwise identical. If we fire a gun with a known elevation and velocity we can easily determine the corresponding range in vacuum from (308),

$$X_{(vacuum)} = \frac{V^2 \sin 2\phi}{g}$$

The value of $X_{(air)}$ is actually measured. The value of Q is found from the relation

$$Q = \frac{X_{(vacuum)}}{X_{(air)}} = \frac{V^2 \sin 2\phi}{gX_{(air)}}. \tag{1643}$$

Ingalls' Table II is now entered with the measured V, and with log Q as found from (1643), and the corresponding value of Z is found. With the latter, C_s is determined from the relation

$$C_s = \frac{X_{(air)}}{Z}$$

and i is then deduced from the relations

$$C_s = \frac{f_a}{\beta} C \tag{513}$$

whence

$$C_s = \frac{f_a w}{\beta \delta i d^2}$$

and

$$i = \frac{f_a w}{\beta \delta C_s d^2}. \qquad (1644)$$

1622. The following is an example of the solution for i by Ingalls' Method.

Determination of i in Ingalls' Method

Given: In the experimental ranging of the 16″2600 f.s. gun the following average results were obtained for a group of seven shots fired at angle of departure 8°: uncorrected observed range, 14,773 yards; observed initial velocity, 2580 f.s.; observed weight of projectiles 2102 lbs.; observed ballistic density, .962. The corrections to be applied to the observed range were found to be: for ballistic wind, (−)31 yards; for height of gun, (−)39 yards; for curvature of the earth, (−) 87 yards.

Find: The value of i that corresponds to the given conditions.

The corrected observed range is

$$X = 14,773 - 31 - 39 - 87 = 14,616 \text{ yards} = 43,848 \text{ feet}.$$

$$Q = \frac{V^2 \sin 2\phi}{gX} \qquad (1643)$$

V = 2580 f.s.	log 3.41162	2 log 6.82324
$\phi = 8°, 2\phi = 16°$		lsin 9.44034 − 10
g = 32.16 f.s.s.	log 1.50732	colog 8.49268 − 10
X = 43,848 feet	log 4.64195	colog 5.35805 − 10
Q,		log 0.11431

From Ingalls' Table II, with $V = 2580$ and $\log Q = 0.11431$.

For 2500 f. s.

$$Z = 3000 + \frac{.00076}{.00417} \times 100 = 3018.2$$

$$H = .28326 + \frac{.00076}{.00417} \times .00123 = .28348$$

For 2600 f. s.

$$Z = 3000 + \frac{.00211}{.00411} \times 100 = 3051.3$$

$$H = .28254 + \frac{.00211}{.00411} \times .00121 = .28316$$

For 2580 f. s.

$Z = 3018.2 + .80 \times 33.1 = 3044.7$

$H = .28348 - .80 \times .00032 = .28322$

$$C_s = \frac{X}{Z}$$

X = 43,848 feet		log 4.64195
Z = 3044.7	log 3.48354	colog 6.51646−10
C_s		log 1.15841

$$y_s = HX \tan \phi \qquad (1639)$$

H = .28322		log 9.45212−10
X = 43,848 feet		log 4.64195
ϕ = 8°		ltan 9.14780−10
y_s = 1745.3 feet		log 3.24187

whence, from the altitude-factor table, with 2/3 y_s = 1164, f_a = 1.035. Also

$$\beta = \sqrt{\sec 8°}, \text{ or } \log \beta = 0.00212.$$

The solution is now completed as follows:

$$i = \frac{f_a w}{\beta \delta C_s d^2} \qquad (1644)$$

f_a = 1.035		log 0.01494
w = 2102		log 3.32263
β	log 0.00212	colog 9.99788−10
δ_b = .962	log 9.98318−10	colog 0.01682
C_s	log 1.15841	colog 8.84159−10
d^2 = 256	log 2.40824	colog 7.59176−10
i = .61041		log 9.78562−10

1623. The process illustrated in the foregoing example can also be used to determine the value of i that corresponds to range-table values. The following example will make this clear.

Determination of i corresponding to range-table values

Given: In the 16″2600 f.s. range table the angle of departure and maximum ordinate for the 10,000-yard trajectory are, respectively, 4°56′.8 and 709 feet.

Find: The value of i, according to Ingalls' Method, that corresponds to the given conditions.

$$Q = \frac{V^2 \sin 2\phi}{gX} \qquad (1643)$$

V = 2600 f.s.	log 3.41497	2 log 6.82994
2ϕ = 9°53′.6		lsin 9.23506−10
g = 32.16 f.s.s.	log 1.50732	colog 8.49268−10
X = 30,000 feet	log 4.47712	colog 5.52288−10
Q		log 0.08056

From Ingalls' Table II, with V = 2600 and log Q = 0.08056

$$Z = 2200 + \frac{.00027}{.00390} \times 100 = 2206.9$$

$$C_s = \frac{X}{Z}$$

$X = 30{,}000$ feet		log 4.47712
$Z = 2206.9$	log 3.34378	colog 6.65622 − 10
C_s		log 1.13334

From the altitude-factor table, with $2/3\ y_s = 473$, $f_a = 1.0142$.
Also,

$$\beta = \sqrt{\sec 4° 56'.8}, \text{ or } \log \beta = 0.00081$$

$$i = \frac{f_a w}{\beta \delta C_s d^2} \tag{1644}$$

$f_a = 1.0142$		log 0.00613
$w = 2100$		log 3.32222
β	log 0.00081	colog 9.99919 − 10
$\delta = 1.00$	log 0.00000	colog 0.00000
C_s	log 1.13334	colog 8.86666 − 10
$d^2 = 256$	log 2.40824	colog 7.59176 − 10
$i = .61088$		log 9.78596 − 10

TABLE II. $V = 2{,}500$ f. s.

$Z = \frac{X}{C}$	A	Δ	H	Δ	$\log B'$	Δ	u	Δ	T'	Δ	$\log Q$	Δ
0	0.00000	52	0.25000	110	0.00000	350	2500.0	296	0.000	40	0.00000	339
100	0.00052	53	0.25110	109	0.00350	350	2470.4	294	0.040	41	0.00339	342
200	0.00105	53	0.25219	108	0.00700	350	2441.0	292	0.081	41	0.00681	345
300	0.00158	54	0.25327	108	0.01050	350	2411.8	290	0.122	42	0.01026	348
400	0.00212	55	0.25435	107	0.01400	350	2382.8	287	0.164	42	0.01374	351
500	0.00267	57	0.25542	106	0.01750	350	2354.1	285	0.206	43	0.01725	353
600	0.00324	57	0.25648	106	0.02100	352	2325.6	282	0.249	43	0.02078	356
700	0.00381	58	0.25754	105	0.02452	354	2297.4	280	0.292	44	0.02434	359
800	0.00439	59	0.25859	105	0.02806	356	2269.4	278	0.336	44	0.02793	362
900	0.00498	60	0.25964	104	0.03162	358	2241.6	276	0.380	45	0.03155	364
1000	0.00558	61	0.26068	104	0.03520	360	2214.0	274	0.425	45	0.03519	368
1100	0.00619	62	0.26172	104	0.03880	361	2186.6	271	0.470	46	0.03887	371
1200	0.00681	63	0.26276	104	0.04241	361	2159.5	269	0.516	47	0.04258	373
1300	0.00744	64	0.26380	104	0.04602	362	2132.6	266	0.563	47	0.04631	376
1400	0.00808	66	0.26484	105	0.04964	363	2106.0	264	0.610	48	0.05007	379
1500	0.00874	66	0.26589	106	0.05327	364	2079.6	262	0.658	48	0.05386	381
1600	0.00940	68	0.26795	107	0.05691	365	2053.4	260	0.706	49	0.05767	384
1700	0.01008	69	0.26802	109	0.06056	366	2027.4	257	0.755	50	0.06151	386
1800	0.01077	70	0.26911	111	0.06422	368	2001.7	255	0.805	50	0.06537	389
1900	0.01147	71	0.27022	113	0 06790	370	1976.2	252	0.855	51	0.06926	392
2000	0.01218	72	0.27135	115	0.07160	372	1951.0	251	0.906	52	0.07318	393
2100	0.01290	74	0.27250	116	0.07532	374	1925.9	248	0.958	52	0.07711	395
2200	0.01364	75	0.27366	117	0.07906	376	1901.1	245	1.010	53	0.08106	398
2300	0.01439	77	0.27483	117	0.08282	378	1876.6	243	1.063	54	0.08504	400
2400	0.01516	78	0.27600	119	0.08660	380	1852.3	240	1.117	54	0.08904	403
2500	0.01594	79	0.27719	120	0.09040	380	1828.3	238	1.171	55	0.09307	404
2600	0.01673	81	0.27839	120	0.09420	380	1804.5	235	1.226	56	0.09711	408
2700	0.01754	82	0.27959	122	0.09800	380	1781.0	233	1.282	57	0.10119	409
2800	0.01836	84	0.28081	122	0.10180	380	1757.7	230	1.339	57	0.10528	413
2900	0.01920	85	0.28203	123	0.10560	380	1734.7	227	1.396	58	0.10941	414
3000	0.02005	87	0.28326	123	0.10940	380	1712.0	223	1.454	59	0.11355	417
3100	0.02092	88	0.28449	123	0.11320	380	1689.7	221	1.513	59	0.11772	420
3200	0.02180	90	0.28572	123	0.11700	380	1667.6	218	1.572	60	0.12192	422
3300	0.02270	92	0.28695	124	0.12080	381	1645.8	215	1.632	61	0.12614	424
3400	0.02362	94	0.28819	125	0.12461	382	1624.3	212	1.693	62	0.13038	427
3500	0.02456	95	0.28944	125	0.12843	382	1603.1	209	1.755	63	0.13465	428
3600	0.02551	97	0.29069	126	0.13225	382	1582.2	206	1.818	64	0.13893	431
3700	0.02648	99	0.29195	126	0.13607	382	1561.6	204	1.882	64	0.14324	434
3800	0.02747	101	0.29321	127	0.13989	381	1541.2	202	1.946	66	0.14758	436
3900	0.02848	102	0.29448	127	0.14370	380	1521.0	200	2.012	66	0.15194	438
4000	0.02950	104	0.29575	128	0.14750	379	1501.0	198	2.078	67	0.15632	441
4100	0.03054	107	0.29703	128	0.15129	378	1481.2	196	2.145	68	0.16073	444
4200	0.03161	109	0.29831	128	0.15507	377	1461.6	193	2.213	68	0.16517	446
4300	0.03270	111	0.29959	129	0.15884	376	1442.3	189	2.281	70	0.16963	448
4400	0.03381	113	0.30088	130	0.16260	376	1423.4	186	2.351	71	0.17411	450
4500	0.03494	115	0.30218	129	0.16636	377	1404.8	183	2.422	72	0.17861	452
4600	0.03609	117	0.30347	130	0.17013	377	1386.5	179	2.494	73	0.18313	454
4700	0.03726	119	0.30477	131	0.17390	377	1368.6	175	2.567	73	0.18767	456
4800	0.03845	122	0.30608	131	0.17767	377	1351.1	172	2.640	74	0.19223	458
4900	0.03967	124	0.30739	131	0.18144	376	1333.9	169	2.714	76	0.19681	461
5000	0.04091	126	0.30870	131	0.18520	373	1317.0	165	2.790	76	0.20142	463

TABLE II. $V = 2{,}600$ f. s.

$Z = \frac{X}{C}$	A	Δ	H	Δ	$\log B'$	Δ	u	Δ	T'	Δ	$\log Q$	Δ
0	0.00000	48	0.25000	91	0.00000	340	2600.0	306	0.000	39	0.00000	344
100	0.00048	49	0.25091	92	0.00340	344	2569.4	303	0.039	39	0.00344	346
200	0.00097	49	0.25183	94	0.00684	347	2539.1	300	0.078	40	0.00690	347
300	0.00146	51	0.25277	95	0.01031	349	2509.1	298	0.118	40	0.01037	349
400	0.00197	51	0.25372	97	0.01380	350	2479.3	295	0.158	41	0.01386	352
500	0.00248	52	0.25469	98	0.01730	350	2449.8	293	0.199	41	0.01738	354
600	0.00300	53	0.25567	100	0.02080	350	2420.5	290	0.240	41	0.02092	356
700	0.00353	53	0.25667	101	0.02430	350	2391.5	288	0.281	42	0.02448	357
800	0.00406	55	0.25768	102	0.02780	350	2362.7	285	0.323	43	0.02805	360
900	0.00461	55	0.25870	104	0.03130	350	2334.2	282	0.366	43	0.03165	362
1000	0.00516	56	0.25974	107	0.03480	351	2306.0	279	0.409	44	0.03527	363
1100	0.00572	57	0.26081	109	0.03831	353	2278.1	277	0.453	44	0.03890	365
1200	0.00629	59	0.26190	110	0.04184	355	2250.4	275	0.497	44	0.04255	367
1300	0.00688	59	0.26300	110	0.04539	357	2222.9	273	0.541	45	0.04622	370
1400	0.00747	60	0.26410	111	0.04896	359	2195.6	271	0.586	46	0.04992	372
1500	0.00807	62	0.26521	112	0.05255	361	2168.5	269	0.632	47	0.05364	374
1600	0.00869	62	0.26636	113	0.05616	363	2141.6	267	0.679	47	0.05738	376
1700	0.00931	63	0.26746	114	0.05979	365	2114.9	265	0.726	47	0.06114	378
1800	0.00994	65	0.26860	116	0.06344	367	2088.4	263	0.773	48	0.06492	381
1900	0.01059	65	0.26976	116	0.06711	369	2062.1	261	0.821	49	0.06873	383
2000	0.01124	67	0.27092	115	0.07080	370	2036.0	259	0.870	49	0.07256	385
2100	0.01191	68	0.27207	115	0.07450	370	2010.1	257	0.919	50	0.07641	388
2200	0.01259	69	0.27322	115	0.07820	370	1984.4	254	0.969	51	0.08029	390
2300	0.01328	71	0.27437	114	0.08190	371	1959.0	251	1.020	51	0.08419	393
2400	0.01399	71	0.27551	115	0.08561	372	1933.9	249	1.071	52	0.08812	395
2500	0.01470	73	0.27666	116	0.08933	373	1909.0	246	1.123	53	0.09207	398
2600	0.01543	75	0.27782	117	0.09306	373	1884.4	243	1.176	54	0.09605	400
2700	0.01618	75	0.27899	118	0.09679	375	1860.1	240	1.230	54	0.10005	402
2800	0.01693	77	0.28017	118	0.10054	377	1836.1	237	1.284	55	0.10407	405
2900	0.01770	78	0.28135	119	0.10431	379	1812.4	234	1.339	55	0.10812	408
3000	0.01848	80	0.28254	121	0.10810	380	1789.0	231	1.394	56	0.11220	411
3100	0.01928	81	0.28375	122	0.11190	380	1765.9	229	1.450	57	0.11631	413
3200	0.02009	82	0.28497	123	0.11570	380	1743.0	226	1.507	58	0.12044	415
3300	0.02091	85	0.28620	123	0.11950	381	1720.4	224	1.565	59	0.12459	418
3400	0.02176	86	0.28743	124	0.12331	382	1698.0	222	1.624	59	0.12877	421
3500	0.02262	87	0.28867	125	0.12713	383	1675.8	220	1.683	60	0.13298	423
3600	0.02349	89	0.28992	126	0.13096	383	1653.8	218	1.743	61	0.13721	425
3700	0.02438	91	0.29118	126	0.13479	383	1632.0	216	1.804	62	0.14146	428
3800	0.02529	92	0.29244	127	0.13862	384	1610.4	213	1.866	63	0.14574	431
3900	0.02621	94	0.29371	128	0.14246	384	1589.1	211	1.929	63	0.15005	433
4000	0.02715	96	0.29499	128	0.14630	384	1568.0	209	1.992	64	0.15438	436
4100	0.02811	97	0.29627	128	0.15014	384	1547.1	206	2.056	65	0.15874	439
4200	0.02908	100	0.29755	129	0.15398	383	1526.5	202	2.121	66	0.16313	441
4300	0.03008	102	0.29884	129	0.15781	382	1506.3	198	2.187	67	0.16754	443
4400	0.03110	103	0.30013	130	0.16163	381	1486.5	195	2.254	67	0.17197	446
4500	0.03213	106	0.30143	130	0.16544	379	1467.0	191	2.321	69	0.17643	448
4600	0.03319	108	0.30273	131	0.16923	377	1447.9	188	2.390	70	0.18091	451
4700	0.03427	110	0.30404	131	0.17300	375	1429.1	184	2.460	70	0.18542	453
4800	0.03537	111	0.30535	132	0.17675	373	1410.7	181	2.530	72	0.18995	455
4900	0.03648	114	0.30667	132	0.18048	372	1392.6	179	2.602	72	0.19450	457
5000	0.03762	115	0.30799	134	0.18420	370	1374.7	177	2.674	73	0.19907	460

APPENDIX B

THE LIMITATIONS OF COLUMNS 7 AND 19 AT VERY SHORT RANGES. DETERMINATION OF HITTING SPACE AT VERY SHORT RANGES.

1701. It is a common error to make direct use of Column 7 of the range table for finding how far beyond a target the point of fall of a shot should occur if the shot pierces the vertical plane of the target at a given height above the water. Column 7 is not designed to give this information and at very short ranges large errors result from using it directly for this purpose. This is illustrated by the following example. Let us suppose that in Figure 50 OMH represents the 2500-yard trajectory of the 16″2600 f.s. gun, and that TM is a 20-foot target so located that the trajectory OMH just touches the top of the target. By definition (art. 914), the distance TH is the *danger space* for the 2500-yard trajectory OMH and the 20-foot target TM. From Column 7 of the 16″2600 f.s. range table we find that at the range 2500 yards the danger space of a 20-foot target is equal to 387 yards.

Limitations of Column 7 for determining hitting space at very short ranges

In Figure 50 we then have $OH = 2500$ yards, $TH = 387$ yards, and $OT = 2500 - 387 = 2113$ yards. It is to be noted, however, that the distance $TH = 387$ yards was found from Column 7 of the range table by entering the

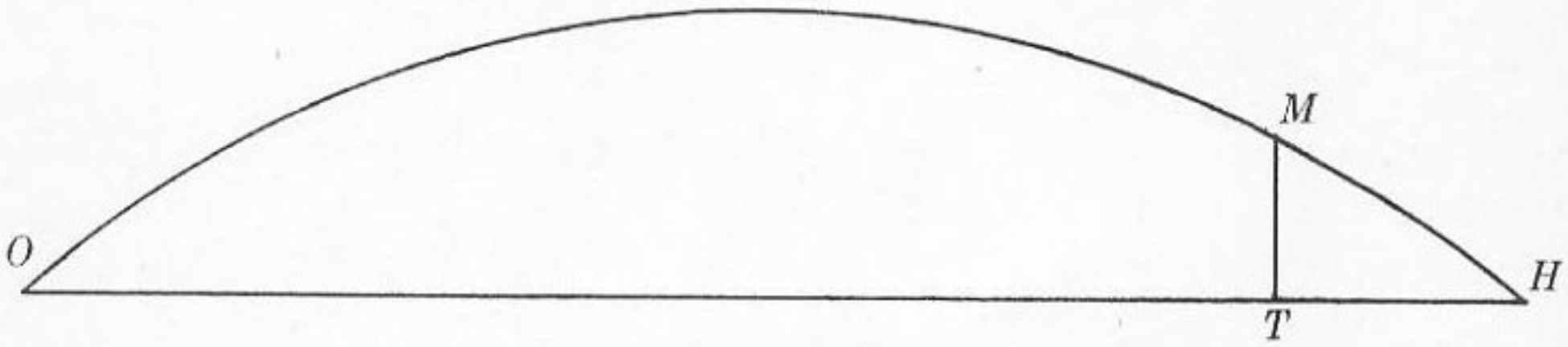


FIGURE 50

latter with 2500 yards, which is the distance from the gun to the point of fall of the trajectory that just touches the top of the target, and not by entering the range table with 2113 yards, which is the distance from the gun to the target. If we should seek to find the distance TH from Column 7 by entering the range table with the *target distance* 2113 yards, we would find the value 489 yards, which we already know is not correct. By definition (art. 1402 (h)), the distance TH in Figure 50 is the *hitting space* for the target TM at the *target distance* OT. For the case illustrated, $TH = 387$ yards is then the hitting space for the 20-foot target at the *target distance* 2113 yards, as compared to the danger space of 489 yards which is found from Column 7 for the same target at the same *target distance.*

The problem of determining the relation between points of impact in the vertical plane of the target and on the surface of the water enters into the construction of spotting diagrams. We have seen above that, with respect to a given *target distance,* this relation is defined by the quantity known as the hitting space rather than by the quantity known as the danger space, and also that at the short

range considered these quantities differ materially. It can be shown that for ranges exceeding about 5000 yards the distinction between danger space and hitting space ceases to have any practical significance; at such ranges Column 7 may be used directly for finding either of these quantities. The particular problem to be dealt with here is the development of a convenient method for determining the hitting space for ranges which occur at Short Range Practice (1600–2100 yards).

1702. One method of determining the hitting space is to deduce it from the danger space in the following manner. In Figure 50, TH is evidently the danger space at range OH and also the hitting space at range OT. Thus if S is the danger space at any given range X, then $S' = S$ is the hitting space at the range $X - S$.* However, the determination of the hitting space from this relation involves an indirect and inconvenient process. Moreover, the values appearing in Column 7 of many of the range tables still in use at the time of this writing were computed by a now obsolete method (see note at bottom of page 119); at short ranges the value of danger space found by this method may be in error by more than 25%, and any values of hitting space deduced from the danger space are subject to the errors of the latter. These undesirable features of the indirect method outlined above are avoided in the following approximate solution for the hitting space which is direct, simple, and sufficiently accurate for any practical purpose.

1703. In Figure 51 a gun is located at O and a target of height h is located at the range $OH = X$. The hitting space is $S' = HH'$.

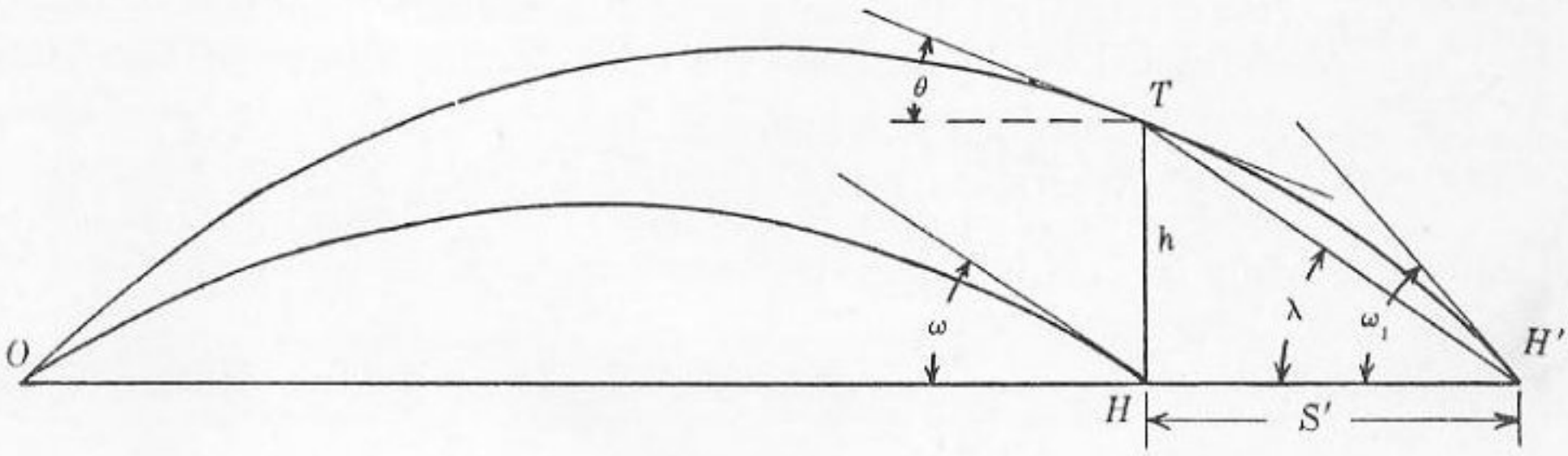


FIGURE 51

The value of S' is given exactly by the relatioı $S' = h \cot \lambda$, in which λ is the angle THH'. This angle is not a known quantity, but it will be observed (Figure 51) that it lies between the angle of fall ω_1 of the trajectory OTH' which just touches the top of the target, and the angle of inclination θ of the same trajectory at the point T where the latter touches the top of the target. The angle of fall ω of the trajectory having the horizontal range OH also lies between the angles ω_1 and θ. Hence it appears that ω is probably very nearly equal to λ, and that the hitting

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* This is, in fact, simply the reverse of the process now used for determining the danger space for short ranges (arts. 917–918). That is to say, the latter process actually determines the hitting space first and then deduces from it the danger space. There is no good reason for not tabulating hitting space in Column 7 in the first place, for at short ranges danger space is an altogether useless quantity, and at long ranges it is useful only by virtue of the fact that at such ranges it is practically equal to hitting space. But the fact that danger space is, nevertheless, tabulated in all of our range tables, makes it necessary to engage in this special study of how to convert it into a useful quantity for short-range problems.

Formula for hitting space at very short ranges

space S' is defined approximately by the relation

$$S' = h \cot \omega \tag{1701}$$

in which ω is the angle of fall corresponding to the target distance X.

Although no rigorous proof of equality between the angles ω and λ (as illustrated in Figure 51) is available, it has been established by exhaustive comparative solutions that these angles are indeed very nearly equal in any practical situation. That is to say, solutions for the hitting space according to the approximate formula $S' = h \cot \omega$ and according to more accurate formulas or processes,* have been compared for a great variety of assumed ranges and target heights, with the result that even at the shortest ranges and for the greatest target heights that are likely to occur in practice, the maximum error of the approximate formula has been found to be about 1%.

1704. We have found in article 921 that Column 19 is computed from the formula

$$\Delta h = \Delta X \tan \omega \tag{907}$$

whence

$$\Delta X = \Delta h \cot \omega.$$

Thus Column 19 gives the height of target which corresponds to a hitting space of 100 yards, and if we divide the given height of target by the value from Column 19 and multiply the result by 100, we will have the hitting space for the given target. But the usefulness of Column 19 in this connection is limited by the fact that the values in this column are tabulated only to the nearest whole foot and hence may be in error by as much as $(\pm)1/2$ foot. For example, in the 4″2900 f.s. range table we find that the value in Column 19 at 1700 yards is 3 feet. Since this may actually be 3.49 feet, the error resulting from the use of the value as tabulated may be as great as $\frac{.49}{3}$, or about 16%. The following example shows how great an error can arise from this cause. At the range 1700 yards for the 4″ 2900 f.s. gun the angle of fall equals 0°44′; according to formula (1701), the correct hitting space for a 20-foot target for this gun at 1700 yards is then

Limitations of Column 19 for determining hitting space at very short ranges

$h = 20$ feet	log 1.30103
$\omega = 0°44'$	lcot 1.89280
$S' = 1563$ feet	log 3.19383
$= 521$ yards	

But by using Column 19 we get

$$\frac{20}{3} \times 100 = 667 \text{ yards}.$$

According to the information obtained by means of Column 19 the spotter for a destroyer at Short Range Practice might then expect that a shot which landed 667 yards behind the 20-foot target should have passed through the top edge

* An accurate solution for the hitting space can be made by solving the trajectory for the abscissa x which corresponds to the ordinate $y = h$. This can be accomplished either by numerical integration or by means of Ingalls' formula (1629).

of the target, whereas, in fact, such a shot would actually have passed well over the top of the target and have landed 146 yards beyond the limit of the hitting space of the 20-foot target. The unsuitability of Column 19 for the determination of the hitting space for short ranges is further illustrated by the following example. In the 4″2900 f.s. range table the value 3 feet appears in Column 19 for ranges from 1300 yards to 1700 yards; hence by use of this column we would get the same hitting space for any range from 1300 yards to 1700 yards. According to formula (1701) we find, however, that the hitting space for a 20-foot target actually varies from 716 yards at the range 1300 yards, to 521 yards at the range 1700 yards. The above examples make it clear that the hitting space for short ranges should always be determined by means of formula (1701).

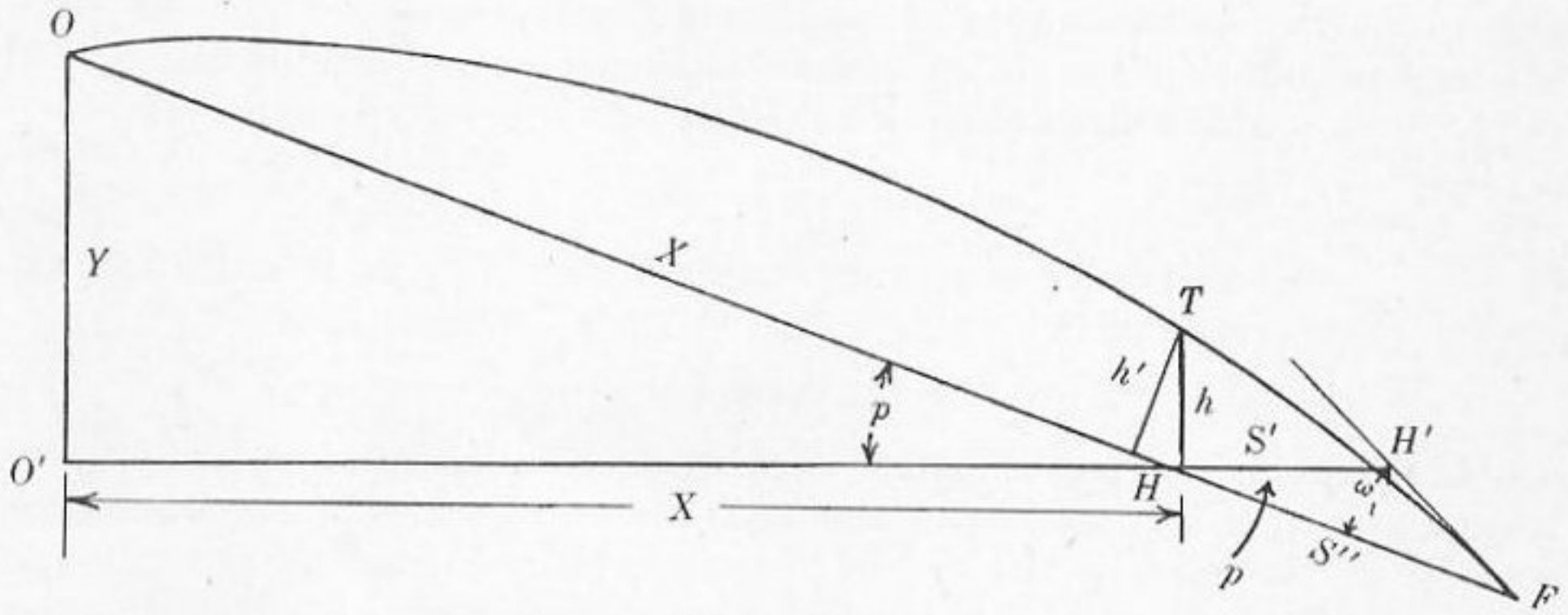


FIGURE 52

Effect of angle of position on hitting space

1705. At very short ranges the hitting space is affected materially by the angle of position, making it necessary to perform further operations on the value of hitting space obtained from formula (1701).

In Figure 52 the position of the gun is at O, at a height Y feet above the surface of the water, and the target is resting on the surface of the water at H; the angle of position is $p = \tan^{-1}\frac{Y}{X}$. In any practical case the angle of position is so small that no distinction need be made between the inclined distance OH and the horizontal distance $O'H$; for example, with the gun as high as 60 feet and the range as short as 1600 yards, the difference between OH and $O'H$ is only about 0.1 yard. Hence we may consider OH to be equal to the horizontal target distance X. Also, the difference between the vertical height h of the target and its perpendicular height h' on the line of position, is negligible; with the gun as high as 60 feet and the range as short as 1600 yards, the difference between h and h', in the case of a 20-foot target, is only about .001 foot.

The hitting space in Figure 52 is $S' = HH'$, H' being the point where the trajectory $OTH'F$ intersects the plane of the water. But if ω is the angle of fall for the range $X = OH$, then the formula $h \cot \omega$ gives, in this case, not the actual hitting space $S' = HH'$, but an imaginary hitting space $S'' = HF$ measured along the line of position OHF. We shall call this imaginary hitting space S'' the *inclined hitting space.* In order to determine the actual hitting space, S', which is measured on the plane of the water, we proceed as follows. Having found S'' from the relation $S'' = h \cot \omega$, we add S'' to X, thus obtaining the range $X + S'' = OF$.

With this new range we find the angle of fall, ω_1, from the range table. Considering the arc $H'F$ to be a straight line, which may be done without material error in any practical case, we have the triangle $H\,H'F$ in which the side $S'' = HF$ and the two angles, p and ω_1, are known. The triangle can now be solved for the side $S' = H\,H'$ by means of the trigonometric formula

$$S' = S'' \sin \omega_1 \operatorname{cosec} (\omega_1 + p). \tag{1702}$$

The procedure for finding the hitting space S' of a target of height h at a horizontal range X, when the gun is elevated Y feet above the water and the target is resting on the water, is then as follows.

Determination of hitting space when gun is above target and target rests on the water

(a) $$S'' = h \cot \omega, \quad \tan p = \frac{Y}{X}$$

in which S'' is the inclined hitting space and ω is the angle of fall at the given range X to the target.

(b) $$X_1 = X + S''$$

and with X_1 the corresponding angle of fall, ω_1, is found from the range table.

(c) $$S' = S'' \sin \omega_1 \operatorname{cosec} (\omega_1 + p).$$

1706. At Short Range Practice the target screen is spread so that its bottom edge is from four to six feet above the water, and in this case the hitting space is not bounded by the foot of the target. This situation is illustrated in Figure 53. The bottom edge of the target is now d feet above the water; other features are the same as in Figure 52. The hitting space S' of the target h is, in this case, evidently equal to the difference between the hitting spaces for target heights equal to d and $(h+d)$, respectively. If the hitting space for the target height d is denoted by S'_1 and the hitting space for the target height $(h+d)$ is denoted by S'_2, the hitting space for the target h is then equal to $S' = S'_2 - S'_1$. The procedure for finding S'_1 and S'_2 is similar to that outlined in the preceding article.

The complete procedure for finding the hitting space S' when both the gun and the target are elevated above the surface of the water, is outlined under Figure 53; the meanings of the symbols used in this outline are clearly illustrated in Figure 53.

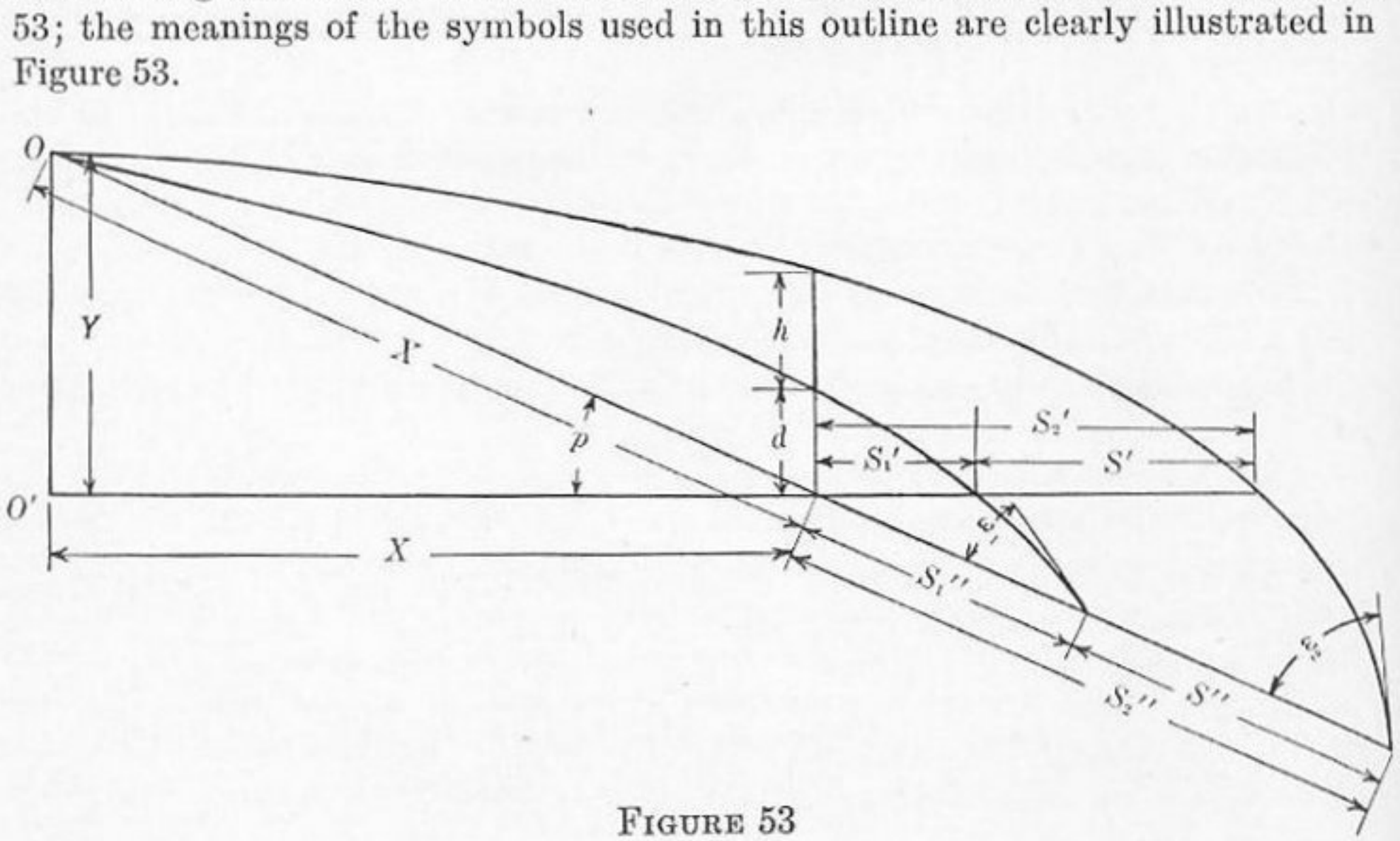


FIGURE 53

Hitting space for a gun elevated above the target, when the target is above the water

(a) $S''_1 = d \cot \omega, \qquad \tan p = \frac{Y}{X}$

$S''_2 = (h+d) \cot \omega$

ω being for the range X to the target.

(b) With range $X+S''_1$, get ω_1 from the range table.
With range $X+S''_2$, get ω_2 from the range table.

(c) $S'_1 = S''_1 \sin \omega_1 \operatorname{cosec} (\omega_1+p)$
$S'_2 = S''_2 \sin \omega_2 \operatorname{cosec} (\omega_2+p)$

(d) $S' = S'_2 - S'_1$

1707. The following example illustrates the application of the process outlined above and also shows how greatly the angle of position affects the hitting space at short ranges.

Given: A 4″2900 f.s. gun is to fire at Short Range Practice; the mean range will be 1633 yards. The gun is elevated 25 feet above the water. The target screen is 15 feet high and is spread so that its bottom edge is 5 feet above the water.

Find: The hitting space for the 15-foot target at the mean range of 1633 yards. What error would result from using Column 19 and neglecting the angle of position? (Use angles to the nearest whole minute.)

Following the procedure outlined in article 1706, we have

(a) $\tan p = \frac{Y}{X}$

$Y = 25$ feet	log 1.39794
$X = 4900$ feet ..... log 3.69020	colog 6.30980–10
$p = 0°\ 18'$	ltan 7.70774–10

$S''_1 = d \cot \omega \qquad S''_2 = (h+d) \cot \omega$

$d = 5$ feet	log	0.69897	$(h+d) = 20$ feet	log	1.30103
$\omega = 0°\ 42'$	lcot	1.91300	$\omega = 0°\ 42'$	lcot	1.91300
$S''_1 = 409.2$ feet	log	2.61197	$S''_2 = 1636.9$ feet	log	3.21403

(b) $X+S''_1 = 5309$ feet or 1770 yards
whence, from the range table, $\omega_1 = 0°\ 46'$.
$X+S''_2 = 6537$ feet or 2179 yards,
whence, from the range table, $\omega_2 = 1°\ 00'$.

(c) $S'_1 = S''_1 \sin \omega_1 \operatorname{cosec} (\omega_1+p)$
$S'_2 = S''_2 \sin \omega_2 \operatorname{cosec} (\omega_2+p)$

$S''_1 = 409.2$ feet	log	2.61197	$S''_2 = 1636.9$ feet	log	3.21403
$\omega_1 = 0°\ 46'$	lsin	8.12647–10	$\omega_2 = 1°\ 00'$	lsin	8.24186–10
$(\omega_1+p) = 1°\ 04'$	lcsc	1.73012	$(\omega_2+p) = 1°\ 18'$	lcsc	1.64422
$S'_1 = 294$ feet	log	2.46856	$S'_2 = 1259$ feet	log	3.10011
$= 98$ yards			$= 420$ yards		

(d) $S' = S'_2 - S'_1$

$S' = 420 - 98 = \underline{322 \text{ yards}}$

Had we used Column 19 and neglected the angle of position, we would have found,

$$S'_1 = \frac{5}{3} \times 100 = 167 \text{ yards},$$

$$S'_2 = \frac{20}{3} \times 100 = 667 \text{ yards},$$

$$S' = 667 - 167 = \underline{500 \text{ yards}},$$

and our result would have been in error 178 yards, or about 55%.

Had we used the hitting-space formula (1701) but neglected the angle of position, our result would have been,

$$S''_2 - S''_1 = 1637 - 409 = 1228 \text{ feet or } \underline{409 \text{ yards}}$$

and this result would have been in error 87 yards, or about 27%.

1708. The above example also illustrates the correct method for determining the relation between points of impact in the vertical plane of the target and on the surface of the water, at very short ranges. The spotter would know, in the case illustrated, that a shot through the top edge of the target should fall at the distance $S_2' = 420$ yards beyond the target, and a shot through the bottom edge at the distance $S_1' = 98$ yards beyond the target. The distance beyond the target at which a shot through the bull's-eye should fall can be found by the same process. The spotter uses this information for constructing a *splash diagram*.

Use of hitting-space by the spotter

1709. A composite graph of hitting-space curves provides a convenient means for finding the data required for the construction of a splash diagram. Such a *hitting-space diagram* may be prepared for each battery carried on a ship, for the mean range of Short Range Practice, and for all heights of gun and target that are likely to occur. The following is an outline of a method by means of which a hitting-space diagram may be constructed.

Construction of the hitting-space diagram

(a) Assume a range which will be the mean range for the given battery at Short Range Practice.

(b) Assume zero height of gun, and compute the hitting space for target heights 0, 3, 5, 10, 15, etc. feet, up to the greatest target height desired. Plot the values of hitting space against height of target, and mark the graph to indicate that it applies to zero height of gun. This graph should be a straight line, i.e., the graph of $h \cot \omega$, ω being the angle of fall corresponding to the assumed mean range.

(c) Assume heights of gun of 5, 10, 15, 20, etc. feet, up to the greatest height of gun likely to occur. For each of these heights of gun repeat the process outlined in (b), and mark the resulting graphs according to the heights of gun to which they apply. For other than zero height of gun these graphs are curves.

(d) Connect the points of the several height-of-gun curves which correspond to the same target heights, thus obtaining a series of height-of-target curves that intersect the height-of-gun curves.

A hitting-space diagram for the 4″2900 f.s. gun at the range 1633 yards is illustrated in Plate V. The problem stated in article 1707 can be solved graphically by means of this diagram, as follows. The gun is 25 feet above the water, and the top of the target 20 feet above the water. On the diagram (Plate V), the 25-foot height-of-gun curve and 20-foot height-of-target curve intersect at A; by projecting this point vertically down to the hitting-space scale we find $S'_2 = 420$ yards. The bottom of the target is 5 feet above the water. The 25-foot height-of-gun curve and 5-foot height-of-target curve intersect at B, whence we find $S'_1 = 98$ yards. The hitting space is therefore $420 - 98 = 322$ yards, which agrees with the result found by computation. The bull's-eye of the target is 12.5 feet above the water. The 25-foot height-of-gun curve and 12.5-foot height-of-target curve intersect at C, whence we find that a shot passing through the bull's-eye will fall 253 yards beyond the target.

Use of the hitting-space diagram

Using the zero height-of-gun curve with the above data we find

$$S'_2 = 546 \text{ yards}, \quad S'_1 = 137 \text{ yards}$$

$$S' = 546 - 137 = 409 \text{ yards}$$

which agrees with the result found in article 1707 when height of gun was neglected.

EXERCISES

1. *Given:* The type of gun, the range, the height of target, depth of target in the line of fire, height of gun above the water, and height of bottom of target above the water.
   Find: The hitting space, using the range table as required.

	Given						Answers
	Gun	Range (yds.)	Target (feet)			Height of gun (feet)	Hitting Space (yds.)
			Height	*Depth	Bottom above water		
A	4″2900 f.s.	2500	14	0	4	26	193
B	6″2600 f.s.	1600	15	0	6	28	293
C	12″2900 f.s.	1650	15	0	5	35	342
D	12″2900 f.s.	20,000	30	90	0	..	62

* See art. 1402 (h).

HITTING SPACE DIAGRAM

4"-2900fs. GUN

SHORT RANGE BATTLE PRACTICE

Mean Range 1633 yards

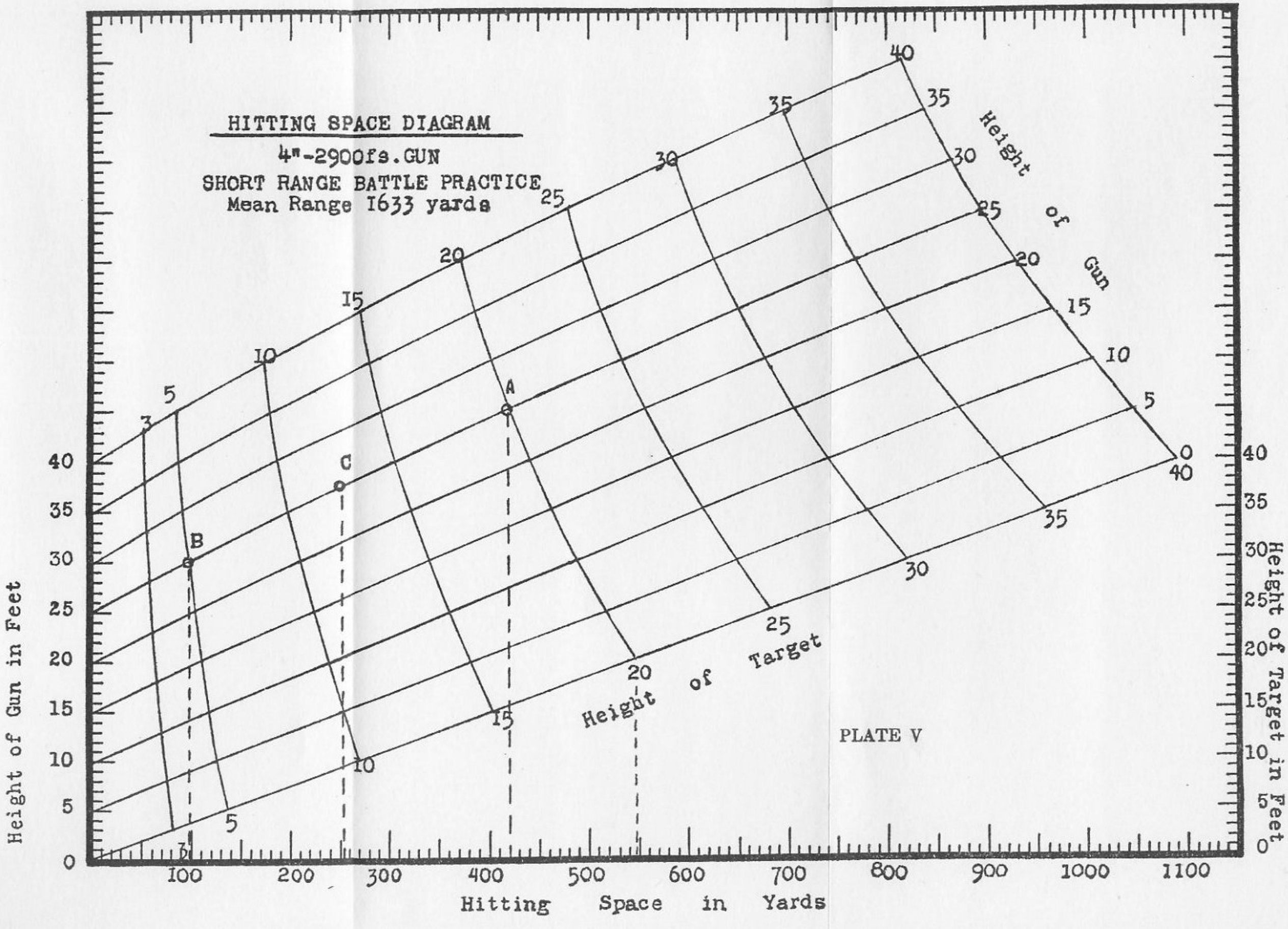


PLATE V

APPENDIX C

THE SOLUTION OF INCLINED TRAJECTORIES. GENERAL FEATURES INVOLVED IN THE CONTROL OF ANTI-AIRCRAFT FIRE AND ILLUMINATION FIRE.

Introductory Note

The purpose of this appendix is to explain the methods by which range tables, or their equivalent, are prepared for antiaircraft fire and illumination fire, and to set forth the theoretical bases on which the control of these types of fire rests. In order that this purpose may be accomplished it is desirable to discuss some practical features of the control problems. Such discussions should, however, be regarded as fundamental in their nature, and not as being designed to acquaint the student with practical details. Moreover, this text is limited to comparatively general references to practical methods actually in use for the control of antiaircraft fire, since many features of such methods are regarded as confidential. The antiaircraft tracking sheet illustrated in this appendix is obsolete, but it is useful, nevertheless, for demonstrating the principles that enter into the construction and use of tracking sheets of more recent issue. It is expected that tracking sheets and other graphical devices in current use will be made available for supplemental classroom study.

1801. The problem of computing antiaircraft trajectories is unique principally in that it involves large angles of position. Numerous approximate methods have been devised for dealing with greatly inclined trajectories, most of them based, in one form or another, upon the principle of tilting the horizontal trajectory.* These methods have generally been discarded in favor of the direct and relatively accurate method based on numerical integration; the principal features of the latter method are outlined below.

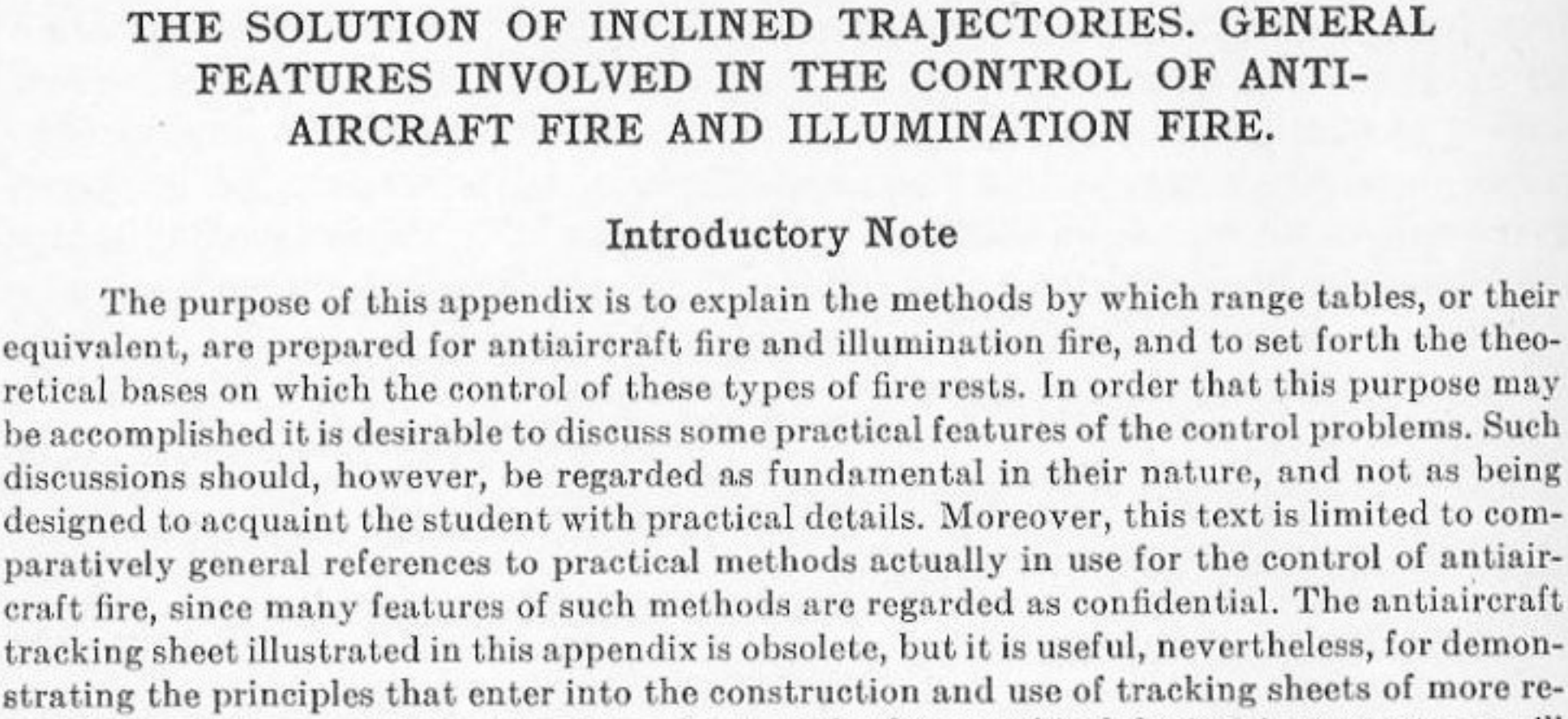


FIGURE 54

1802. In Figure 54, OMH represents a horizontal trajectory with angle of departure 45° and horizontal range $X = OH$. Let us suppose that this trajectory

* Ref. Chapter VIII, *Handbook of Ballistics*, Vol. I, Cranz and Becker.

has been computed by the method of numerical integration and plotted to large scale. (The trajectory OMH in Figure 54 represents, approximately to scale, the 45° trajectory of the 5″3150 f.s. gun, the horizontal range being equal to about 23,000 yards and the maximum ordinate to about 24,000 feet.) The lines of position OP_1, OP_2, OP_3, OP_4, make with the horizontal the angles of position, respectively, 15°, 20°, 25°, and 30°. Each of these lines of position cuts off a segment of the horizontal trajectory, and each segment is an inclined trajectory. For example, the line of position OP_1 cuts off the inclined trajectory OMP_1; the angle of position of the latter trajectory is equal to 15°, and the *angle of elevation* ϕ' is evidently the angle DOP_1, which is equal to $45°-15°=30°$. The *slant range*, $X'=OP_1$, can be measured graphically, or it can be determined from the relation $X'=x \sec p$, x being the abscissa of the point P_1. The time of flight (t), angle of inclination (θ), and remaining velocity (v) corresponding to the point P_1 on the horizontal trajectory OMH being known, we have the terminal elements of the inclined trajectory OMP_1.

Solution of inclined trajectories for antiaircraft fire

By the same process we can determine the terminal elements of a number of inclined trajectories, each corresponding to a different angle of position. All of these trajectories are, of course, portions of the same 45°, horizontal trajectory, but, considered as inclined trajectories, each of them has a different angle of elevation. From the one horizontal trajectory and several lines of position plotted in Figure 54, we can derive the following inclined trajectories:

For $p=15°$ and $\phi'=30°$, $X'=OP_1$
For $p=20°$ and $\phi'=25°$, $X'=OP_2$
For $p=25°$ and $\phi'=20°$, $X'=OP_3$
For $p=30°$ and $\phi'=15°$, $X'=OP_4$

These results are illustrated in the figure. By plotting at suitable intervals along the horizontal trajectory OMH the values of t, θ, and v found in the step-by-step solution of that trajectory by numerical integration, the terminal values of these elements for any inclined trajectory whose terminal point lies on the given horizontal trajectory can be determined very conveniently.

1803. A single horizontal trajectory yields, of course, only one inclined trajectory corresponding to any given angle of position, i.e., only one value of ϕ' corresponding to each assumed value of p. Additional inclined trajectories, for

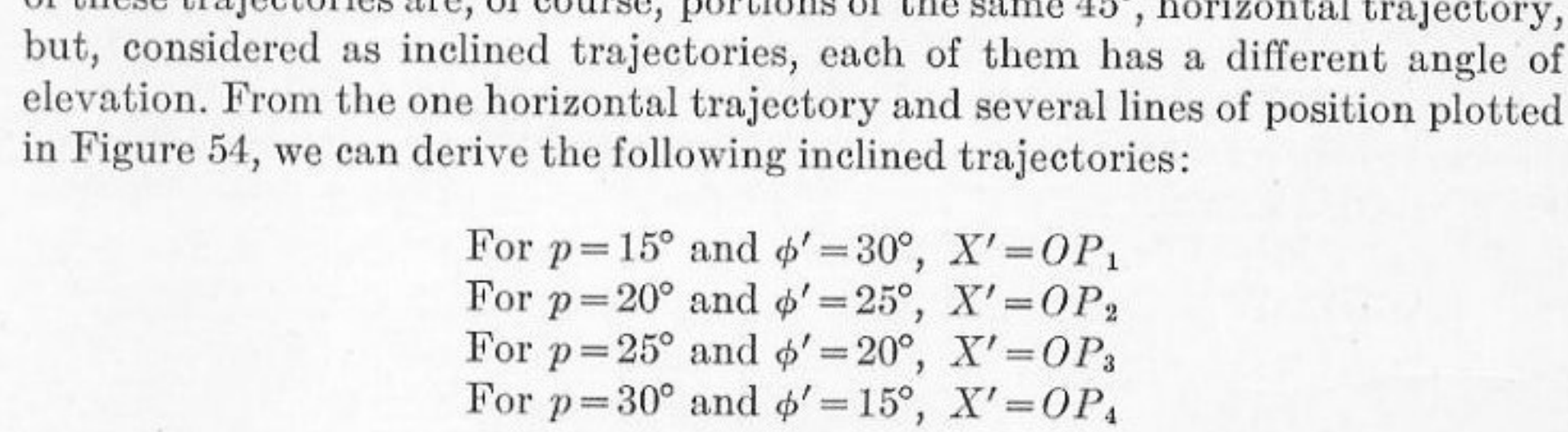


FIGURE 55

any assumed angle of position, can be obtained by plotting additional horizontal trajectories, as shown in Figure 55. This figure represents a plot (not to scale) of horizontal trajectories having angles of departure 30°, 35°, 40°, and 45°, and of a 15° line of position. From this plot we can evidently derive the following inclined trajectories:

For $p=15°$ and $\phi'=15°$, $X'=OP_1$
For $p=15°$ and $\phi'=20°$, $X'=OP_2$
For $p=15°$ and $\phi'=25°$, $X'=OP_3$
For $p=15°$ and $\phi'=30°$, $X'=OP_4$

The other terminal elements for each of these trajectories can be found as already described in the foregoing article.

1804. We have already seen (art. 819) how a range table for horizontal fire is built up by a process of interpolation between computed points. A range table for inclined fire could be built up by a similar process, but such a table would have to include angle of position as an additional argument and would be too complicated for practical use. A more practical arrangement of data pertaining to antiaircraft trajectories is obtained as follows.

Graphs of data for antiaircraft trajectories

Horizontal trajectories are computed and plotted for angles of departure up to 90°, at intervals of 5°. Lines of position are plotted for angles of position up to 90°, at intervals of 10°. The process described in article 1803 and illustrated in Figure 55 is carried out for each plotted line of position. This yields, for each angle of position, a series of corresponding slant ranges and angles of elevation, all pertaining to that angle of position. For example, the point where the 10° line of position cuts the 15° trajectory gives the slant range corresponding to the angle of elevation 5°; the point where the 10° line of position cuts the 20° trajectory gives the slant range corresponding to the angle of elevation 10°; etc. For each angle of position, the various slant ranges thus obtained are plotted against their corresponding angles of elevation, and a fair curve is drawn through the plotted points. This process yields a series of graphs, each graph representing the range-elevation relation for one angle of position. The angle of elevation corresponding to *any* slant range and any *plotted* value of angle of position can then be taken directly from one of the graphs, and the angle of elevation corresponding to *any* slant range and *any* angle of position can be found by interpolating between two graphs. Graphs showing the relation of time of flight to slant range and angle of position are prepared similarly.

1805. The effects of wind, and of variations from standard in initial velocity, weight of projectile, and atmospheric density are determined by the method of differential variations (see note on p. 124). As any inclined trajectory may be considered to be a portion of a horizontal trajectory (as illustrated in Figure 54), the determination of these variations for an inclined trajectory can be simplified by the use of weighting factors based on the horizontal trajectory which includes the given inclined trajectory. Graphs are prepared to show the effects of variations from standard conditions on angle of elevation and time of flight.

1806. The burning rate of powder-train time fuzes varies with atmospheric density and pressure, and hence with altitude; consequently the fuze settings for this type of fuze do not necessarily agree with actual times of flight. The relation between fuze setting and actual time of flight is determined experimentally for each type of powder-train time fuze; graphs are prepared to show directly the relation of fuze setting to slant range and angle of position, under standard atmospheric conditions. Additional graphs are prepared for the determination of corrections to these fuze settings for variations from standard in atmospheric density and pressure. The burning rate of the powder train depends also on the velocity of the projectile. A variation in the initial velocity therefore requires a change in fuze

Relation between time of flight and fuze setting

setting due to its effect on the burning rate of the fuze, as well as a change due to its effect on the time of flight.

1807. The drift formulas given in Chapter 9 (art. 911 and note adjacent thereto) are of such nature that they are applicable only to the terminal point of a horizontal trajectory; that is to say, the drift for an inclined trajectory may not be found from these formulas by substituting in them the values of elements for the intermediate point of a horizontal trajectory which corresponds to the terminal point of the given inclined trajectory. The drift for intermediate points of a horizontal trajectory can be determined satisfactorily only by step-by-step integration of analytical expressions for this element. It is necessary, of course, to determine constants for use in these expressions by experimental firing.* Having thus determined the drift for various points along a horizontal trajectory, the drift for inclined trajectories whose terminal points lie on this horizontal trajectory can be determined as already described for other elements (art. 1802).

Drift for inclined trajectories

A very rough approximation of the drift for an inclined trajectory is given by the relation

$$D_{\text{inclined}} = D_{\text{surface}} \times \cos p$$

in which D_{inclined} is the drift for an inclined trajectory, D_{surface} is the drift for a horizontal trajectory having an angle of elevation equal to that of the inclined trajectory, and p is the angle of position of the inclined trajectory. This is a purely arbitrary relation, based on the consideration that when $p = 90°$ the fire is vertical and there is no drift, for under this condition there is no curvature to the trajectory and consequently no moment causing precession. It is assumed, then, that since the drift is reduced to zero at the zenith, it is related to the angle of position by a function which is equal to unity for zero degrees and to zero for ninety degrees. Cos p is used merely because it satisfies these conditions. Although the approximate relation given above was formerly used for determining the drift of inclined trajectories, it is the present practice to solve for this element by the method outlined in the foregoing paragraph.

1808. A sight scale graduated in terms of range evidently is unsuitable for antiaircraft fire, for in this type of fire the angle of elevation is governed by the angle of position as well as by the range. Sight scales for antiaircraft fire are accordingly graduated in minutes of arc, and are set directly in terms of angle of elevation, or *sight angle*, rather than in terms of range. (The *sight angle* is the angle, measured in the plane of elevation, between the line of sight of a gun and the axis of the bore of the gun.)

Sight scales for antiaircraft fire

Sight angle

The various graphs described above constitute the equivalent of a range table for an antiaircraft gun, and by means of them the angle of elevation required for a given slant range and angle of position, under any specified conditions, can be determined by methods similar to those followed in determining the sight-bar range for surface fire from a range table. The graph of angle of elevation vs. slant range and angle of position, gives the angle of elevation corresponding to any specified slant range and angle of position under standard conditions. Other

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* Neither of the formulas given in Chapter 9 is in shape for this process. Analytical expressions in shape for solution by numerical integration are given on p. 353, *Handbook of Ballistics*, Vol. I, Cranz and Becker, and on pp. 119–120, *Computation of Firing Tables for the U. S. Army*, H. P. Hitchcock.

graphs give the corrections to the angle of elevation required for variations from standard in initial velocity, atmospheric density, and for wind, etc. The deflection for antiaircraft fire can also be determined by methods similar to those already described for surface fire, the essential difference being that the required data are found from graphs instead of from the columns of a range table.

1809. It is obviously impracticable to use the graphs directly, in the manner outlined above, for continuous determination of the angle of elevation, deflection, and fuze setting under the condition of a rapidly changing slant range and angle of position. The only wholly satisfactory solution of the problem of determining these elements with the required rapidity lies in a mechanical system designed to generate them continuously. Such systems are now installed on most vessels.

Mechanical systems for control of antiaircraft fire

The general principles of methods which were developed prior to the adoption of mechanical systems are described below. These methods still serve as the primary methods of antiaircraft fire control for vessels not equipped with mechanical systems, and as stand-by methods for vessels having the mechanical equipment.

Manual systems for control of antiaircraft fire

1810. Plate VI is a reproduction of an *antiaircraft tracking sheet* for a 3″ antiaircraft gun. The construction of this diagram is similar to that of Figure 55. On it are plotted horizontal trajectories for angles of departure from 5° to 85° at intervals of 1°. The trajectories corresponding to whole multiples of 5° are labeled where they meet the inner margin of the diagram, and along each of these trajectories are marked times of flight at intervals of $\frac{1}{10}$ second, labeled at each $\frac{1}{2}$ second. A scale for measuring angles of position is laid off on the outer margin. Circles concentric with the origin (lower left-hand corner) indicate the slant range to any point on any trajectory; these range circles are labeled along the bottom edge of the diagram. Horizontal lines across the diagram indicate altitude; they are labeled along the left-hand edge of the diagram.

Antiaircraft tracking sheet

1811. The antiaircraft tracking sheet serves two purposes; it is a convenient tracking sheet for plotting the approach of the aircraft, and it provides a means for deducing directly from the plotted track the angle of elevation and fuze setting required to hit the aircraft at any point in this track.

In order to plot the track of the approaching aircraft it is necessary to measure its angle of position and slant range (or altitude) at several points, as, for example, at P_1, P_2, and P_3. Plate VI shows the track of a plane approaching at an altitude of 5000 feet. By measuring the distances covered by the plane between successive positions along its track, and comparing these with the times elapsed between these positions, an estimate of the plane's speed may be made; the speed thus found is, of course, the plane's relative speed with respect to the firing ship.

Now let us suppose that the plane is to be fired at when it reaches point P_4. The line of position, which is the line drawn from the origin through P_4, intersects the marginal scale at 27°, indicating that at point P_4 the angle of position of the plane will be 27° ($p=27°$). The point P_4 also lies on the 30° trajectory (as indicated at the inner right-hand margin). Therefore the angle of elevation required to hit the plane at P_4 is $\phi'=\phi-p=30°-27°=3°$. Also, at P_4 the time of flight is 7.5 seconds.

But if we should wait until the plane actually reaches the point P_4 before opening fire, the plane would, during the time of flight of the projectile, reach a point considerably beyond P_4; for example, if the plane were traveling at 90 knots

Determination of angle of elevation from the antiaircraft tracking sheet

it would cover 375 yards during the time of flight of 7.5 seconds. Putting the plane 375 yards *back* along its track locates it at point D, where its angle of position is $24\frac{1}{2}°$. Then in order to hit the plane at P_4 we must fire upon it when it reaches point D; since P_4 still lies on the 30° trajectory, while D lies on the $24\frac{1}{2}°$ line of position, the required angle of elevation is $30° - 24\frac{1}{2}° = 5\frac{1}{2}°$, or 330′. What we have done is to find the trajectory that will *intercept* the target, rather than the one that will pass through the position occupied by the target at the instant of fire. The problem is, of course, similar to that of firing at a moving target on the surface. If the sight angle 330′ is set on the sight scale and the gun is aimed at the target when the latter occupies point D, the bore will be elevated as required to cause the projectile to follow the trajectory which will intercept the target at point P_4.

The times plotted along the trajectories of Plate VI are actual times of flight and hence can be used directly as fuze settings only for mechanical time fuzes. Fuze settings applicable to a particular type of powder-train time fuze can also be plotted along the trajectories. For the purposes of the illustrations to be given here we shall assume that the times plotted on Plate VI represent fuze settings. The fuze setting required to cause a burst at point P_4 is then 7.5 seconds.

1812. The great rapidity with which the problem must be solved renders direct use of the diagram, as outlined above, impracticable. An understanding of the direct, graphical solution on the antiaircraft tracking sheet, as outlined above, is essential, however, to a proper understanding of the methods actually employed. These methods are based on the use of tabulated solutions for the sight angle and fuze setting corresponding to various combinations of target speed and target altitude.

Referring to Plate VI, it will be noted that after the track of the target has been established, altitude and angle of position define the target's position at any point in this track. Thus if the target is flying at an altitude of 5000 feet and its angle of position is 27°, it must occupy the point P_4. For tabulated solutions, altitude is a more convenient argument than slant range, for the target's altitude often is nearly constant whereas its slant range always changes rapidly.

A table of sight angles and fuze settings corresponding to a single altitude and various target speeds can be prepared as follows. For this illustration we shall assume an altitude of 4000 feet and a target speed of 90 knots (90 knots corresponds to 50 yards per second). Now let us assume for target positions a series of points defined by certain fuze settings, say 12, 10, and 8 seconds. On Plate VI these positions are plotted on the 4000-foot altitude line and are marked 12, 10, and 8.

In order to hit the target at these positions, we must fire on it before it reaches them. In order to hit the target at the 12-second point we must fire at it when it is still $12 \times 50 = 600$ yards from this point, or at the position marked Δ and labeled Z-12. In order to be hit at the 10-second and 8-second points the target must be fired at when it occupies the positions Z-10 and Z-8, respectively; Z-10 is located $10 \times 50 = 500$ yards from the 10-second point, and Z-8 is located $8 \times 50 = 400$ yards from the 8-second point.*

---

* The times to be used for determining these distances should, of course, be the *times of flight* corresponding to the given fuze settings; in accordance with the assumption stated in article 1811 (par. 5), times of flight are here taken to be equal to fuze settings.

The sight angles required to hit the target at the 12-, 10-, and 8-second points can now be found by the process described in article 1811. The 12-second point lies on the 21°40′ trajectory, and the angle of position at Z-12 is equal to 13°50′; hence the sight angle required for the 12-second point is equal to 21°40′ − 13°50′ = 7°50′, or 470′. By the same process the sight angle for the 10-second point is found to be equal to 22°10′ − 15°40′ = 6°30′, or 390′, and for the 8-second point 23°45′ − 18°25′ = 5°20′, or 320′.

1813. All of the above solutions pertain to a 90-knot target flying at an altitude of 4000 feet. Now let us refer to the positions for which we have made solutions, as *zones;* that is, let us call the positions Z-12, Z-10, and Z-8, respectively, Zone 12, Zone 10, and Zone 8. These zones have been so chosen and designated that the zone description of a given zone agrees with the fuze setting required to hit a target which is fired upon when it occupies that zone; the sight angles for the several zones* have been found as shown above. The information thus found may now be arranged in tabular form as follows:

Zones

Antiaircraft sight-setting table

Altitude 4000 feet

Zone	2	3	4	5	6	7	8	9	10	12
Target Speed										
70										
80										
90							320		390	470
100										
110										

The above table, which is called a *sight-setting table,* represents a convenient form for tabulating all of the solutions pertaining to the 4000-foot altitude. The three solutions actually made in the foregoing article have been entered in the table. In order to complete the table, solutions must be made for each zone and each target speed included in the arguments of the table. It must be borne in mind that all of the values included in a single table pertain to a single altitude. A complete outfit of sight-setting tables includes a separate table for each 1000 feet of altitude up to the greatest altitude likely to be required.

1814. The sight-setting table is used as follows. As soon as the altitude of the target has been determined, the appropriate sight-setting table is defined. As soon as the speed of the target has been determined, the appropriate line of the sight-setting table is defined, and the sight angles corresponding to the various zones through which the target has to pass are then immediately available. For example, let us suppose that the altitude and speed of an approaching bomber have been determined, by tracking, to be 4000 feet and 90 knots, and that it has been decided that the position Z-12 (Plate VI) is the earliest at which it is practicable to open fire. From the sight-setting table for the altitude 4000 feet it is found that the sight angle corresponding to Zone 12 and target speed 90 knots, is 470′; fuzes are then set at 12 seconds and sight scales at 470′, and fire is opened as the target approaches the position Z-12. After the target has passed beyond Z-12, the fuze setting and sight angle are reset for a new zone, say Zone 10, and the fire is continued.

In determining the proper instants at which to open fire for the zone initially selected, and at which to change the settings for a new zone, due allowance must

* The reason for referring to these *points* as *zones* is explained in article 1814.

Allowance for dead time

be made for *dead time*. This can be done readily by advancing the actually plotted target positions a distance equal to the product of the dead time and target speed. For example, if the dead time is equal to 10 seconds and the target speed is equal to 90 knots, the target is considered to have reached a given position when, according to the plotted track, it is still $10 \times 50 = 500$ yards from that position.

Allowance must be made also for the length of the interval during which the settings are to remain unchanged. Thus if the settings are to be changed at intervals corresponding to a change of one second in fuze setting, fire for a given zone should be opened, with the settings for that zone, when the target reaches a position about half-way between the given zone and the next preceding zone, and continued until the target reaches a position about half-way between the given zone and the next following zone (allowance for dead time being made, of course, as indicated above). The size of interval to be used depends on the rate of fire of the battery and on the rapidity with which the required settings can be found, transmitted, and applied. Intervals corresponding to a change of only one second in fuze setting will probably be much too short when the target speed is great; even in the case illustrated on Plate VI, the intervals corresponding to changes of one second in fuze setting are equal to only about seven seconds of time. It appears, then, that it is impracticable to assign definite limits to zones; Zone 10, for example, may extend from the point corresponding to the fuze setting 10.5 seconds to the point corresponding to the fuze setting 9.5 seconds, or it may extend from the 11-second point to the 9-second point, or from the 11.5-second point to the 8.5-second point, etc., depending upon the interval chosen by the control party. It is for this reason that the term *zone* is applied to a *point*, such as the points *Z*-8, *Z*-10, and *Z*-12 (Plate VI). Actually these points represent the middle points of their respective zones, and the width of the zones depends upon the intervals to be used.

Width of zones

1815. It is to be noted that the points *Z*-8, *Z*-10, and *Z*-12, as plotted on Plate VI, define the positions of Zone 8, Zone 10, and Zone 12, respectively, for the single target speed of 90 knots, as well as for the single target altitude of 4000 feet. In order that the diagram may be used in the manner outlined in the foregoing article, it is necessary to plot on it zone positions for all target speeds and altitudes that are likely to occur in practice. The method for locating the position of a given zone for a given target speed and altitude has already been explained in article 1812. Let us suppose that the positions for Zone 8, for the target speed 90 knots, have been located on the 1000-foot, 2000-foot, 3000-foot, etc. altitude lines, just as the point *Z*-8 has been located on the 4000-foot altitude line. A curve connecting these positions will be the locus of Zone 8 for the target speed 90 knots and *any altitude* (within the limits of the diagram). The loci of other zones can be plotted similarly. The complete diagram will then include a group of curves for each zone, each group containing curves which are the loci of positions corresponding to the same fuze setting (or zone description) and to various target speeds.*

Provisions for handling various altitudes and target speeds on the tracking sheet

1816. It is to be understood, of course, that the diagram described above, and its accompanying sight-setting tables, are based on standard conditions, i.e.,

* It is expected that the diagrams actually in use in the service will be exhibited in the classroom in connection with the study of this appendix. Reproduction of these diagrams in this textbook is not permitted.

range-table conditions, with the exception that target speed has been taken into account in plotting the various zone loci on the diagram and in determining the corresponding sight angles for the sight-setting tables. Other ballistic corrections can be found from the graphs described in articles 1806–1808, but in view of the form in which the firing data have been prepared it is very inconvenient to apply corrections to them in the usual manner. One way of handling such corrections is to express them in terms of time of flight and to apply them, in this form, to the zone description. This can be done (in advance of firing, of course,) by comparing the sight-angle and fuze-setting corrections corresponding to variations from standard initial velocity and atmospheric density, with the data of the sight-setting tables for the conditions of altitude and target speed likely to be encountered, and thus deducing appropriate corrections in terms of zone description.

Ballistic corrections for antiaircraft fire

Since the target is tracked with reference to a fixed origin on the diagram (point *O*, Plate VI), the target speed found is the target's *relative* speed with respect to the firing ship; that is, the speed found is the resultant of target speed and firing-ship speed. This means that the firing ship's speed is accounted for in the target speed, and consequently it is accounted for *as* target speed. According to the rules stated in article 1219, corrections for wind should then be based on the *apparent wind*. (This applies, of course, only to the stand-by methods; antiaircraft range-keepers handle target speed and firing-ship speed separately, and the considerations governing the choice of true or apparent wind for use with an antiaircraft range-keeper are the same as for any other range-keeper (art. 1220).)

Ballistic corrections for wind

1817. In a bombing attack, the motion of the attacking bomber must be such as to cause the component of his own motion across the line of fire to be practically the same in amount, and in the same direction, as the component of the firing ship's motion across the line of fire. If the correction for wind across the line of fire is based on the *apparent* wind, the corrections for target motion and firing-ship motion across the line of fire then practically cancel each other, and the deflection need include corrections only for drift and apparent wind. The deflection problem in a bombing attack is therefore relatively simple. In a torpedo attack, the motion of the attacking torpedo-plane may be such as to have a large and rapidly varying component across the line of fire, and the deflection problem in this type of attack is therefore much more difficult.

Deflection for antiaircraft fire

CONTROL OF ILLUMINATION FIRE.

1818. The object of illumination fire is to release a source of light above and behind a target in order that the latter may be silhouetted against an illuminated water area and thus rendered visible to the firing vessel at night. An illuminating projectile, or starshell, contains an illuminating element, or star, which is attached to a parachute, and an expelling charge which, when set off by a time fuze, ignites the star and releases it and its parachute. The duration of burning of the star and its rate of descent are such that the star should be released at an altitude of about 1000 feet in the case of 3″ starshell, and at an altitude of about 1500 feet in the case of 4″, 5″, and 6″ starshell. It has been found by experience that the point of release should be sufficiently far behind the target to prevent the illumination of water in front of the

General principles of illumination fire

target; the purpose of the star is to silhouette the target, and illumination in front of the target reduces the silhouette effect. Range tables for illumination fire are based on a release point 1000 yards beyond the target. The general principles involved in the preparation of these range tables are outlined below.

1819. In Figure 56, a gun is located at O and a target at T. The trajectory of the starshell must be such that it will have an ordinate $P'P$, equal to about 1500 feet (assuming a 5″ gun), where its abscissa is OP', equal to OT plus 1000 yards. Let us suppose, now, that a trajectory has been plotted for the angle of departure ϕ, giving the horizontal range OH. We may determine graphically at what point, P, this trajectory has an ordinate of 1500 feet, and also find the abscissa OP' of

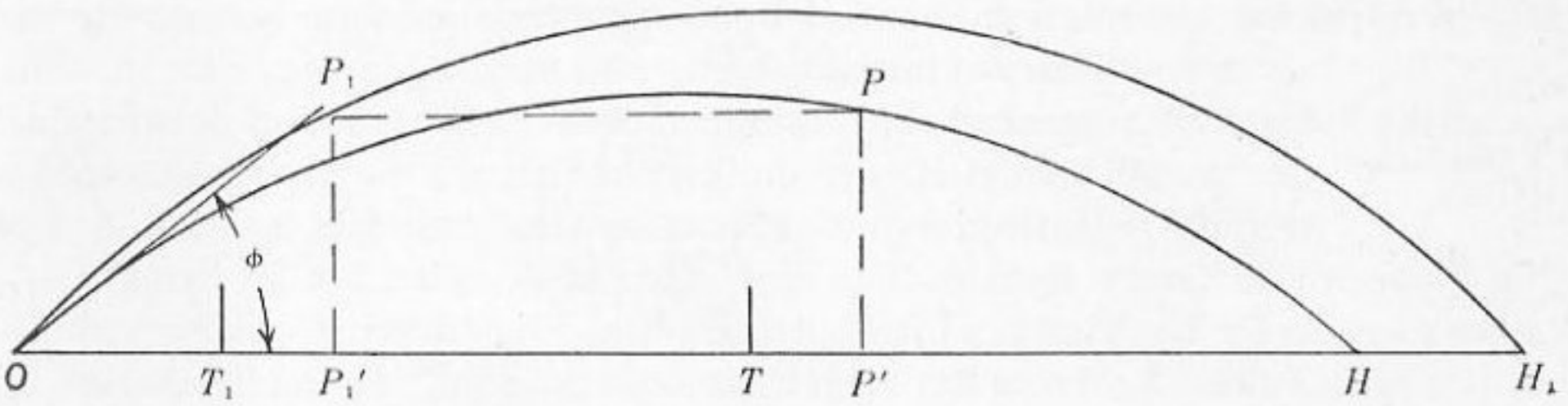


FIGURE 56

this point. This trajectory will then be the proper one for a target located at the range $OT = OP' - 1000$ yards. From the time of flight to the point P the fuze setting may be determined.

The entering argument (Column 1) for starshell range tables is the *burst range*, i.e., the abscissa of the trajectory for the point having an ordinate of 1500 feet (or 1000 feet in the case of 3″ projectiles), rather than the horizontal range of the trajectory. Column 2 gives the angle of departure of the trajectory, Column 3 the corresponding sight-bar range, and Column 4 the time of flight, for the burst range given in Column 1.

Burst range

If a target at a range of 5000 yards is to be illuminated, and it is desired that the star be released at a point 1000 yards beyond the target and 1500 feet above the surface, i.e., at a point having an abscissa of 6000 yards and an ordinate of 1500 feet, the range table must be entered at range 6000 yards in Column 1 and the corresponding sight-bar range found in Column 3. This sight-bar range will then cause the projectile to travel in a trajectory whose ordinate will be 1500 feet at the burst range of 6000 yards. The fuze setting required to cause a burst at this point is given in Column 5. The variation in the burning rate of the fuze-train due to change of altitude is accounted for in the tabulated fuze settings.

General features of starshell range tables

The more recent practice is to prepare the data for illumination fire in the form of graphs rather than in the form of tables. In addition to graphs from which the sight-bar range, time of flight, and fuze setting corresponding to a given burst range may be obtained, graphs are provided also for the determination of corrections to the sight-bar range and fuze setting for variations in initial velocity and atmospheric density.

Graphs for illumination fire

1820. A peculiar feature of illumination fire is that the sight-bar range may increase while the target distance decreases. The reason for this will be apparent from an examination of Figure 56. If the target were located at T_1 instead of at T,

the trajectory would have to reach the 1500-foot ordinate at P_1' instead of at P', and in order to accomplish this the sight-bar range would have to be *increased* from OH to OH_1. It will be observed that if the target lies beyond the maximum ordinate of the trajectory, an *increase* in target distance will require an *increase* in sight-bar range; but if the target lies short of the maximum ordinate, a *decrease* in target distance will require an *increase* in sight-bar range. It is as a result of this feature that in range tables for illumination fire the sight-bar range (Col. 3) decreases with increases of burst range (Col. 1) up to a certain point, while beyond that point sight-bar range increases with increases of burst range. The fuze setting, however, always increases with increases of burst range.

Ballistic corrections for illumination fire

1821. Ballistic corrections are used in illumination fire chiefly for the purpose of adjusting the relation between sight-bar range and fuze setting for the given conditions of initial velocity and atmosphere, in order that the burst may occur at the required altitude. The nature of illumination fire is such that it is very unlikely that it will be practicable to determine ballistic corrections for other elements, such as wind, firing-ship motion, and target motion. The first stage of illumination fire usually involves a search for the target, which is accomplished by covering wide arcs of the horizon with starshell. When the target has been located, the determination of the sight-bar range and deflection required for efficient illumination is largely a spotting problem. Knowledge of the proper relation between sight-bar range and fuze setting is important, however, in order that the burst points may be maintained at the proper altitude as the sight-bar range is changed.

Deflection for illumination fire

1822. At some target practices it is desirable to have a preliminary estimate of the starshell deflection. The problem of determining the deflection required to cause the starshell to burst directly in line with the target is essentially the same as that of determining the deflection required to hit the target. However, a point directly in line with the target is usually not the most satisfactory position for the point of burst, for after the star has been released and is suspended in the air, its position with relation to the line of sight is altered by the motion of the target and firing ship, as well as by the wind. Thus if the burst occurs directly in line with the target (as viewed from the firing ship), it will probably soon drift away from this line, especially if there is a strong cross wind or if the target's motion across the line of fire is great. The burst therefore should occur at such a point that the star will drift across the line of sight at approximately the middle of the burning period of the star. The deflection required to give a burst at this point (assuming, of course, that the gun is aimed at the target as usually) can be obtained by combining with the deflection for a burst point directly in line with the target, additional corrections for the effects of true wind and motion of target and firing ship during one-half the burning period of the star. Starshell range tables (or graphs) do not include data as to the effects of wind or motion of gun and target, but approximate values of these effects can be obtained from the regular range table by entering the latter with the time of flight corresponding to the burst point, provided that the form of the starshell does not differ greatly from that of the projectile for which the regular range table has been prepared. The following example illustrates the features outlined above.

Let us suppose that it is desired to determine approximately the starshell deflection for the 5″ long-point starshell under the following conditions; the firing ship and target are abeam of each other, port to port, on parallel courses, each steaming at 10 knots; a true wind of 10 knots is blowing from directly astern of

the firing vessel; the target distance is 7000 yards; service charges (3150 f.s.) are to be used. From the graphs for the 5″ long-point starshell it is found that the time of flight corresponding to a burst range of 7000+1000=8000 yards is 13 seconds (the corresponding fuze setting is 17 seconds, but we are not concerned with the latter in this problem). The 5″ long-point starshell is sufficiently similar to the 5″ service projectile to permit the use of the regular 5″3150 f.s. range table for determining approximate values of the drift, and of the effects of target and firing-ship motion and of wind.

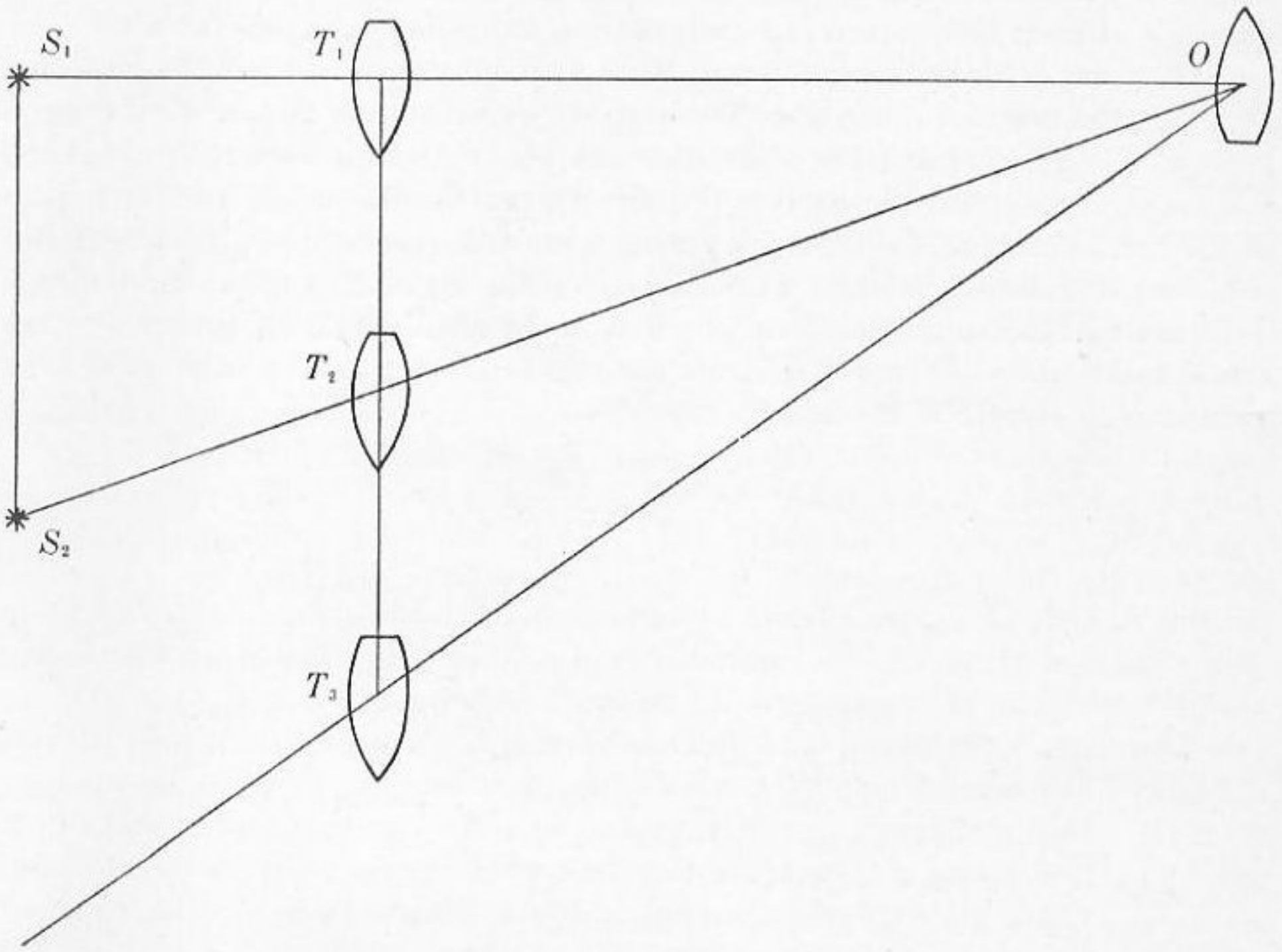


FIGURE 57

We shall first compute the deflection required to give a burst directly in line with the target. Entering the 5″3150 f.s. range table where time of flight 13 seconds appears in Column 4, which occurs at a range of about 8400 yards, we find the following data:

Deflection for a burst point in line with the target

	Error (yds.)
Drift (Col. 6)	41 R
Wind (Col. 16)	27 R
Firing-ship motion (Col. 17)	46 R
Target motion (Col. 18)	73 R
Total	187 R

Since at the burst range of 8000 yards 1 mil is the equivalent of 8 yards, the total deviation of 187 yards to the right requires a correction of about Left 23 mils (Scale 77).

Let us now examine, in turn, the effects of wind, and of motion of target and firing ship, on the position of the star with respect to the line of sight; and let

us assume that the burning period of the starshell is 40 seconds. In 40 seconds the 10-knot wind will carry the star a distance of 222 yards to the right. In order that the motion imparted to the star by the wind may cause the star to drift across the line of sight at the middle of the burning period, the burst should then occur 111 yards to the left of the line of sight; in other words, the error of the position directly in line with the target is 111 yards *right* due to *wind* alone.

Effect of wind during burning period

The effect of target motion alone during the burning period is illustrated in Figure 57, in which O and T_1, respectively, represent the positions of the firing ship and target at the instant at which a burst, S_1, occurs directly in line with the target. During the burning period of the star the target moves to T_3, causing the burst at S_1 to be much too far to the right. In order that the *relative* motion of the star with respect to the target may be such as to cause the star to cross the line of sight at the middle of the burning period, the burst should then occur at S_2, or in line with the position that the target will occupy at the middle of the burning period. The distance T_1T_2 is equal to the travel of the target, at its speed of 10 knots, during one-half the burning period (20 seconds), or it is equal to 111 yards. We have $OT_1 = 7000$ yards, and $T_1S_1 = 1000$ yards. Also

Effect of target motion during burning period

$$\frac{S_1S_2}{111} = \frac{8000}{7000}$$

whence $S_1S_2 = 127$ yards. Thus it is found that the burst should occur 127 yards to the left of the line of sight, and hence the error of the position directly in line with the target is 127 yards *right* due to *target motion* alone.

The effect of firing-ship motion alone during the burning period is illustrated in Figure 58, in which O_1 and T, respectively, represent the positions of the firing ship and target at the instant at which a burst, S_1, occurs directly in line with the target. During the burning period of the star the firing vessel moves to O_3, causing the burst at S_1 to be too far to the right. In order that the relative motion of the star with respect to the target may be such as to cause the star to cross the line of sight at the middle of the burning period, the burst should then occur at S_2, or in line with the position that the firing ship will occupy at the middle of the burning period. We have, in this case, $O_1O_2 = 111$ yards, $O_1T = 7000$ yards, and $TS_1 = 1000$ yards, whence

Effect of firing-ship motion during burning period

$$\frac{S_1S_2}{111} = \frac{1000}{7000}$$

and $S_1S_2 = 16$ yards. Thus it is found that the burst should occur 16 yards to the left of the line of sight, and hence the error of the position directly in line with the target is 16 yards *right* due to *firing-ship motion* alone.

The deflection required for a burst point that will cause the star to cross the line of sight at the middle of the burning period may now be found by combining the several errors found above with those previously found. Let us compare the resultant starshell deflection with the deflection that would be required under the stated conditions for fire directed at the target itself. The elements for the target deflection are found from the 5″3150 f.s. range table for the range 7000 yards.

Comparison of starshell and target-shell deflections

	Error (yds.)	
	Starshell	Target shell
Drift (Col. 6)	41 R	23 R
Wind (Col. 16)	27 R	18 R
Firing-ship motion (Col. 17)	46 R	38 R
Target motion (Col. 18)	73 R	56 R
Displacement for wind, ½ burning period	111 R	—
Displacement for firing-ship motion, ½ burning period	16 R	—
Displacement for target motion, ½ burning period	127 R	—
Total error (yds.)	441 R	135 R
Correction (mils)	Left 55	Left 19

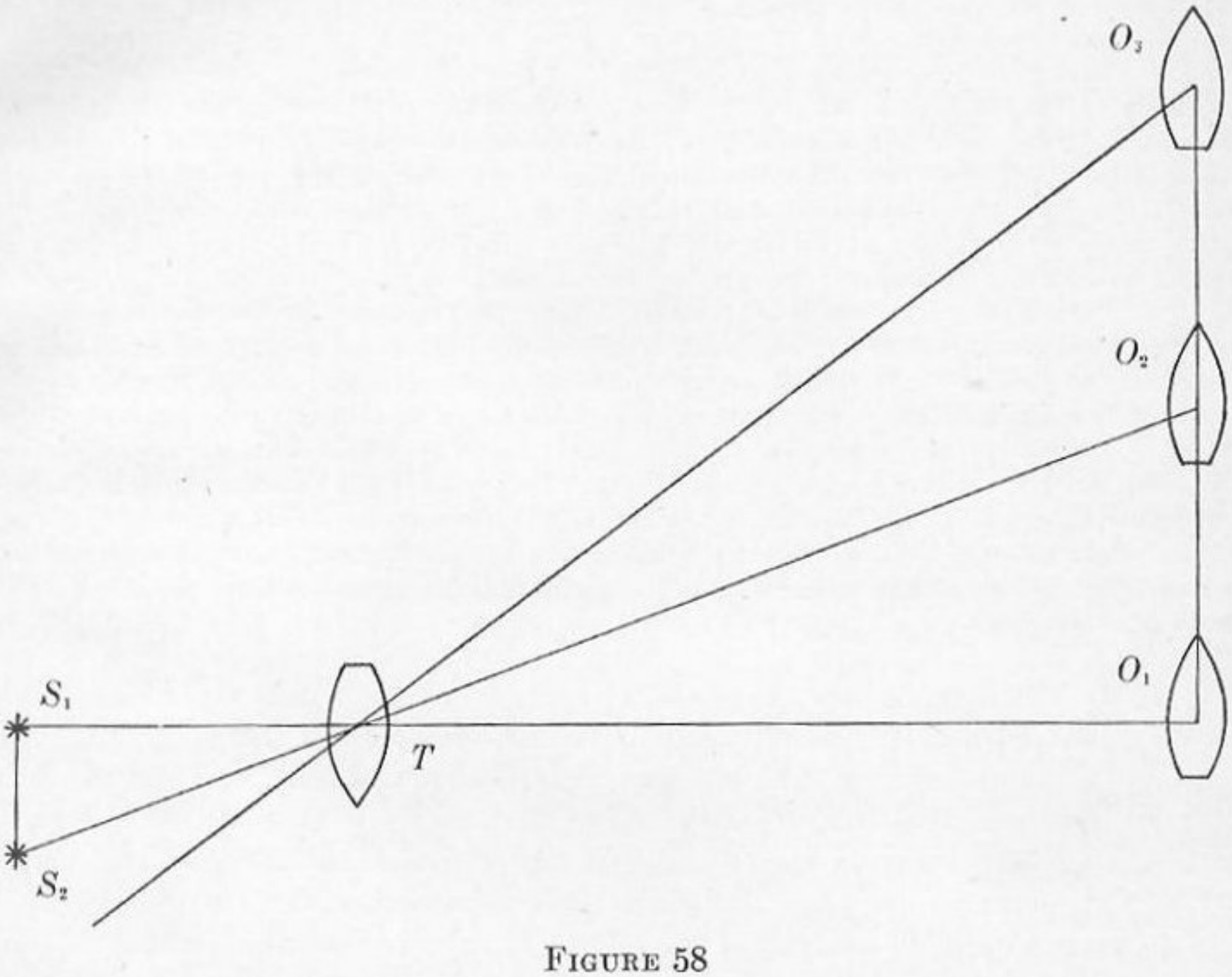
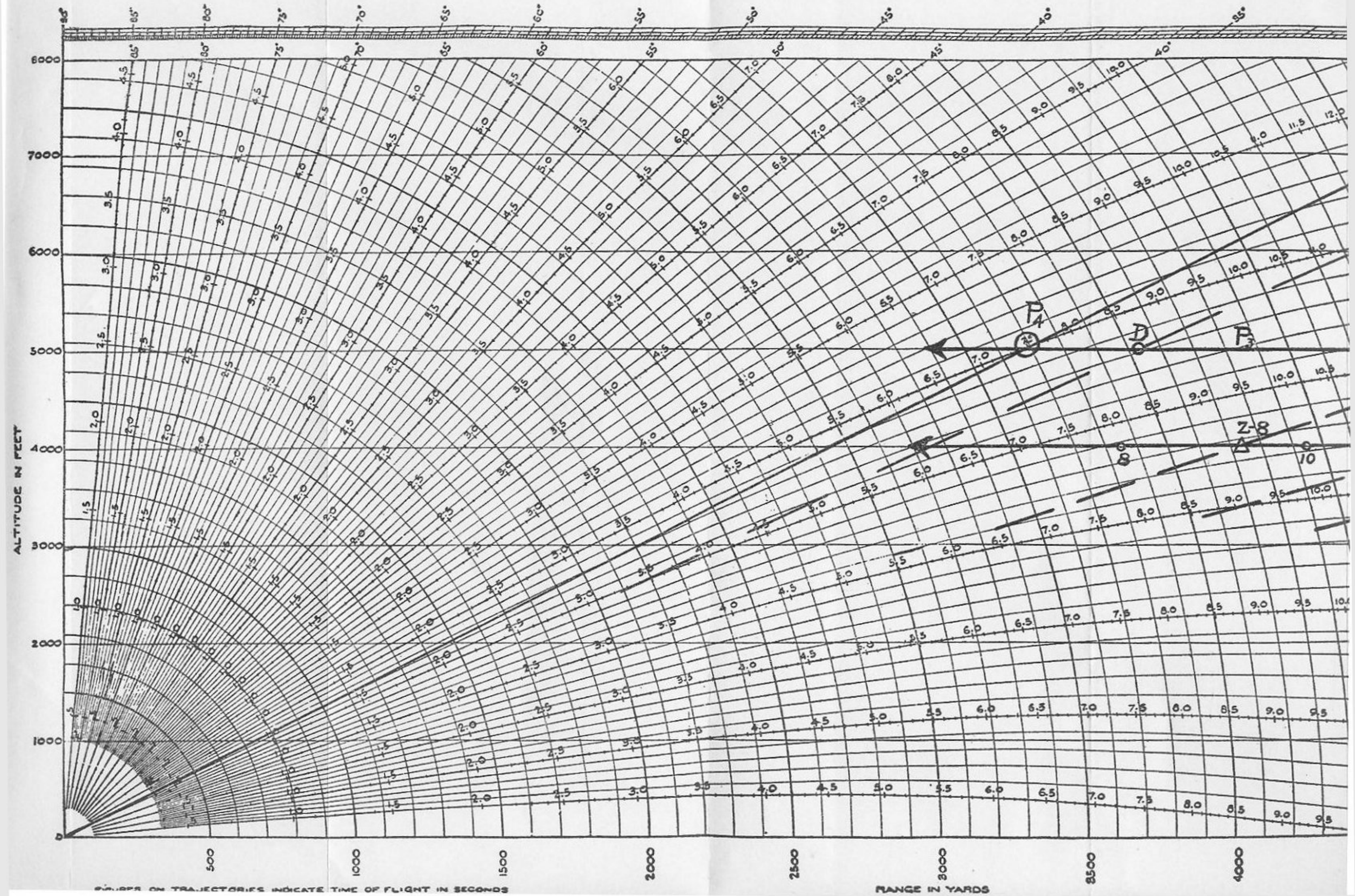


FIGURE 58

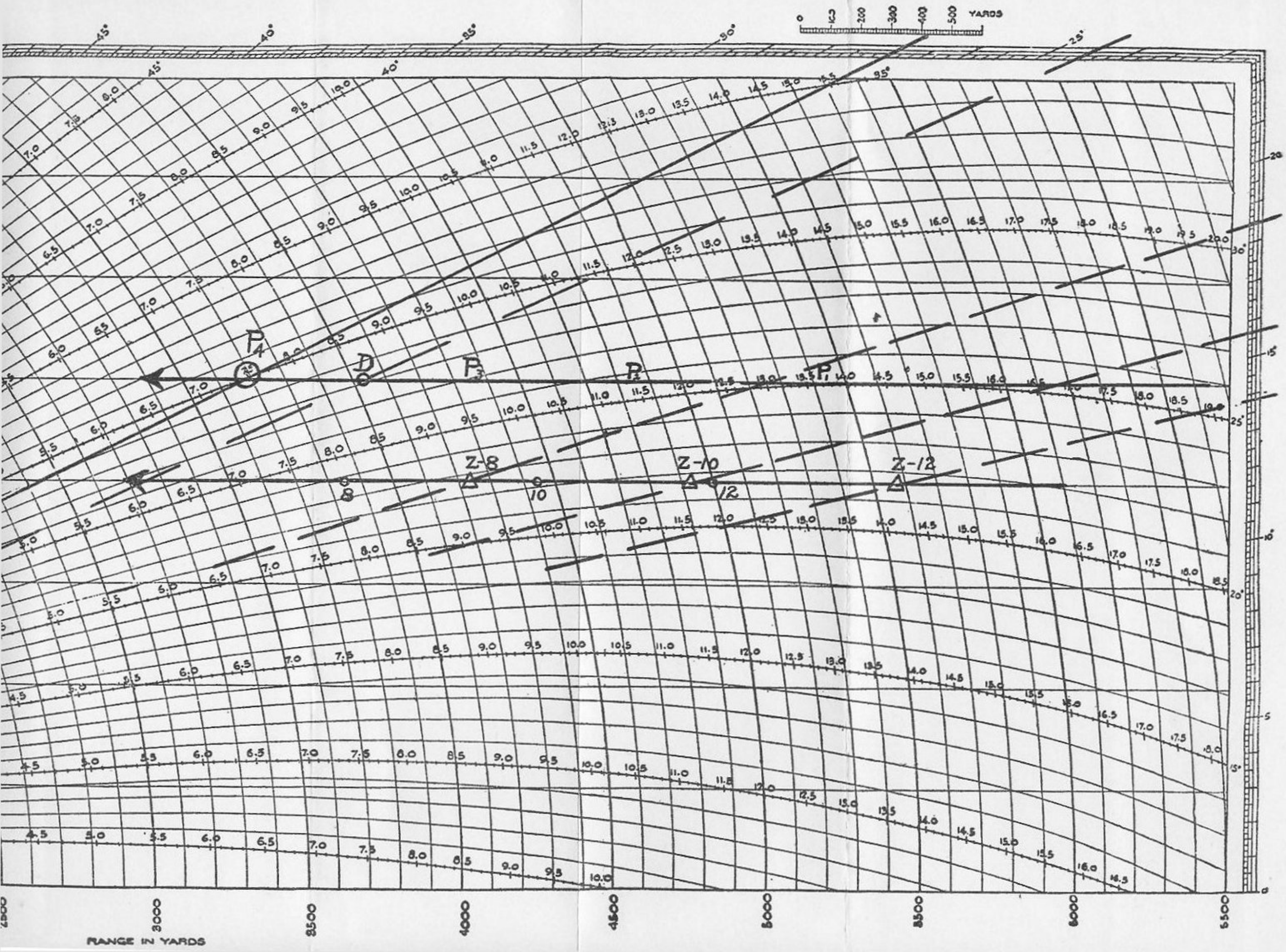
The determination of starshell deflection in the manner outlined above is, of course, a highly artificial procedure insofar as its application to a specific case is concerned. But a knowledge of the elements that enter into the deflection for illumination fire will enable the spotter better to anticipate the extraordinary deflections that often occur in this type of fire. Analysis of starshell deflections for typical situations, by the method outlined above, has been put to practical use in the design of devices for the control of deflection in illumination fire.

ANTIAIRCRAFT TRACKING SHEET
(See introductory note to this appendix)



ANTIAIRCRAFT TRACKING SHEET
(See introductory note to this appendix)

PLATE VI



APPENDIX D

THE EFFECT OF WIND ON THE PRECESSION OF A ROTATING PROJECTILE. THE EFFECT OF CROSS WIND ON THE RANGE OF THE PROJECTILE.

Introductory Note

Although it is a well established fact that cross winds affect the range appreciably at the ranges of Short Range Practice, proving-ground data as to the amount of this effect are totally lacking, and corrections therefor are not given in the range tables. Corrections of an arbitrary nature, designed to account for observed differences between the hitting ballistics of the starboard and port batteries at Short Range Practice, were used perhaps as long as fifteen or more years ago, but it has been only comparatively recently that these differences have been definitely associated with the precessional forces set up by cross winds. However, up to the present time very little has been accomplished toward developing fully the theory governing the action of wind on a rotating projectile, and practically nothing on this subject is to be found in many otherwise complete works on ballistics.

The principal purpose of this appendix is to outline methods for analysis of the effect of cross wind on range and for determination of corrections therefor. Comparison of hitting ballistics for firings to starboard and port at Short Range Practice constitutes, at the present time, practically the sole source of dependable information as to the amount of this effect; the methods outlined herein are accordingly based on such information, and the examples given are confined to short ranges. A brief discussion of the theory governing the action of wind on a rotating projectile is also given, but the purpose of this is merely to give a logical explanation of the observed effect and thus to promote a better understanding of the methods for dealing with it, rather than to provide a basis for theoretical determination of the amount of the effect.

General inadequacy of wind corrections given in range tables

1901. The formulas for the effects of range and cross winds which have been deduced in Chapter 11, and on which Columns 13 and 16 of the range table are based, do not take into account the effects of such winds on the precession of the projectile. Wind obviously operates to alter the pressure which is exerted against the projectile in its flight, and hence operates to alter the projectile's precessional motion and its drift. Complete analysis of the effects of wind on the precessional motion of the projectile is beset with the same difficulties that have already been discussed in connection with the causes of the normal drift (Chapter 9). A further obstacle results from practical difficulties involved in determining experimentally, ballistic effects that may definitely be associated with wind. On the whole, it must be accepted that our knowledge as to the effects of wind on the flight of a rotating projectile is very incomplete, and that the existing wind formulas, which neglect the precessional effects, are in some cases but rough approximations.*

The discussion which follows is limited to a brief examination of the effects of

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* Brief mention of the problem involved in calculating the effect of wind on a rotating projectile is made in §5.13, *The Aerodynamics of a Spinning Shell*, by Fowler, Gallop, Lock and Richmond (Philosophical Transactions of the Royal Society of London, Vol. 221, 1921). A practical method for calculating this effect is given in a paper entitled, *The Approximate Motions of Projectiles*, by Commander W. A. Lee, U. S. Navy (Battleships Gunnery Conferences, 1932).

both range and cross winds on the precessional motion of the projectile, and is designed principally to give a physical explanation of certain observed results which cannot otherwise be accounted for.

1902. Let us consider first the situation that exists near the beginning of the trajectory of a projectile which is fired into an opposing range wind (i.e., a horizontal wind contrary to the direction of fire). The wind exerts a pressure against the under side of the projectile and tends to cause a clockwise precession of the projectile's axis about the horizontal (assuming that the projectile has right-handed rotation). In any practical case, however, the precessional moment set up by the wind is small in comparison with the precessional moment set up by the air resistance, and the revolution about the horizontal which the wind tends to set up is opposed by the forces tending to keep the axis of the projectile nearly in coincidence with the tangent to the trajectory, which result from the action of the air resistance. Consequently the wind continues to exert itself against the under side of the projectile as long as the latter's axis is pointed above the horizontal, i.e., as long as the projectile is in the ascending branch of the trajectory. The wind pressure, although not great enough to cause complete precessional revolutions of the projectile about the axis of the wind, continuously superimposes upon the motion of the projectile a component tending to drive the latter to the right (since the pressure, being exerted continuously against the under side of the nose, tends to turn the latter to the right). An opposing range wind therefore causes an *increase* in the normal drift in the *ascending* branch of the trajectory. The effect of the wind in thus increasing the drift is greatest near the origin, where the inclination of the projectile's axis with respect to the horizontal wind is greatest, and it vanishes at the summit.

Effect of a range wind on precession of the projectile

In the descending branch, with an opposing wind, the wind pressure is exerted against the upper side of the projectile. This pressure tends, as before, to cause clockwise precession of the projectile about the horizontal, which is opposed by the forces tending to keep the axis of the projectile nearly in coincidence with the tangent. The situation is therefore the same as in the ascending branch, except that the wind pressure is now exerted continuously against the projectile's upper side, in consequence of which there is superimposed on the latter's motion a component tending to drive it to the left and hence to *decrease* the drift.

The situation in the case of an *opposing* range wind then is that the drift is increased in the ascending branch of the trajectory and decreased in the descending branch. The total *decrease* of drift which occurs in the descending branch exceeds the total *increase* which occurs in the ascending branch, the effect in the descending branch being the greater due to the greater time of flight, greater average inclination, and lesser average velocity of this branch as compared to the ascending branch. Considering that the total effects in each of the two branches are of a relatively small order of magnitude, it is a reasonable assumption that the difference between the effects for the two branches, or the resultant effect for the entire horizontal trajectory, is small enough to be neglected in any practical case.

It may be reasoned similarly that in the case of a range wind which blows *in* the direction of fire, the drift is *decreased* in the *ascending* branch of the trajectory and *increased* in the *descending* branch, and that the resultant effect for an entire horizontal trajectory is a slight *increase* in the drift.

1903. In the case of a horizontal cross wind, the wind pressure is exerted continuously against one side or the other of the projectile throughout the tra-

Effect of cross wind on precession of the projectile

jectory, and tends to set up a clockwise precession of the projectile's axis about the direction *from* which the wind is blowing. If unopposed, this precession would cause the projectile to revolve end over end in a vertical circle, the point moving downward when the wind meets the left-hand side of the projectile and upward when the wind meets the right-hand side of the projectile. In any practical case, however, the precessional moment set up by the cross wind is small in comparison with the precessional moment set up by the air resistance, and the revolution which the wind tends to set up is opposed by the forces tending to keep the axis of the projectile nearly in coincidence with the tangent to the trajectory, which result from the action of the air resistance. Consequently the wind exerts itself continuously, throughout the trajectory, against the left-hand side of the projectile in the case of a wind blowing from left to right, and against the right-hand side of the projectile in the case of a wind blowing from right to left.*

The pressure of the cross wind, although not great enough to cause complete precessional revolutions of the projectile about the axis of the wind, continuously superimposes upon the motion of the projectile a component which, in the case of a wind blowing from left to right tends to depress the projectile's nose, and in the case of a wind blowing from right to left tends to elevate the projectile's nose. The action of the cross wind thus results in what may be termed a *vertical drift*, the latter being downward and hence shortening the projectile's flight and range when the cross wind strikes the projectile from the left, and upward and hence sustaining the projectile's flight and increasing its range when the cross wind strikes the projectile from the right.

Vertical drift due to cross wind

1904. The conclusions arrived at above therefore are that *a cross wind blowing from left to right decreases the range and a cross wind blowing from right to left increases the range.* It is noteworthy that the effect of a cross wind is of constant direction throughout the trajectory, and also that the wind continues to meet the axis of the projectile practically perpendicularly, and hence with maximum effect, throughout the trajectory. It follows, then, that the change in point of fall of a horizontal trajectory, due to the precessional effect of wind, is much greater in the case of a cross wind than in the case of a range wind. The latter conclusion has been substantiated by practical experience at least to the extent that material changes of range which are clearly attributable to the effects of cross winds have been observed at some target practices, whereas similar evidence as to the effects of range winds on the drift is totally lacking.

Cross-wind effect on range

1905. Experimental data pertaining to changes in range due to cross winds have been derived principally from analysis of firings at Short Range Practice. This practice lends itself especially well to analysis of the effects of cross winds, for the firings of the starboard and port batteries usually involve large changes in the cross-wind components, without material changes in the many other factors that influence the result. It is a well known fact that at Short Range Practice the hitting ballistics (in range) of the starboard and port batteries, firing under otherwise nearly identical conditions, usually differ materially. The explanation of this rests in the fact that the cross-wind components for the two batteries are usually opposite to each other in direction.

Sources of data as to effects of cross wind on range

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* The terms *right* and *left*, as used hereafter throughout this discussion, are to be considered as signifying directions referred to the direction of fire.

In the case of fire from a moving ship, the projectile is acted upon by an *apparent wind* which is the resultant of the true wind and of the ship's motion (art. 1217). At Short Range Practice ships usually select firing courses which bring the apparent wind a few points on the bow. Under these conditions the direction of the cross wind is from left to right for the starboard battery, and from right to left for the port battery; other things being equal, it is then to be expected that greater sight-bar ranges will be required for firing to starboard than for firing to port, and this is, indeed, found to be the case. In other words, when the apparent wind is from ahead the cross-wind effect requires that the ballistic corrections in range include an additional *up correction* for the starboard battery, and an additional *down correction* for the port battery.

It is to be noted, however, that the above difference between the ballistic corrections in range for the starboard and port batteries depends directly upon the amount and direction of the cross wind for each battery, and that it is associated with the difference in location between the two batteries only insofar as the latter affects the direction of the line of fire with respect to the direction of the apparent wind.* Due to the forward motion of the firing ship, the apparent wind is more often from forward of the beam than from abaft the beam, and it follows that the condition requiring up corrections for the starboard battery and down corrections for the port battery (for cross-wind effect) occurs more often than that requiring the reverse. It must be borne in mind, however, that when the apparent wind blows from a direction abaft the beam, down corrections are required to starboard and up corrections to port.

Analysis of cross-wind effect on range from S.R.B.P. firing

1906. The following examples illustrate a convenient method of analyzing target-practice results for the purpose of obtaining values of the correction in range due to the effect of cross wind.

I. *Given:* The post-firing computed *gun ballistics* in range (Sheet 10) for the Secondary Battery groups of a battleship, for Short Range Practice, were as follows: starboard battery, *Up* 117 yards; port battery, *Up* 128 yards. The *hitting navigational-range ballistics* for the same groups were: starboard battery, *Up* 207 yards; port battery, *Up* 90 yards. The cross winds (i.e., components of apparent wind perpendicular to the line of fire) were as follows for these groups: starboard battery, 14 knots *from the left;* port battery, 26 knots *from the right.*

Find: The *correction* in range (in yards per knot) due to cross wind, and the residual arbitrary ballistic, corresponding to the above data.

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* The fact that Short Range Practice is usually fired with the apparent wind from ahead, under which condition a projectile fired to port actually travels farther than one fired to starboard (other things being equal), has led to a popular misconception that this is *always* the case. Evidence of this is to be seen in the term "port-side effect" which is commonly used in referring to the difference between the hitting ballistics of the two batteries. This term is misleading in that it associates the difference between the two batteries with their locations on the ship, whereas, in fact, the difference depends entirely on the direction of the cross wind, as noted above.

The term "Magnus Effect" is also commonly used in referring to the change of range caused by cross wind. However, the "Magnus Effect" due to cross wind causes vertical drift upward, and hence increase of range, for a cross wind blowing from left to right,—and vertical drift downward, and hence decrease of range, for a cross wind blowing from right to left (see art. 904). The effects actually observed in practice occur in the opposite directions, and hence cannot be attributed to "Magnus Effect."

We have, to begin with, the following differences between the hitting (or actual) ballistics and the computed (or predicted) ballistics for the two batteries:

	Port battery	Starboard battery
Hitting ballistic	+ 90 yds.	+207 yds.
Computed ballistic	+128 yds.	+117 yds.
Difference	− 38 yds.	+ 90 yds.
Cross wind*	− 26 kts.	+ 14 kts.

It is at once apparent that these entire differences are not proportional to their respective cross-wind components, and that the differences must include something besides the changes due to cross-wind effect. Let us assume, then, that the difference for each battery includes, (1) a correction in range due to cross-wind effect, proportional in amount to the component of cross wind for that battery, and (2) a residual arbitrary correction which is common to both batteries. Let us denote by x (in yards) the *correction* in range due to *1 knot* of cross wind; according to the conclusions stated in articles 1904 and 1905, x will be negative for wind from the right and positive for wind from the left. Let us denote by y (in yards) the residual arbitrary ballistic (which is, by assumption, the same for both batteries). We may now set up the following relations:

Starboard battery . . . . . $y+14x=90$
Port battery . . . . . $y-26x=-38$

whence we find, $x=3.2$ yards per knot, and $y=45$ yards.

In other words, we have found that the *correction* in range for the effect of cross wind is 3.2 yards per knot (of cross wind), and that the residual arbitrary ballistic is *Up* 45 yards. The data of this example are fairly typical for the firing of a 5″2300 f.s. gun (reduced charge) at Short Range Practice, at ranges of 1600 to 1700 yards.

The following is an example of the analysis for a case in which the cross wind is in the same direction, with respect to the line of fire, for both batteries; this condition may arise if the firing vessel is obliged to change the base course between the firings of the two batteries.

II. *Given:* The post-firing computed *gun ballistics* in range (Sheet 10) for the Secondary Battery groups of a battleship, for Short Range Practice, were as follows: starboard battery, *Up* 96 yards; port battery *Up* 82 yards. The *hitting navigational-range ballistics* for the same groups were: starboard battery, *Up* 173 yards; port battery, *Up* 91 yards. The cross winds were as follows for these groups: starboard battery, 32 knots *from the left;* port battery, 12 knots *from the left.*

Find: The *correction* in range (in yards per knot) due to cross wind, and the residual arbitrary ballistic, corresponding to the above data.

Proceeding just as in the first example, we have:

	Port battery	Starboard battery
Hitting ballistic	+91 yds.	+173 yds.
Computed ballistic	+82 yds.	+ 96 yds.
Difference	+ 9 yds.	+ 77 yds.
Cross wind*	+12 kts.	+ 32 kts.

* The signs used with the cross-wind components here agree with the signs of the corresponding range *corrections.*

Starboard battery . . . . . $y+32x=77$
Port battery . . . . . $y+12x=9$

whence we find $x=3.4$ yards per knot, and $y=-32$ yards. In other words, the *correction* in range for the effect of cross wind is 3.4 yards per knot (of cross wind), and the residual arbitrary ballistic is *Down* 32 yards.

Cross-wind effect on range for various guns

1907. Analysis of Short Range Practice results has shown beyond doubt that at the ranges of this practice (1600–2100 yards) cross wind affects the range materially, especially in the case of the smaller calibers. Values of from 3.0 to 3.5 yards per knot are generally found in the case of the 5″/51 2300 f.s. gun.* Other typical values are: about 2.5 yards per knot for the 5″/25 2200 f.s. gun; about 1.5 yards per knot for the 6″/53 2300 f.s. gun; about 2 yards per knot for the 4″/50 2900 f.s. gun; about 1 yard per knot for major calibers at their target-practice velocities (14″ and 16″).

Comparison between effects on range of winds *in* and *across* line of fire

It is noteworthy that at the ranges of Short Range Practice the effect on *range* of wind *in* the line of fire is practically negligible in comparison with the effect on *range* of wind *across* the line of fire. For example, Column 13 of the 5″/51 2300 f.s. range table shows that at 1700 yards a 10-knot wind *in* the line of fire causes a change of range of 3 yards; analysis of firings at this range shows that a 10-knot wind *across* the line of fire causes a change of range of about 30 or 35 yards, or a change at least ten times as great as that due to wind *in* the line of fire. The ratio in the cases of other guns is also of the order of about ten to one. It should be understood, however, that the situation here described exists particularly at very short ranges. The situation for long ranges will be discussed later.

Determination and application of range corrections for wind at S.R.P.

1908. The following examples illustrate the method of determining and applying corrections to the range for the components of wind in and across the line of fire.

I. *Given:* The Secondary Battery of a battleship is to fire its 5″/51 guns, at Short Range Practice, at the reduced velocity of 2300 f.s. The ballistic corrections in range have been computed in advance of the practice for all items except apparent wind, and the total ballistic (less wind) for each battery has been found to be as follows: starboard battery, *Up* 122 yards; port battery, *Up* 116 yards. From analysis of previous firings of this battery it has been determined that a correction in range of 3.2 yards per knot is required for the effect of cross wind. The value in Column 13 of the 5″2300 f.s. range table,† at 1700 yards, is 3 yards. Just prior to the firing of each battery the apparent wind was observed to be as follows: starboard battery, 30 knots from 15° relative; port battery, 27 knots from 340° relative. The mean relative bearing of the target was expected to be practically 90° for the starboard side, and 270° for the port side.

Find: The total ballistic corrections in range (including wind) for the two batteries.

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* This applies to the 5″/51 guns with uniform-twist rifling of 1 turn in 35 calibers; values around 4 yards per knot were found for the 5″/51 gun with increasing-twist rifling (final twist 1 in 25) which was formerly in use.

† Note that this is not the 5″ range table which is given in *Range and Ballistic Tables, 1935*.

Let us first resolve the wind, for each battery, into its components *in* and *across* the line of fire (ref. arts. 1215–1216). We have,

	Port battery	Starboard battery
Wind in line of fire	$27 \cos 70° = 9$ kts. (against)	$30 \cos 75° = 8$ kts. (against)
Wind across line of fire	$27 \sin 70° = 25$ kts. (from right)	$30 \sin 75° = 29$ kts. (from left)

The *corrections* for the components *in* the line of fire are, using the given value from Column 13,

Starboard battery $\dfrac{8}{10} \times 3 = +2$ yards

Port battery $\dfrac{9}{10} \times 3 = +3$ yards

The *corrections* for the components *across* the line of fire are, using the given value of 3.2 yards per knot,

Starboard battery $(+)29 \times 3.2 = +93$ yards
Port battery $(-)25 \times 3.2 = -80$ yards

The required total corrections then are,

Corrections	Port battery	Starboard battery
Computed (less wind)	+116 yds.	+122 yds.
Wind in line of fire	+ 3 yds.	+ 2 yds.
Wind across line of fire	− 80 yds.	+ 93 yds.
Total	+ 39 yds.	+217 yds.

II. *Given:* The same data as for example I of this article, except that the apparent wind observed just prior to the firing of each battery was as follows: starboard battery, 12 knots from 150° relative; port battery, 30 knots from 300° relative.

Find: The total ballistic corrections in range (including wind) for the two batteries.

Proceeding just as in the first example, we have,

	Port battery	Starboard battery
Wind in line of fire	$30 \cos 30° = 26$ kts. (against)	$12 \cos 60° = 6$ kts. (against)
Wind across line of fire	$30 \sin 30° = 15$ kts. (from right)	$12 \sin 60° = 10$ kts. (from right)

Corrections for components *in* line of fire,

Starboard battery $\dfrac{6}{10} \times 3 = +2$ yards

Port battery $\dfrac{26}{10} \times 3 = +8$ yards

Corrections for components *across* line of fire,

Starboard battery.....(−)10×3.2=−32 yards
Port battery.....(−)15×3.2=−48 yards

Required total corrections,

Corrections	Port battery	Starboard battery
Computed (less wind).....	+116 yds.....	+122 yds.
Wind in line of fire.....	+ 8 yds.....	+ 2 yds.
Wind across line of fire.....	− 48 yds.....	− 32 yds.
Total.....	+ 76 yds.....	+ 92 yds.

1909. Analysis of cross-wind effects for long ranges, from target-practice results, is rarely practicable, for the conditions attending long-range practices are generally such as to render uncertain a number of the factors upon which the ballistic corrections depend. Considering the degree of approximation with which aloft winds are generally known, and with which the ballistic wind is determined therefrom, it is difficult, in the first place, to discriminate between errors which may be due to the use of incorrect values of the wind itself, and those which may be due to incorrect values as to the effect of the wind. Another fundamental difficulty lies in the uncertainty which usually exists in the measurement of the actual ranges obtained. Analysis of cross-wind effects depends upon accurate knowledge of hitting ballistics under materially varying conditions of cross wind; this, in turn, involves accurate knowledge of hitting gun ranges and navigational ranges for different legs, or phases, of a practice. It is doubtful whether the determination of these values for long-range practices, even by careful post-firing analysis, is ever accomplished with sufficient accuracy to yield dependable values of the cross-wind effect at long ranges.

Cross-wind effect on range for long ranges

Such information as has been obtained from analysis of long-range target practices indicates that at such ranges the cross-wind effect on range is relatively small, or, at least, too small to be clearly identified, under the limitations of analysis noted above, among the ordinary errors attending long-range fire. Although this finding appears rather surprising, considering the relatively great effect noted at short ranges, it is, in fact, substantiated by the theory governing the action of the projectile under the influence of cross wind. As noted in article 1903, cross wind causes a vertical drift of the projectile. Since the direction of the cross wind with respect to the direction of the projectile remains practically constant throughout the trajectory, the vertical drift increases approximately in proportion to the time of flight.* But the change of range due to a given amount of vertical drift is equal, approximately, to the vertical drift multiplied by the cotangent of the angle of fall. As the range increases, the cotangent of the angle of fall decreases more rapidly than the time of flight increases, whence it follows that, even although the *vertical drift increases* with range, the *change of range* due thereto *decreases* with range. In the case of the 5″2300 f.s. gun it is likely that the cross-wind effect on range at 10,000 yards does not exceed about 2 yards per knot, as compared to 3 or 3.5 yards per knot at 1700 yards. The effect for long ranges may

* Change in stability of the projectile as it loses velocity, while the cross wind remains constant, alters the rate of vertical drift somewhat. Commander Lee, in his theoretical analysis of the cross-wind effect for the 5″ 2300 f.s. and 14″ 2000 f.s. guns (see footnote on page 290) finds that the vertical drift in these cases increases somewhat more rapidly than the time of flight, but that the resultant changes of range nevertheless decrease with range.

therefore be regarded as of relatively small importance, in comparison with range-finding errors, errors in measurement of winds and densities, etc., at long ranges.

EXERCISES

I. *Given:* The post-firing computed gun ballistics in range, hitting navigational-range ballistics, and observed apparent winds, for the starboard and port batteries at Short Range Practice; the mean relative bearing of target was 90° for the starboard battery, and 270° for the port battery.

Find: The correction in range (in yards per knot) due to cross wind, and the residual arbitrary ballistic.

	Given									Answers	
	Gun	Port battery				Starboard battery				Cross-wind effect	Residual arbitrary ballistic
		Computed gun ballistic	Hitting ballistic	Wind		Computed gun ballistic	Hitting ballistic	Wind			
				Velocity	Rel. brg.			Velocity	Rel. brg.		
		yds.	yds.	kts.	deg.	yds.	yds.	kts.	deg.	yds./kt.	yds.
A	4″/2900 f.s.	Up 80	Up 49	28	330	Up 88	Up 172	30	20	2.2	Up 22
B	5″/2200 f.s.	Up 65	Down 3	24	350	Up 57	Up 98	21	30	2.6	Down 6
C	5″/2300 f.s.	Up 85	Up 177	8	210	Up 75	Up 235	28	10	3.2	Up 70
D	6″/2300 f.s.	Up 112	Up 74	33	340	Up 120	Up 125	6	160	1.7	Up 15

II. *Given:* The pre-firing ballistic corrections in range (less wind), correction in range due to cross wind (in yards per knot), and the apparent winds observed just prior to firing, for the starboard and port batteries at Short Range Practice; the mean relative bearing of target for the starboard battery was expected to be 90°, and for the port battery 270°.

Find: The total ballistic corrections in range (including wind) for the two batteries.

	Given									Answers	
	Gun	Cross-wind effect on range	Range table, Col. 13*	Port battery			Starboard battery			Total ballistic corrections in range	
				Computed ballistic less wind	Wind		Computed ballistic less wind	Wind		Port	Stbd.
					Velocity	Rel. brg.		Velocity	Rel. brg.		
		yds./kt.	yds.	yds.	kts.	deg.	yds.	kts.	deg.	yds.	yds.
A	4″/2900 f.s.	2.0	2	Up 90	33	340	Up 104	6	160	Up 30	Up 92
B	5″/2200 f.s.	2.5	3	Up 85	8	210	Up 80	28	10	Up 104	Up 152
C	5″/2300 f.s.	3.2	3	Up 65	24	350	Up 68	21	30	Down 11	Up 129
D	6″/2300 f.s.	1.5	2	Up 70	28	330	Up 88	30	20	Up 37	Up 132

* The values given are for 10-knot components.

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