

STRESSES IN WIRE-WRAPPED GUNS AND IN GUN CARRIAGES

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PREFACE TO SECOND EDITION.

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COLDEN L'H. RUGGLES.

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PREFACE.

In this text the author has endeavored to explain and illustrate a number of the important engineering principles underlying the design of wire-wrapped guns and of gun carriages, and in its preparation he has made free use of the methods of officers of the Ordnance Department, U. S. Army, who have been engaged on such work and of the various publications on the subject written by them or translated by them from foreign sources.

The deductions of the formulas relating to wire-wrapped guns and their application to a 12-inch gun on which the wire is wrapped under constant tension have been taken from *Notes on the Construction of Ordnance Nos. 38 and 87* written by General William Crozier, Chief of Ordnance, U. S. Army, when a junior officer of the Ordnance Department, the formulas giving the tensions in the wire envelope and the pressures produced by it being, as stated by General Crozier, mainly those of Longridge, an English engineer. Some shortening of the mathematical work involved in the deductions of the formulas has been effected by the author of this text by starting with the assumption that the modulus of elasticity is the same for the steel wire as for the steel tube of the gun, and the formulas have been slightly extended to include the radial stresses in the wire envelope.

In the preparation of Chapter II much assistance was derived from the ordnance pamphlet entitled *A Discussion of the Methods Proposed to Increase the Rapidity of Fire of Field Guns* by Captain (now Lieutenant Colonel) Charles B. Wheeler and Captain (now Major) William H. Tschappat, Ordnance Department; and from the original calculations made in the Office of the Chief of Ordnance in connection with the design of the 3-inch field carriage, model of 1902, and the 5-inch barbette carriage, model of 1903.

Much assistance was likewise derived in the preparation of

Chapter III from the original calculations made in the Office of the Chief of Ordnance in connection with the design of the 6-inch disappearing carriage, model of 1905 M1. The computations relating to the throttling grooves of this carriage given in the text (which are the same in principle as those made for the throttling grooves of a number of earlier models of disappearing carriages) are practically identical with those made for this carriage in the Office of the Chief of Ordnance by Captain James B. Dillard, Ordnance Department, acting under the direction of Major John H. Rice, Ordnance Department, the present chief of the gun-carriage division of that office.

The sources of the formulas used in Chapter IV are given in the text.

In the preparation of Chapter V the author has freely consulted various standard works on applied mechanics and mechanical engineering, the greatest assistance having been derived from the works of the International Library of Technology.

The methods outlined in Chapter VI are largely based upon the practice of the Ordnance Department.

The thanks of the author are due to Major Tracy C. Dickson, Ordnance Department, for advice and information in connection with the preparation of the text; and to Captain Otho V. Kean, Ordnance Department, 1st Lieutenant Ned B. Rehkopf, 2nd Field Artillery, and 1st Lieutenant George R. Allin, 6th Field Artillery, instructors in the Department of Ordnance and Science of Gunnery, U. S. Military Academy, for suggestions tending to add to the clearness of the text, for checking and correcting where necessary the results of the computations, and for reading the proofs.

The author wishes to thank also Sergeant Carl A. Schopper, Detachment of Ordnance, U. S. Military Academy, for the skill and care with which he has prepared the many drawings for the figures appearing in the text.

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Stresses in Wire-Wrapped Guns and in Gun Carriages.

CHAPTER I.

ELASTIC STRENGTH OF WIRE-WRAPPED GUNS.

1. General Construction. — Wire-wrapped guns consist of (a) an inner steel tube which forms the support on which the wire is wrapped and in which the rifling grooves are cut; (b) the layers of wire wrapped upon the tube to increase its resistance by the application of an exterior pressure as well as to add to the strength of the structure by their own resistance to extension under fire; and (c) one or more layers consisting of a steel jacket and hoops placed over the wire with or without shrinkage. The jacket is generally relied upon to furnish the longitudinal strength of the gun since this feature is lacking in the wire envelope. The breech-block is therefore ordinarily screwed into the jacket, or into a breech bushing screwed into the jacket, in order that the latter may resist the longitudinal stress due to the pressure at the bottom of the bore, the consequent rearward acceleration of the gun in recoil, and the resistance to recoil of the recoil brake applied to the gun through the trunnions or through a lug forming part of the jacket.

2. An Important Principle. — A very important principle in wire gun construction is that enunciated in paragraph 121, page 216, Lissak's Ordnance and Gunnery, to the effect that the stresses produced by any pressure applied to a compound cylinder are exactly the same as would be produced by the same pressure applied to a single cylinder of the same dimensions. This refers to the stresses and strains produced by the pressure under consideration only, and if before the application of this pressure there existed in the compound cylinder stresses and

strains produced by shrinkage or otherwise, the resulting stresses are the algebraic sum of those previously existing and those induced by the application of the pressure. It follows from this that if we know the tangential and radial stresses and strains existing at any radius r in a compound cylinder before the application of an interior or exterior pressure, the resultant tangential and radial stresses and strains therein at any radius r when an interior or exterior pressure or both are acting can be readily obtained by adding algebraically to those previously existing the stresses and strains computed from the appropriate one of equations (9) and (10), paragraph 104, page 195, Lissak's Ordnance and Gunnery, which are as follows:

$$El_t = S_t = \frac{2}{3} \frac{P_0 R_0^2 - P_1 R_1^2}{R_1^2 - R_0^2} + \frac{4}{3} \frac{R_0 R_1^2 (P_0 - P_1)}{R_1^2 - R_0^2} \frac{1}{r^2} \quad (1)$$

$$El_p = S_p = \frac{2}{3} \frac{P_0 R_0^2 - P_1 R_1^2}{R_1^2 - R_0^2} - \frac{4}{3} \frac{R_0^2 R_1^2 (P_0 - P_1)}{R_1^2 - R_0^2} \frac{1}{r^2} \quad (2)$$

in which R_0 and R_1 are the inner and outer radii, respectively, of the compound cylinder and r is the radius of any point within the wall of the cylinder. If the compound cylinder is composed of any number of layers n and its radii are designated as $R_0, R_1, R_2, \dots, R_n$, R_0 in formulas (1) and (2) will be the R_0 of the compound cylinder but the R_1 of these formulas will be the R_n of the compound cylinder. r in the formulas may have any value between R_0 and R_n of the compound cylinder.

Similarly if we know the resultant tangential and radial stresses and strains at any radius r in a compound cylinder when acted upon by an interior or exterior pressure, or by both, the tangential and radial stresses and strains that will remain at any radius r when these pressures are removed can be readily obtained by subtracting algebraically from the resultant stresses and strains those computed from the appropriate one of equations (1) and (2) as due to the pressures that have been removed.

3. Difference Between Tangential Tension (or Tension Simply) and Tangential Stress and Strain; and Between Radial Pressure and Radial Stress and Strain. — Equations (7) and (8), page 194, Lissak's Ordnance and Gunnery, which are as follows:

* The equations in this discussion will be renumbered consecutively as used regardless of their number in Lissak's Ordnance and Gunnery.

$$t = \frac{P_0 R_0^2 - P_1 R_1^2}{R_1^2 - R_0^2} + \frac{R_0^2 R_1^2 (P_0 - P_1)}{R_1^2 - R_0^2} 1/r^2 \quad (3)$$

$$p = -\frac{P_0 R_0^2 - P_1 R_1^2}{R_1^2 - R_0^2} + \frac{R_0^2 R_1^2 (P_0 - P_1)}{R_1^2 - R_0^2} 1/r^2 \quad (4)$$

differ from equations (1) and (2) in that they give the tangential tension t and the radial pressure p at any radius r produced by the application of interior and exterior pressures to a cylinder whose inner and outer radii are R_0 and R_1 , respectively. In the deduction of these equations tensile forces and tensile strains have been considered positive and compressive forces and compressive strains negative. The radial pressure in the walls of a gun is always a compressive force. The difference between the tension and the tangential stress and strain, and between the radial pressure and the radial stress and strain should be carefully noted. The tangential strain l_t is due both to the tangential tension (or compression) and the radial pressure and is equal to

$$1/E [t - (-p/3)] \quad \text{or} \quad 1/E (t + p/3)$$

obtained from the first of equations (4), page 192, Lissak's Ordnance and Gunnery, by making $q = 0$; and the tangential stress El_t is equal to this strain multiplied by the modulus of elasticity E , or to $(t + p/3)$. If the tension and the pressure have different signs, that is, if one is a tensile and the other a compressive force, the tangential stress will be greater numerically than the tension by one-third of the radial pressure; but if they have like signs, that is, if both are tensile or compressive forces the tangential stress will be equal numerically to the difference between the tangential tension and one-third of the radial pressure. Similarly the radial strain l_p is due both to the tangential tension and the radial pressure and is equal to

$$1/E (-p - t/3) = -1/E (p + t/3)$$

obtained from the second of equations (4), page 192, Lissak's Ordnance and Gunnery, by making $q = 0$; and the radial stress El_p is equal to this strain multiplied by the modulus of elasticity E , or to $-(p + t/3)$. If the tension and the pressure have different signs the radial stress will be numerically greater than the radial pressure by one-third of the tangential tension; but if they have like signs the radial stress will be equal numerically

to the difference between the radial pressure and one-third of the tangential tension.

4. The Elastic Strength of a Gun is Reached when Either El_t or El_p is Equal to the Elastic Limit of the Material. — Tension and Pressure at any Radius r of a Compound Cylinder, System in Action or at Rest. — While as a matter of fact, neglecting the longitudinal force, the only forces acting on a particle in the walls of a gun are the tangential tension and the radial pressure, the strain produced by these forces acting together is, in the tangential or radial direction, the same as would be produced by a force El_t or El_p , respectively, acting alone. It is generally accepted that if the material is subjected to a compressive or tensile strain equal to that occurring at the elastic limit when a single force is applied in the direction of the strain, it will yield no matter how the strain may be produced; and therefore the stresses El_t and El_p corresponding to the strains l_t and l_p instead of the tangential tension and the radial pressure are considered as determining whether or not the metal of the gun is being worked within the elastic limit.

As in the case of the stresses and strains, the tensions and pressures produced by any interior or exterior pressure or both applied to a compound cylinder are exactly the same as would be produced by the same pressure or pressures applied to a single cylinder of the same dimensions, and if we know the tension and pressure at any radius r of the compound cylinder before the application of the pressures, the resultant tension and pressure at that radius when the pressures are acting may be determined by adding algebraically to those previously existing, those due only to these pressures, computed from equations (3) and (4). Also if we know the resultant tension and pressure which exist at any radius r of a compound cylinder when interior or exterior pressures are acting, those which will remain at the radius r when the interior or exterior pressures are removed can be obtained by subtracting algebraically from the resultant tension and pressure those computed by equations (3) and (4) as due only to the pressures which have been removed.

5. The Elastic Strength of a Compound Cylinder, Properly Assembled to Secure the Maximum Resistance to an Interior Pressure, Depends only on the Sum of the Elastic Limits for

Compression and Tension of the Material of the Tube and the Thickness of the Wall in Calibers. — Compression of the Tube, System at Rest, Beyond its Elastic Limit. — It follows from the above discussion that if a compound cylinder has been so assembled as to compress the inner surface of the inner cylinder to the elastic limit ρ for that cylinder, the elastic strength of the compound cylinder can be determined from equation (1) by finding that interior pressure which acting alone on the compound cylinder will cause a stress $\rho + \theta$ on its interior surface, which stress will overcome the initial compression ρ and cause a tangential extension equal to θ , the elastic limit for tension of the material.

Making $P_1 = P_n = 0$, $R_1 = R_n$, $r = R_0$, and $S_t = \rho + \theta$ in equation (1) and solving for P_0 , we have

$$P_0 = \frac{3(R_n^2 - R_0^2)(\rho + \theta)}{4R_n^2 + 2R_0^2} \quad (5)$$

which gives the maximum interior pressure which the compound cylinder can withstand without exceeding the elastic limit for tension of the inner cylinder. In this discussion it is assumed that the compound cylinder whether wire-wrapped or of the built-up construction has been properly assembled, in which case the stresses in the layers outside the inner cylinder will not exceed the elastic limit whether in the state of rest or action. As shown in paragraph 121, page 217, Lissak's Ordnance and Gunnery, the greatest value of P_0 , corresponding to $R_n = \infty$, is $.75(\rho + \theta)$. Making $R_n = 3R_0$, corresponding to a thickness of wall of one caliber,

$$P_0 = .63(\rho + \theta)$$

and, therefore, comparatively little advantage results from increasing the thickness of wall beyond one caliber. On the other hand, whatever be the mode of assembling the compound cylinder, by wrapping the tube with wire or otherwise, the elastic strength of the cylinder, if properly assembled to secure the maximum resistance to an interior pressure, will depend only on the sum of the elastic limits for compression and tension of the tube and the thickness of the wall in calibers.

Many designers of wire-wrapped guns have compressed the tube in the state of rest beyond the elastic limit for tangential

compression, and on the assumption that the elastic limit for tension has not been lowered thereby, have computed the elastic strength of the gun from equation (1), substituting for ρ the value of the actual stress of compression of the bore of the tube. Many experiments, however, indicate that if the metal is compressed beyond the elastic limit, its elastic limit for tension is lowered, and vice versa. Therefore, if, when such a gun is fired, the stress at the bore is raised to the elastic limit for tension θ as determined in the testing machine, it will in reality have passed the actual elastic limit, due to the over-compression at rest. As a result the tube will be worked beyond its elastic limit both in tension and compression. Such treatment is known to be most injurious if the repetitions of stress are sufficiently numerous, but owing to the comparatively limited number of rounds fired in any gun no trouble has so far resulted in wire-wrapped guns from over-compression of the tube.

6. Two Principal Methods of Wrapping Wire on a Gun Tube. Special Formulas Relating to Layers of Wire Wrapped on a Gun Tube. — Let Fig. 1 represent a section of a wire-wrapped gun

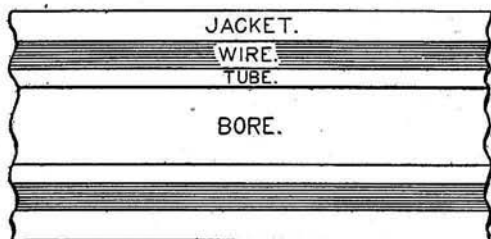


Fig. 1.

consisting of tube, wire envelope, and jacket. Represent the inner and outer radii of the tube by R_0 and R_1 , respectively, the radius of the outer surface of the wire envelope by R_2 and that of the outer surface of the jacket by R_3 . When wire is wrapped under tension around a tube the effect is to cause a pressure on its exterior surface which compresses the metal tangentially, the greatest stress occurring as has been shown (paragraph 108, page 200, Lissak's Ordnance and Gunnery) at the bore. The pressure on the tube and the resulting compression in a tangential direction increase with the number of layers of wire applied. As each

layer of wire is applied its tension is that of wrapping, but this is gradually reduced by the compression of this layer due to the pressure on its surface caused by the application of the succeeding layers of wire. There are two principal methods of wrapping wire on a tube; one is to wrap the wire at constant tension and the other is to wrap it at such varying tension that, when the gun is fired with the prescribed pressure, all the layers of wire shall be subjected to the same tangential stress. The latter method is theoretically the better, but owing to the greater convenience of wrapping the wire at constant tension and to its great elastic strength, which permits the tube to be compressed to its elastic limit by wrapping wire thereon at constant tension without causing the stress in the wire to approach too close to its elastic limit when the gun is fired, the former method is generally used. Formulas have been deduced giving

- (a) the uniform tension T at which an envelope of wire of thickness equal to $R_2 - R_1$ must be wrapped on a tube of radii R_0 and R_1 to produce a pressure P_1' on its exterior.
- (b) the pressure at any radius r of the envelope of wire when the gun is in the state of rest due to the wrapping thereon of the outer layers of wire.
- (c) the resultant tension at any radius r of the envelope of wire due to the tension of wrapping and to the compression caused by the wrapping of the layers from that radius out.
- (d) the tangential and radial stresses and strains at any radius r of the wire envelope after the wrapping has been completed.
- (e) the uniform tangential stress $El_t = S_t$ which must occur in all the layers of a wire envelope of thickness equal to $R_2 - R_1$ to produce a pressure P_1 on the exterior of a tube of radii R_0 and R_1 when the gun is fired.
- (f) the variable tension of winding which must be used in order that when the gun is fired all the layers of wire will be subjected to a uniform tangential stress S_t .
- (g) the pressure, the tension, and the radial stress and strain which exist at any radius r of the wire envelope when the gun is fired.

The formulas referred to under (a) to (d), inclusive, pertain only to the case where the wire is wrapped under constant ten-

sion and relate exclusively to the gun in the state of rest. Those referred to under (e) to (g), inclusive, pertain only to the case where the wire is wrapped at such varying tension that when the gun is fired with the prescribed powder pressure all layers of wire shall be subjected to the same tangential stress; and they relate exclusively to the gun in the state of action.

CASE I. WIRE WRAPPED UNDER CONSTANT TENSION.

7. General Method of Design. — Suppose it is desired to construct a wire-wrapped gun as shown in Fig. 1, in such manner as to give to it the maximum elastic strength permitted by the quality of the metal in the tube and the total thickness of wall $R_3 - R_0$, with the condition that the wire shall be wrapped on the tube under constant tension. The first thing to determine is the maximum interior pressure which the gun can support in action on the assumption that in the state of rest the inner surface of the tube is compressed to its elastic limit. This is given by equation (5). The exterior pressure on the tube which will cause its interior surface to be compressed to the elastic limit when in the state of rest is then determined from equation (29), page 206, Lissak's Ordnance and Gunnery, which, after replacing α by its value R_1^2/R_0^2 is as follows:

$$P_{1p} = \frac{R_1^2 - R_0^2}{2 R_1^2} \rho \quad (6)$$

Part of this pressure P_{1p} will be due to the wire envelope and part to the shrinkage of the jacket on the wire. If the jacket is assembled without shrinkage, as is sometimes the case, all of the pressure P_{1p} will be due to the pressure exerted by the wire envelope. Supposing the jacket assembled with shrinkage, the amount of the shrinkage must be determined and from that the pressure on the exterior of the tube due to such shrinkage. Subtracting this pressure from P_{1p} there results the pressure which must be exerted by the wire envelope. To utilize the full strength of the jacket its inner surface must be extended in the state of action to the elastic limit. Part of this extension will be due to the action of P_0 and the remainder to the shrinkage. The part due to P_0 is obtained from equation (1) by making $P_1 = 0$, $R_1 = R_3$, and $r = R_2$. The difference between this part and the tensile

elastic limit θ_3 is due to the shrinkage. Calling this difference S'_{t3} , the pressure P'_2 to produce it is obtained from equation (1) by making $P_1 = 0$, $S_t = S'_{t3}$, $P_0 = P'_2$, $R_1 = R_3$, $R_0 = R_2$, and $r = R_2$, and solving for P'_2 , whence

$$P'_2 = \frac{3(R_3^2 - R_2^2)}{4R_3^2 + 2R_2^2} S'_{t3} \quad (7)$$

The absolute shrinkage to produce this pressure is given by equation (54), page 218, Lissak's Ordnance and Gunnery, which, after replacement of the radius ratios by their values in terms of radii and making $R_1 = R_2$, $R_2 = R_3$, $P_1 = P'_2$, and $S_1 = S_2$, becomes

$$S_2 = \frac{4R_2^3(R_3^2 - R_0^2)P'_2}{E(R_2^2 - R_0^2)(R_3^2 - R_2^2)} \quad (8)$$

The pressure P'_2 due to the shrinkage of the jacket on the wire envelope causes a pressure p''_1 on the exterior of the tube which is obtained from equation (4) by making $P_0 = 0$, $P_1 = P'_2$, $R_1 = R_2$, and $r = R_1$.

Whence

$$p''_1 = \frac{P'_2 R_2^2}{R_2^2 - R_0^2} \left(1 - \frac{R_0^2}{R_1^2}\right) \quad (9)$$

The pressure p'_1 to be produced by the wire envelope is, therefore,

$$p'_1 = P_{1p} - p''_1$$

8. Intensity of the Constant Tension of Wrapping Necessary to Produce a Given Pressure on the Exterior of the Tube, System at Rest.—It now remains to deduce the formulas referred to under (a) to (d), inclusive, article 6. To do this let us suppose the wrapping of the wire on the tube, Fig. 1, to be finished to a radius r producing a pressure on the exterior of the tube p_t and then to be continued to any other radius r' , the pressure on the exterior of the tube changing to p'_t ; the additional wrapping will also produce a pressure p_r on the wire at r . The pressure p_r on the wire at r then produces a pressure $p'_t - p_t$ on the tube. Applying equation (4), making $p = p'_t - p_t$, $P_0 = 0$, $P_1 = p_r$, $r = R_1$, $R_1 = r$, we obtain

$$p'_t - p_t = \frac{p_r r^2}{r^2 - R_0^2} - \frac{p_r r^2 R_0^2}{(r^2 - R_0^2) R_1^2} \quad (10)$$

which, by reduction becomes,

$$p'_t - p_t = \frac{(R_1^2 - R_0^2) r^2 p_r}{R_1^2 (r^2 - R_0^2)} \quad (11)$$

Let τ be the mean tension of the wires between r and r' , then $p_r \times 2r = \tau \times 2(r' - r)$ whence

$$p_r = \frac{r' - r}{r} \tau$$

Substituting this value for p_r in equation (11) and dividing by $r' - r$, we get

$$\frac{p'_t - p_t}{r' - r} = \frac{(R_1^2 - R_0^2) r \tau}{R_1^2 (r^2 - R_0^2)} \quad (12)$$

Passing to the limit of both members, we have, since τ then becomes the uniform tension of wrapping T ,

$$dp_t/dr = \frac{(R_1^2 - R_0^2) r T}{R_1^2 (r^2 - R_0^2)} \quad (13)$$

Multiplying by dr and integrating between the limits r and R_2

$$p'_{tR_2} = \frac{R_1^2 - R_0^2}{2 R_1^2} T \log_e \frac{R_2^2 - R_0^2}{r^2 - R_0^2} \quad (14)$$

which gives the pressure on the tube produced by wrapping from r to R_2 . Making in this $r = R_1$ we have the total pressure of the wire envelope on the tube or

$$p'_1 = \frac{R_1^2 - R_0^2}{2 R_1^2} T \log_e \frac{R_2^2 - R_0^2}{R_1^2 - R_0^2} \quad (15)$$

and solving this for T we have for the uniform tension of wrapping to produce the required pressure p'_1 on the exterior of the tube

$$T = \frac{2 R_1^2 p'_1}{(R_1^2 - R_0^2) \log_e \frac{R_2^2 - R_0^2}{R_1^2 - R_0^2}} \quad (16)$$

If the jacket is assembled without shrinkage on the wire envelope, or if there is no jacket, p'_1 becomes the total pressure $P_{1\rho}$ required on the exterior of the tube to compress its inner surface to the elastic limit when the gun is in the state of rest.

9. Intensity of Pressure at any Radius r of the Wire Envelope Due to Wrapping, System at Rest. — In equation (11) p_r is the

pressure at r due to wrapping from r to any other radius r' . If r' be taken equal to R_2 , p_r will be the pressure at r due to wrapping all the wire from r out, or the final pressure at rest at r due to the wrapping, and $p'_t - p_t$ will be the pressure on the exterior of the tube produced by the same wrapping. This pressure is the p'_{tR_2} of equation (14). Substituting for $p'_t - p_t$ in equation (11) the value of p'_{tR_2} from equation (14) we have

$$\frac{R_1^2 - R_0^2}{2 R_1^2} T \log_e \frac{R_2^2 - R_0^2}{r^2 - R_0^2} = \frac{(R_1^2 - R_0^2) r^2 p_r}{R_1^2 (r^2 - R_0^2)}$$

and solving for p_r

$$p_r = \frac{r^2 - R_0^2}{2 r^2} T \log_e \frac{R_2^2 - R_0^2}{r^2 - R_0^2} \quad (17)$$

which gives the pressure p_r due to the wrapping at any radius r of the wire envelope when the gun is in the state of rest. If, in equation (17) r becomes R_1 , p_r becomes p'_1 the total pressure of the wire envelope on the tube. Making these substitutions in equation (17)

$$p'_1 = \frac{R_1^2 - R_0^2}{2 R_1^2} T \log_e \frac{R_2^2 - R_0^2}{R_1^2 - R_0^2}$$

the same as given in equation (15), as it should be.

10. Intensity of Tension at any Radius r of the Wire Envelope Due to Wrapping, System at Rest. — Let t_r be the tension at r due to the wrapping after it is completed. Then calling τ the mean tension of the wires between r and r' ,

$$p'_r r' - p_r r = - \tau (r' - r) \quad \text{or} \quad \frac{p'_r r' - p_r r}{r' - r} = - \tau$$

or passing to the limit, when τ becomes $-t_r$,

$$\frac{d(p_r r)}{dr} = -t_r \quad (18)$$

Substituting in equation (18) the value of p_r from equation (17) we have

$$-t_r = d \left[\frac{r^2 - R_0^2}{2 r} T \log_e \frac{R_2^2 - R_0^2}{r^2 - R_0^2} \right]_{dr}$$

Performing the differentiation indicated

$$-t_r = \frac{T}{dr} \left[\log_e \frac{R_2^2 - R_0^2}{r^2 - R_0^2} \left\{ \frac{4r^2 dr - (r^2 - R_0^2) 2 dr}{4r^2} \right\} - \left\{ \frac{r^2 - R_0^2}{2r} \right\} \left\{ \frac{R_2^2 - R_0^2}{(r^2 - R_0^2)^2} 2r dr \frac{r^2 - R_0^2}{R_2^2 - R_0^2} \right\} \right]$$

and reducing

$$t_r = T \left[1 - \frac{r^2 + R_0^2}{2r^2} \log_e \frac{R_2^2 - R_0^2}{r^2 - R_0^2} \right] \quad (19)$$

Equation (19) gives the tension due to the wrapping at any radius r of the wire envelope when the gun is in the state of rest.

11. Tangential and Radial Stresses and Strains at any Radius r of the Wire Envelope Due to Wrapping, System at Rest. — The tangential stress at any radius r of the wire envelope due to the wrapping, system at rest, is

$$S_{tw} = t_r + p_r / 3 \quad (20)$$

and substituting in this the values of t_r and p_r from equations (19) and (17)

$$S_{tw} = T \left[1 - \frac{r^2 + R_0^2}{2r^2} \log_e \frac{R_2^2 - R_0^2}{r^2 - R_0^2} + \frac{r^2 - R_0^2}{6r^2} \log_e \frac{R_2^2 - R_0^2}{r^2 - R_0^2} \right]$$

reducing

$$S_{tw} = T \left[1 - \frac{r^2 + 2R_0^2}{3r^2} \log_e \frac{R_2^2 - R_0^2}{r^2 - R_0^2} \right] \quad (21)$$

The radial stress at any radius r of the wire envelope due to the wrapping, system at rest, is

$$S_{pw} = -p_r - t_r / 3 \quad (22)$$

and substituting in this the values of p_r and t_r from equations (17) and (19)

$$S_{pw} = -\frac{T}{3} \left[1 + \frac{r^2 - 2R_0^2}{r^2} \log_e \frac{R_2^2 - R_0^2}{r^2 - R_0^2} \right] \quad (23)$$

The tangential and radial strains can be obtained by dividing S_{tw} and S_{pw} , respectively, by the modulus of elasticity E .

12. Stresses in Jacket, Wire Envelope, and Tube, System at Rest and in Action. — Equations (17), (19), (21), and (23) give the radial pressure, the tangential tension, the tangential stress,

and the radial stress due to the wrapping, at any radius r of the wire envelope when the system is at rest, in terms of the uniform tension of wrapping T and the radii of the tube and wire envelope. The shrinkage of the jacket also produces radial pressure, tangential tension, and tangential and radial stresses in the wire envelope which must be added algebraically to those produced by the wrapping to obtain the final state of the wire envelope, system at rest. The stresses in the jacket, system at rest, are due only to the shrinkage pressure P'_2 and those in the tube, system at rest, only to the exterior pressure on the tube P_{1p} . The radial pressure, tangential tension, and tangential and radial stresses in both jacket and tube, system at rest, can be calculated from equations (4), (3), (1), and (2), respectively, after proper substitutions therein.

The tangential tension, radial pressure, and stresses in the tube, wire envelope, and jacket having been calculated for the gun in the state of rest, the tensions, pressures, and stresses in action corresponding to any pressure P_0 may be obtained by adding algebraically to those in the state of rest the tensions, pressures, and stresses computed from equations (3), (4), (1), and (2), respectively, by considering the gun as a single cylinder acted on only by an interior pressure P_0 .

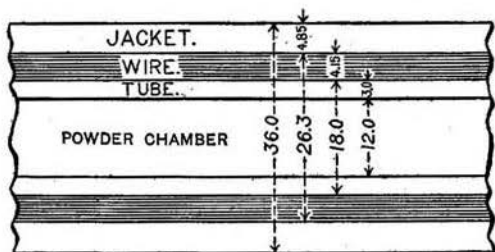


Fig. 2.

EXAMPLE.

13. Figure 2 represents a section through the powder chamber of a 12-inch wire-wrapped gun.

The material and physical qualities of the tube, wire envelope and jacket are shown in the following table:

Part	Material	Elastic limit in tension θ	Elastic limit in compression ρ	Modulus of elasticity E
Tube	Forged steel	45,000	60,000	30,000,000
Wire	Steel wire	100,000	100,000	30,000,000
Jacket	Cast steel	30,000	30,000	30,000,000

It is required to determine,

- the maximum powder pressure which the gun may withstand without the tangential stresses in any part thereof exceeding its elastic limit;
- the shrinkage of the jacket;
- the uniform tension of wrapping of the wire envelope;
- the stresses at the interior and exterior surfaces of each part in the state of rest; and
- the stresses at the interior and exterior surfaces of each part in the state of action corresponding to the maximum permissible powder pressure and also to a powder pressure of 42,000 lbs. per sq. in.

Maximum Permissible Powder Pressure — Shrinkage of Jacket — Uniform Tension of Wrapping. — From Fig. 2

$$R_0 = 6; R_1 = 9; R_2 = 13.15; R_3 = 18.$$

From equation (5) the maximum permissible powder pressure is

$$P_0 = \frac{3[(18)^2 - (6)^2][60,000 + 45,000]}{4(18)^2 + 2(6)^2} = 66,316 \text{ lbs. per sq. in.} \quad (24)$$

The pressure on the exterior of the tube to compress its interior surface in the state of rest to the elastic limit in compression, 60,000 lbs. per sq. in., is, from equation (6),

$$P_{1p} = \frac{(9)^2 - (6)^2}{2(9)^2} 60,000 = 16,667 \text{ lbs. per sq. in.} \quad (25)$$

Part of this pressure is due to the shrinkage of the jacket. To determine this shrinkage it is first necessary to find the intensity of stress at the interior of the jacket in the state of action due to the powder pressure of 66,315 lbs. per sq. in., and to sub-

tract this stress from the elastic limit in tension of the material in the jacket. The difference will be due to the shrinkage pressure and from it the shrinkage pressure can be determined. Having the shrinkage pressure, the pressure which it transmits to the exterior of the tube may be obtained. Substituting in equation (1) $R_3 = 18$ for R_1 , $R_2 = 13.15$ for r , and making $P_1 = 0$ and $P_0 = 66,316$, the tangential stress at the inner surface of the jacket due to P_0 is

$$S''_{t3} = \frac{2}{3} \frac{66,316 (6)^2}{(18)^2 - (6)^2} + \frac{4}{3} \frac{(6)^2 (18)^2 66,316}{[(18)^2 - (6)^2] (13.15)^2} \\ = 26,235 \text{ lbs. per sq. in. tension.} \quad (26)$$

The elastic limit in tension of the jacket being 30,000 lbs. per sq. in., 30,000 - 26,235 or 3765 lbs. per sq. in. will be the stress due to the shrinkage pressure. Substituting in equation (1), $R_2 = 13.15$ for R_0 , $R_3 = 18$ for R_1 , making $P_1 = 0$, $P_0 = P'_2$, and $S_t = 3765$, and solving for P'_2 , the shrinkage pressure,

$$P'_2 = \frac{3 [(18)^2 - (13.15)^2]}{4 (18)^2 + 2 (13.15)^2} 3765 = 1039 \text{ lbs. per sq. in.} \quad (27)$$

The absolute shrinkage to produce this pressure is from equation (8)

$$S_2 = \frac{4 (13.15)^3 [(18)^2 - (6)^2] 1039}{30,000,000 [(13.15)^2 - (6)^2] [(18)^2 - (13.15)^2]} \\ = .00439 \text{ inch.} \quad (28)$$

The pressure on the exterior of the tube due to the shrinkage pressure of the jacket from equation (9), derived from equation (4), is

$$p''_1 = \frac{1039 (13.15)^2}{(13.15)^2 - (6)^2} [1 - (6)^2 / (9)^2] = 729 \text{ lbs. per sq. in.} \quad (29)$$

Subtracting this from the total exterior pressure on the tube, equation (25), when the system is at rest, we have $p'_1 = 16,667 - 729 = 15,938$ lbs. per sq. in. as the part of the pressure on the exterior of the tube, system at rest, which must be due to the wire envelope. Substituting this value of p'_1 in equation (16)

$$T = \frac{2 (9)^2 15,938}{[(9)^2 - (6)^2] \log_e \frac{(13.15)^2 - (6)^2}{(9)^2 - (6)^2}} = 51,566 \text{ lbs. per sq. in.} \quad *(30)$$

* The Napierian logarithm of a number is equal to the common logarithm of the number divided by .4343.

which is the uniform tension under which the wire must be wrapped on the tube.

STRESSES AT REST.

(See figures 3 and 4.)

14. Inner Surface of Tube.—The tangential stress, see equation (25), is one of compression equal to 60,000 lbs. per sq. in. The radial stress obtained from equation (2) by making $P_0 = 0$, $P_1 = 16,667$, $r = R_0 = 6$, and $R_1 = 9$, is

$$S_r = \frac{2}{3} \frac{-16,667 (9)^2}{(9)^2 - (6)^2} - \frac{4}{3} \frac{-(9)^2 16,667}{(9)^2 - (6)^2} = +20,000 \text{ lbs. per sq. in.} \quad (31)$$

and since the result is positive the stress is one of tension.

Outer Surface of Tube.—The tangential stress, obtained from equation (1) by making $P_0 = 0$, $P_1 = 16,667$, $R_0 = 6$, and $r = R_1 = 9$, is

$$S_t = \frac{2}{3} \frac{-16,667 (9)^2}{(9)^2 - (6)^2} + \frac{4}{3} \frac{-(6)^2 16,667}{(9)^2 - (6)^2} = -37,777 \text{ lbs. per sq. in.} \quad (32)$$

and since the result is negative the stress is one of compression. The radial stress, obtained by making the same substitutions in equation (2), is

$$S_r = \frac{2}{3} \frac{-16,667 (9)^2}{(9)^2 - (6)^2} - \frac{4}{3} \frac{-(6)^2 16,667}{(9)^2 - (6)^2} = -2223 \text{ lbs. per sq. in.} \quad (33)$$

and the stress is one of compression.

Inner Surface of Wire Envelope.—The tangential stress due to the layers of wire only, obtained from equation (21) by making $R_0 = 6$, $r = R_1 = 9$, $R_2 = 13.15$, and $T = 51,566$, see equation (30), is

$$S'_{tw} = 51,566 \left\{ 1 - \frac{(9)^2 + 2(6)^2}{3(9)^2} \log_e \frac{(13.15)^2 - (6)^2}{(9)^2 - (6)^2} \right\} = +15,439 \text{ lbs. per sq. in.} \quad (34)$$

and the stress is one of tension.

This stress is decreased by that due to the shrinkage pressure of the jacket which is obtained from equation (1) by making

$$R_0 = 6, R_1 = R_2 = 13.15, r = R_1 = 9, P_1 = P'_2 = 1039, \text{ and } P_0 = 0.$$

Whence

$$S''_{tw} = \frac{2}{3} \frac{(1039)(13.15)^2}{(13.15)^2 - (6)^2} + \frac{4}{3} \frac{(6)^2(13.15)^2(1039)}{(13.15)^2 - (6)^2} \times \frac{1}{(9)^2} \\ = -1653 \text{ lbs. per sq. in. compression.} \quad (35)$$

The final tangential stress at rest is, therefore,

$$15,439 - 1653 = 13,786 \text{ lbs. per sq. in. tension.}$$

The radial stress due to the layers of wire only, obtained from equation (23) by making the same substitutions as in equation (21) for the tangential stress, is

$$S'_{rw} = -\frac{51,566}{3} \left\{ 1 + \frac{(9)^2 - 2(6)^2}{(9)^2} \log_e \frac{(13.15)^2 - (6)^2}{(9)^2 - (6)^2} \right\} \\ = -19,314 \text{ lbs. per sq. in. compression.} \quad (36)$$

To this must be added algebraically the radial stress due to the shrinkage of the jacket which is obtained from equation (2) by making the appropriate substitutions therein.

Whence

$$S''_{rw} = \frac{2}{3} \frac{(1039)(13.15)^2}{(13.15)^2 - (6)^2} - \frac{4}{3} \frac{(6)^2(13.15)^2(1039)}{(13.15)^2 - (6)^2} \times \frac{1}{(9)^2} \\ = -97 \text{ lbs. per sq. in. compression.} \quad (37)$$

The final radial stress at rest is, therefore,

$$-19,314 - 97 = -19,411 \text{ lbs. per sq. in. compression.}$$

Outer Surface of Wire Envelope.—Before the assembling of the jacket the tangential stress in 51,566 lbs. per sq. in., the tension of wrapping. This is decreased by the compression due to the shrinkage of the jacket which is obtained from equation (1) by making the proper substitutions therein.

Whence

$$S''_{tw} = \frac{2}{3} \frac{(1039)(13.15)^2}{(13.15)^2 - (6)^2} + \frac{4}{3} \frac{(6)^2(13.15)^2(1039)}{(13.15)^2 - (6)^2} \\ \times \frac{1}{(13.15)^2} = -1240 \text{ lbs. per sq. in. compression.} \quad (38)$$

The final tangential stress at rest is, therefore,

$$51,566 - 1240 = 50,326 \text{ lbs. per sq. in. tension.}$$

Before assembling the jacket the radial stress is minus one-third the tension of wrapping = $-51,566/3 = -17,188$ lbs.* per sq. in. compression. The radial stress due to the shrinkage of the jacket, obtained from equation (2) by making the proper substitutions therein, is

$$S''_{pw} = \frac{2}{3} \frac{(1039) (13.15)^2}{(13.15)^2 - (6)^2} - \frac{4}{3} \frac{(6)^2 (13.15)^2 (1039)}{(13.15)^2 - (6)^2} \times \frac{1}{(13.15)^2} \\ = -510 \text{ lbs. per sq. in. compression.} \quad (39)$$

The final radial stress at rest is, therefore,

$$-17,188 - 510 = -17,698 \text{ lbs. per sq. in. compression.}$$

Inner Surface of Jacket. — All stresses in the jacket, system at rest, are due to the shrinkage. The tangential stress at its inner surface, see equation (27), is 3765 lbs. per sq. in. tension. The radial stress, obtained from equation (2) by making the proper substitution therein, is

$$S_{pi} = \frac{2}{3} \frac{(1039) (13.15)^2}{(18)^2 - (13.15)^2} - \frac{4}{3} \frac{(13.15)^2 (18)^2 (1039)}{(18)^2 - (13.15)^2} \times \frac{1}{(13.15)^2} \\ = -2178 \text{ lbs. per sq. in. compression.} \quad (40)$$

Outer Surface of Jacket. — The tangential stress, obtained from equation (1) by making the proper substitutions therein, is

$$S_{ti} = \frac{2}{3} \frac{(1039) (13.15)^2}{(18)^2 - (13.15)^2} + \frac{4}{3} \frac{(13.15)^2 (18)^2 (1039)}{(18)^2 - (13.15)^2} \times \frac{1}{(18)^2} \\ = +2378 \text{ lbs. per sq. in. tension.} \quad (41)$$

The radial stress, obtained from equation (2) by making the proper substitutions therein, is

$$S_{pi} = \frac{2}{3} \frac{(1039) (13.15)^2}{(18)^2 - (13.15)^2} - \frac{4}{3} \frac{(13.15)^2 (18)^2 (1039)}{(18)^2 - (13.15)^2} \times \frac{1}{(18)^2} \\ = -793 \text{ lbs. per sq. in. compression.} \quad (42)$$

* This result can also be obtained from equation (23) by proper substitutions therein.

STRESSES IN ACTION.

(See figures 3 and 4.)

15. Maximum Permissible Powder Pressure $P_0 = 66,315$. — Making in equations (1) and (2) $P_0 = 66,315$, $P_1 = 0$, $R_0 = 6$, and $R_1 = R_3 = 18$, they become, respectively,

$$S_t = \frac{66,315 (6)^2}{(18)^2 - (6)^2} \left\{ \frac{2}{3} + \frac{4}{3} \frac{(18)^2}{r^2} \right\} = 5526 + \frac{[6.55400]^*}{r^2} \quad (43)$$

and

$$S_p = \frac{66,315 (6)^2}{(18)^2 - (6)^2} \left\{ \frac{2}{3} - \frac{4}{3} \frac{(18)^2}{r^2} \right\} = 5526 - \frac{[6.55400]^*}{r^2} \quad (44)$$

By substituting for r in equations (43) and (44) its values corresponding to the surfaces of the various parts of the gun, the stresses due to the action of P_0 on the gun considered as a single cylinder may be determined, and the algebraic sums of these stresses and those previously determined for the state of rest are the stresses in action.

Inner Surface of Tube. $r = R_0 = 6$. — The tangential stress system in action, see equation (24), is 45,000 lbs. per sq. in. tension. The radial stress is

$$+20,000 + 5526 - \frac{[6.55400]}{(6)^2} = +20,000 - 93,946 = -73,946 \text{ lbs.}^\dagger \\ \text{per sq. in. compression.} \quad (45)$$

Outer Surface of Tube. $r = R_1 = 9$. — The tangential stress is

$$-37,777 + 5526 + \frac{[6.55400]}{(9)^2} = -37,777 + 49,735 = +11,958 \text{ lbs.} \\ \text{per sq. in. tension.} \quad (46)$$

The radial stress is

$$-2223 + 5526 - \frac{[6.55400]}{(9)^2} = -2223 - 38,683 = -40,906 \text{ lbs.} \\ \text{per sq. in. compression.} \quad (47)$$

* The figures in brackets are logarithms of the numbers.

† This exceeds the elastic limit for compression.

Inner Surface of Wire Envelope. $r = R_1 = 9$. — The tangential stress is

$$+13,786 + 5526 + \frac{[6.55400]}{(9)^2} = +13,786 + 49,735 = +63,521 \text{ lbs.} \\ \text{per sq. in. tension.} \quad (48)$$

The radial stress is

$$-19,411 + 5526 - \frac{[6.55400]}{(9)^2} = -19,411 - 38,683 = -58,096 \text{ lbs.} \\ \text{per sq. in. compression.} \quad (49)$$

Outer Surface of Wire Envelope. $r = R_2 = 13.15$. — The tangential stress is

$$+50,326 + 5526 + \frac{[6.55400]}{(13.15)^2} = +50,326 + 26,234 = +76,560 \text{ lbs.} \\ \text{per sq. in. tension.} \quad (50)$$

The radial stress is

$$-17,698 + 5526 - \frac{[6.55400]}{(13.15)^2} = -17,698 - 15,182 = -32,880 \text{ lbs.} \\ \text{per sq. in. compression.} \quad (51)$$

Inner Surface of Jacket. $r = R_2 = 13.15$. — The tangential stress is, see equations (26) and (27),

$$+3765 + 26,234 = +29,999 \text{ lbs. per sq. in. tension.} \quad (51\frac{1}{2})$$

The radial stress is

$$-2178 + 5526 - \frac{[6.55400]}{(13.15)^2} = -2178 - 15,182 = -17,360 \text{ lbs.} \\ \text{per sq. in. compression.} \quad (52)$$

Outer Surface of Jacket. $r = R_3 = 18$. — The tangential stress is

$$+2378 + 5526 + \frac{[6.55400]}{(18)^2} = +2378 + 16,578 = +18,956 \text{ lbs.} \\ \text{per sq. in. tension.} \quad (53)$$

The radial stress is

$$-793 + 5526 - \frac{[6.55400]}{(18)^2} = -793 - 5526 = -6319 \text{ lbs.} \\ \text{per sq. in. compression.} \quad (54)$$

The tangential stresses, system at rest, and in action when $P_0 = 66,315$ lbs. per sq. in. are shown graphically in Fig. 3, and the radial stresses in Fig. 4.

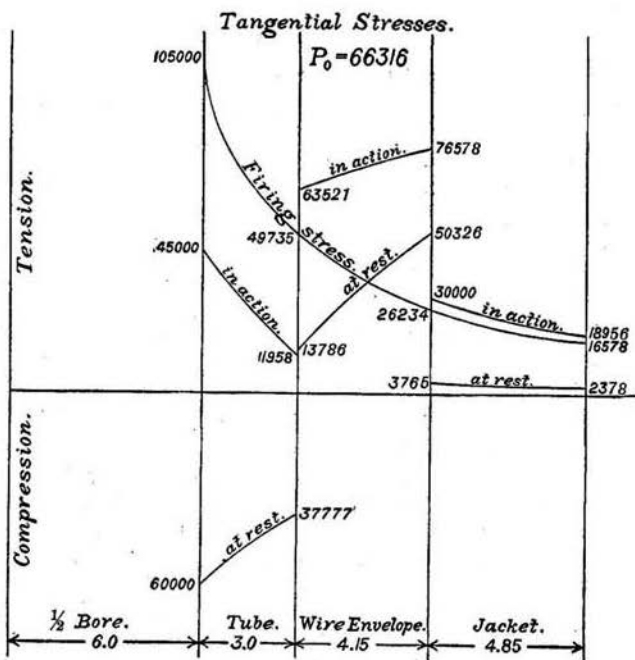


Fig. 3.

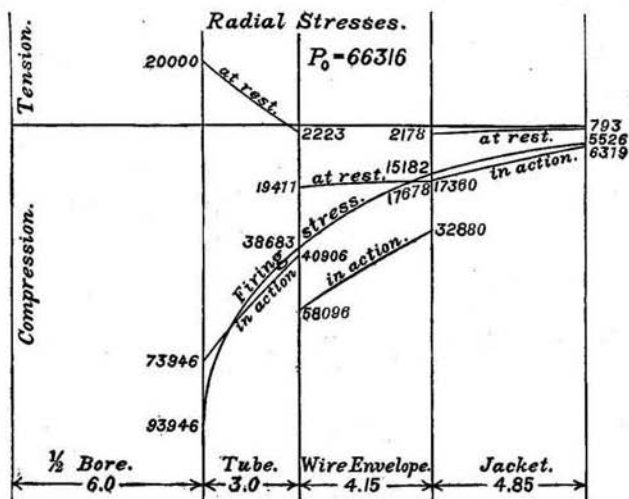


Fig. 4.

STRESSES IN ACTION

(See figures 5 and 6.)

16. $P_0 = 42000$ lbs. per sq. in. — By reference to equations (1) and (2), or to equations (43) and (44), it will be seen that the stresses due to the action of an interior pressure on a gun considered as a single cylinder are directly proportional to the intensity of the interior pressure. The stresses at the inner and outer surfaces of the various parts of this gun, system in action, under an interior powder pressure of 42,000 lbs. per sq. in. will, therefore, be the stresses at rest plus $42,000/66,316$ of the increases in stress in the various parts due to the action on the gun, considered as a single cylinder, of the maximum permissible powder pressure of 66,316 lbs. per sq. in. These increases in stress have already been calculated and are given in equations (45) to (54), inclusive, excepting the increase in tangential stress at the bore of the tube which had been previously determined to be 105,000 lbs. per sq. in.

Inner Surface of Tube.— The tangential stress is

$$-60,000 + \frac{42,000}{66,316} \times 105,000 = -60,000 + 66,501 = +6501 \text{ lbs.}$$

per sq. in. tension (55)

The radial stress is

$$+20,000 + \frac{42,000}{66,316} \times (-93,946) = +20,000 - 59,500 = -39,500 \text{ lbs.}$$

per sq. in. compression. (56)

Outer Surface of Tube.— The tangential stress is

$$-37,777 + \frac{42,000}{66,316} \times 49,735 = -37,777 + 31,499 = -6278 \text{ lbs.}$$

per sq. in. compression. (57)

The radial stress is

$$-2223 + \frac{42,000}{66,316} \times (-38,683) = -2223 - 24,500 = -26,723 \text{ lbs.}$$

per sq. in. compression. (58)

Inner Surface of Wire Envelope.— The tangential stress is

$$+13,786 + \frac{42,000}{66,316} \times 49,735 = +13,786 + 31,499 = +45,285 \text{ lbs.}$$

per sq. in. tension. (59)

The radial stress is

$$-19,411 + \frac{42,000}{66,316} \times (-38,683) = -19,411 - 24,500 = -43,911 \text{ lbs.} \\ \text{per sq. in. compression.} \quad (60)$$

Outer Surface of Wire Envelope. — The tangential stress is

$$+50,326 + \frac{42,000}{66,316} \times 26,234 = +50,326 + 16,615 = +66,941 \text{ lbs.} \\ \text{per sq. in. tension.} \quad (61)$$

The radial stress is

$$-17,698 + \frac{42,000}{66,316} \times (-15,182) = -17,698 - 9616 = -27,314 \text{ lbs.} \\ \text{per sq. in. compression.} \quad (62)$$

Inner Surface of Jacket. — The tangential stress is

$$+3765 + \frac{42,000}{66,316} \times 26,234 = +3765 + 16,615 = +20,380 \text{ lbs.} \\ \text{per sq. in. tension.} \quad (63)$$

The radial stress is

$$-2178 + \frac{42,000}{66,316} \times (-15,182) = -2178 - 9616 = -11,794 \text{ lbs.} \\ \text{per sq. in. compression.} \quad (64)$$

Outer Surface of Jacket. — The tangential stress is

$$+2378 + \frac{42,000}{66,316} \times 16,578 = +2378 + 10,502 = +12,880 \text{ lbs.} \\ \text{per sq. in. tension.} \quad (65)$$

The radial stress is

$$-793 + \frac{42,000}{66,316} \times (-5526) = -793 - 3500 = -4293 \text{ lbs.} \\ \text{per sq. in. compression.} \quad (66)$$

Figs. 5 and 6 show graphically the tangential and radial stresses, respectively, system in action, when $P_0 = 42,000$ lbs. per sq. in.

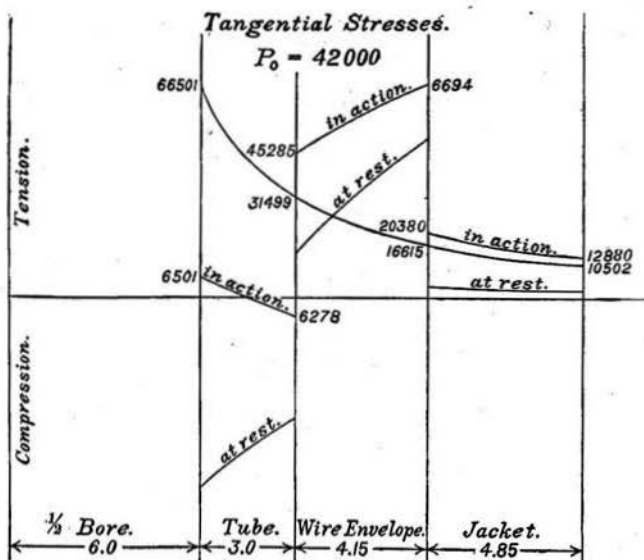


Fig. 5.

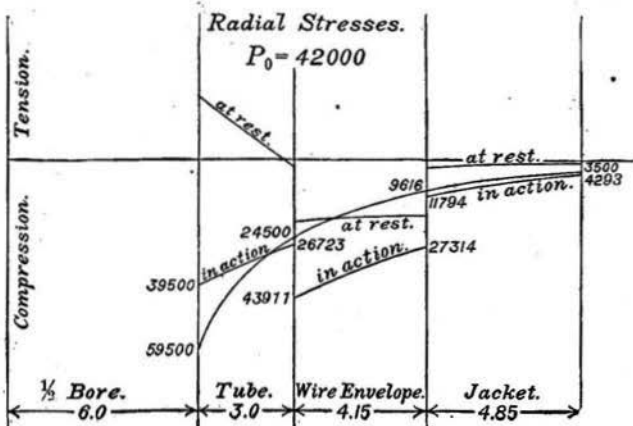


Fig. 6.

17. Problem. — Fig. 7 shows a longitudinal section through a part of the 6-inch wire-wrapped gun model 1908 just in front of the forcing cone.

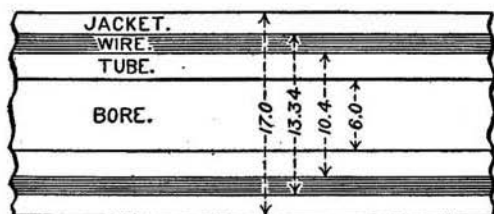


Fig. 7.

Part	Prescribed elastic limit $p = \theta$
Tube	50,000
Wire envelope	140,000
Jacket	50,000

The jacket is assembled with an absolute shrinkage of .007 inch. The wire is wrapped under a constant tension of 50,000 lbs. per sq. in.

Find:

- the maximum powder pressure which the gun can withstand without the tangential stress on any part exceeding the elastic limit in tension or compression;
- the tangential and radial stresses at the inner and outer surfaces of each part, system at rest;
- the tangential and radial stresses at the inner and outer surfaces of each part, system in action, when subjected to the powder pressure determined under (a).

CASE II. — WIRE WRAPPED UNDER SUCH VARYING TENSION THAT WHEN THE GUN IS FIRED WITH THE PRESCRIBED MAXIMUM POWDER PRESSURE ALL LAYERS OF WIRE WILL BE SUBJECTED TO THE SAME TANGENTIAL STRESS.

18. General Method of Design. — Let it be required to construct a wire-wrapped gun as shown in Fig. 1 or 2 in such manner

as to give it the maximum elastic strength permitted by the quality of the metal in the tube and the total thickness of wall $R_3 - R_0$, and with the condition that when the gun is fired with the prescribed maximum powder pressure all layers of wire shall be subjected to the same tangential stress.

First determine from equation (5) the maximum interior pressure which the gun can support in action on the assumption that in the state of rest the inner surface of the tube is compressed to its elastic limit. From equation (6) determine the pressure P_{1p} on the exterior of the tube which will compress its inner surface in the state of rest to its elastic limit. With the values of P_0 and P_{1p} from equations (5) and (6) determine from equation (48), page 215, Lissak's Ordnance and Gunnery, the corresponding value of the pressure on the exterior of the tube, system in action. The last equation, after replacing the radius ratios by their values in terms of the radii and substituting R_3 for R_2 since there are three parts to the gun instead of two, becomes

$$P_1 = P_{1p} + \frac{R_0^2 (R_3^2 - R_1^2) P_0}{R_1^2 (R_3^2 - R_0^2)} \quad (67)$$

The next steps require the use of formulas (e) to (g), inclusive, referred to in article 6 which will now be deduced. Let S_{twa} be the uniform tangential stress in the wire, system in action; S_{pwa} the radial stress at any radius r of the wire envelope, system in action; t_{ra} the tension at any radius r of the wire envelope, system in action; p_{ra} the pressure at any radius r of the wire envelope, system in action; and T_r the tension of wrapping.

19. Radial Pressure at any Radius r of the Wire Envelope, System in Action. Constant Tangential Stress in the Wire Envelope, System in Action.—Substituting for t_{ra} in the expression

$$p_{ra} dr + r dp_{ra} = -t_{ra} dr$$

its value, $t_{ra} = S_{twa} - p_{ra}/3$ obtained from the first of equations (4), page 192, Lissak's Ordnance and Gunnery, by considering $q = 0$, we have

$$p_{ra} dr + r dp_{ra} = -(S_{twa} - p_{ra}/3) dr \quad (68)$$

Whence

$$\frac{dp_{ra}}{S_{twa} + 2 p_{ra}/3} = -dr/r \quad (69)$$

Integrating

$$\frac{3}{2} \log_e (S_{twa} + \frac{2}{3} p_{ra}) = \log_e C / r \quad (70)$$

Since the wire envelope is covered by a jacket there will, when the system is in action, be a pressure P_2 on the exterior of the wire envelope, which pressure, if the full elastic strength of the jacket is to be utilized, must be such as to extend its inner surface, system in action, to its elastic limit. Therefore, when $r = R_2$, $p_{ra} = P_2$; and the constant of integration C is

$$(S_{twa} + \frac{2}{3} P_2)^{3/2} R_2$$

Substituting this value of C in equation (70)

$$\frac{3}{2} \log_e [S_{twa} + \frac{2}{3} p_{ra}] = \log_e [(S_{twa} + \frac{2}{3} P_2)^{3/2} R_2 / r]$$

or

$$(S_{twa} + \frac{2}{3} p_{ra})^{3/2} = (S_{twa} + \frac{2}{3} P_2)^{3/2} R_2 / r$$

and solving for p_{ra}

$$p_{ra} = \frac{3}{2} [(S_{twa} + \frac{2}{3} P_2) (R_2 / r)^{2/3} - S_{twa}] \quad (71)$$

which gives the radial pressure at any radius r of the wire envelope, system in action. Making in this $r = R_1$, the pressure on the exterior of the tube, system in action, is

$$P_1 = \frac{3}{2} [(S_{twa} + \frac{2}{3} P_2) (R_2 / R_1)^{2/3} - S_{twa}] \quad (72)$$

and solving for S_{twa} we obtain the constant tangential stress in the wire envelope, system in action, required to produce a given pressure P_1 on the exterior of the tube, system in action.

Whence

$$S_{twa} = \frac{2}{3} \frac{P_1 - P_2 (R_2 / R_1)^{2/3}}{(R_2 / R_1)^{2/3} - 1} \quad (73)$$

20. Tangential Tension and Radial Stress at any Radius r of the Wire Envelope, System in Action. — The tangential tension t_{ra} at any radius r of the wire envelope, system in action, is $t_{ra} = S_{twa} - p_{ra}/3$ and substituting in this the value of p_{ra} from equation (71)

$$t_{ra} = \frac{3}{2} S_{twa} - \frac{1}{2} (S_{twa} + \frac{2}{3} P_2) (R_2 / r)^{2/3} \quad (74)$$

The tangential stress S_{twa} throughout the wire envelope, system in action, is constant by hypothesis. The radial stress at any radius r of the wire envelope, system in action, is, from the

second of equations (4), page 192, Lissak's Ordnance and Gunnery, when q is taken as zero,

$$S_{pwa} = -(p_{ra} + t_{ra}/3)$$

and substituting for p_{ra} and t_{ra} their values from equations (71) and (74), respectively,

$$S_{pwa} = -\frac{2}{3} (S_{twa} + \frac{2}{3} P_2) (R_2/r)^{3/2} + S_{twa} \quad (75)$$

The tangential and radial strains can be obtained by dividing the corresponding stresses by 30,000,000, the modulus of elasticity E for steel.

21. Variable Tension of Wrapping at any Radius r of Wire Envelope.—Equation (74) gives the tangential tension t_{ra} at any radius r of the wire envelope, system in action. The increase in tension at the radius r , system in action, over the tension of wrapping T_r is due to the interior pressure P_0 and the pressure p_{ra} , system in action, neither of which was acting when the layer at the radius r was applied. Considering the part of the gun between the interior radius R_0 and the radius r of the wire envelope as a single cylinder, the increase in tension at the radius r over the tension of wrapping due to the pressures P_0 and p_{ra} is obtained from equation (3) by making $P_1 = p_{ra}$ and $R_1 = r$.

Whence

$$t = \frac{2 P_0 R_0^2 - p_{ra} (r^2 + R_0^2)}{r^2 - R_0^2}$$

Subtracting this increase in tension from the tension at the radius r of the wire envelope, system in action, there results the tension of wrapping at any radius r

$$T_r = t_{ra} - t = t_{ra} - \frac{2 P_0 R_0^2 - p_{ra} (r^2 + R_0^2)}{r^2 - R_0^2}$$

Substituting in this the values of t_{ra} and p_{ra} from equations (74) and (71), respectively,

$$T_r = \frac{2}{3} S_{twa} - \frac{1}{2} (S_{twa} + \frac{2}{3} P_2) (R_2/r)^{3/2} - \frac{2 P_0 R_0^2 - \frac{2}{3} [(S_{twa} + \frac{2}{3} P_2) (R_2/r)^{3/2} - S_{twa}] (r^2 + R_0^2)}{r^2 - R_0^2}$$

and reducing

$$T_r = \frac{(S_{twa} + \frac{2}{3} P_2) (R_2/r)^{3/2} (r^2 + 2 R_0^2) - R_0^2 (3 S_{twa} + 2 P_0)}{r^2 - R_0^2} \quad (76)$$

The tension of wrapping at any radius r may be obtained from equation (76) by substituting therein the particular value of r .

22. Stresses in the Jacket, the Wire Envelope, and the Tube, System in Action and at Rest. — The radial pressure, constant tangential stress, tangential tension, and radial stress in the wire envelope, system in action, may be obtained by proper substitutions in equations (71), (73), (74), and (75), respectively. The stresses in the jacket, system in action, are due only to the pressure P_2 on its inner surface; and those in the tube, system in action, are due to the pressure P_0 on its inner surface and the pressure P_1 on its exterior. The tangential tension, radial pressure, and the tangential and radial stresses in both jacket and tube, system in action, may be calculated from equations (3), (4), (1), and (2), respectively, after proper substitutions therein. The tangential tensions, radial pressures, and stresses actually existing in the various parts of the gun, system in action, corresponding to an interior powder pressure P_0 having been calculated, those at rest may be obtained by subtracting algebraically from the tensions, pressures, and stresses in action those computed from equations (3), (4), (1), and (2), respectively, by considering the gun as a single cylinder acted on only by an interior pressure P_0 .

EXAMPLE.

23. Let it be required to determine for the wire-wrapped gun shown in Fig. 2

- (a) the maximum powder pressure which it can withstand without the tangential stress in any part exceeding its elastic limit;
- (b) the shrinkage of the jacket;
- (c) the varying tension of wrapping of the wire envelope so that each layer shall be subjected to the same tangential stress, system in action, under the maximum powder pressure determined under (a);
- (d) the stresses at the interior and exterior surfaces of each part, system in action, under the maximum powder pressure;
- (e) the stresses at the interior and exterior surfaces of each part, system at rest; and
- (f) the stresses at the interior and exterior surfaces of each part, system in action, under a powder pressure of 42000, lbs. per sq. in.

Maximum Permissible Powder Pressure. — Pressure on the Exterior of the Tube and on the Interior of the Jacket, System in Action. — Shrinkage of the Jacket. — The maximum permissible interior pressure in this case is the same as when the wire was wrapped under constant tension and is

$$P_0 = 66,316 \text{ lbs. per sq. in., from equation (24).}$$

The value of P_{1p} from equation (25) is 16,667 lbs. per sq. in. Substituting this value of P_{1p} and the value of $P_0 = 66,316$ in equation (67), the value of the exterior pressure on the tube, system in action, is

$$P_1 = 16,667 + \frac{(6)^2[(18)^2 - (9)^2] 66,316}{(9)^2 [(18)^2 - (6)^2]} = 41,535 \text{ lbs. per sq. in. (77)}$$

The elastic limit of the jacket being 30,000 lbs. per sq. in. the pressure P_2 in action between the jacket and the wire envelope which will extend the inner surface of the jacket to its elastic limit is from equation (1), after making the proper substitutions therein and solving for P_2 ,

$$P_2 = \frac{3 [(18)^2 - (13.15)^2] 30,000}{4 (18)^2 + 2 (13.15)^2} = 8281 \text{ lbs. per sq. in. (78)}$$

From equation (26) the tangential stress at the inner surface of the jacket due to the action of the powder pressure of 66,316 lbs. per sq. in. was found to be 26,235 lbs. per sq. in., leaving a stress of 3765 lbs. per sq. in. to be produced by the shrinkage of the jacket on the wire envelope. The shrinkage pressure to produce this stress was, from equation (27), found to be 1039 lbs. per sq. in., and the corresponding absolute shrinkage from equation (28) was .00439 inch.

24. Constant Tangential Stress in Wire Envelope, System in Action. — Variable Tension of Wrapping. — The constant tangential stress in the layers of the wire envelope, system in action, may now be obtained from equation (73) by substituting therein the values of P_1 and P_2 from equations (77) and (78), respectively; and then the variable tension of wrapping from equation (76) by substituting therein the values obtained for S_{twa} , P_2 , P_0 , and the value of r for each layer of the wire envelope.

Solving equation (73) for S_{twa} after substituting for P_1 and P_2 the values 41,535 and 8281 from equations (77) and (78), respectively,

$$S_{twa} = \frac{\frac{2}{3} \left\{ 41,535 - 8281 \left(\frac{13.15}{9} \right)^{\frac{3}{2}} \right\}}{\left(\frac{13.15}{9} \right)^{\frac{3}{2}} - 1} = +71,578 \text{ lbs. per sq. in.} \quad \text{tension.} \quad (79)$$

which is the constant tangential stress in all the layers of the wire envelope, system in action.

Substituting in equation (76) the values of S_{twa} , P_0 , P_2 , R_0 , and R_2 , and reducing

$$T_r = \frac{77,099 \left(\frac{13.15}{r} \right)^{\frac{3}{2}} (r^2 + 72) - 12,541,004}{r^2 - 36} \quad (80)$$

By substituting in equation (80) the value of r for any layer of wire, the tension of wrapping for that layer may be obtained. The wire used on this gun was of square cross section .1 inch on a side. For the first layer of wire the value of r , taken at the center of the thickness of the wire, is 9.05 and the corresponding value of $T_r = 58,425$ lbs. per sq. in.

For the middle layer of wire, the value of r taken as 11.1 gives $T_r = 49,420$ lbs. per sq. in.

For the outside layer, the value of r taken as 13.10 gives $T_r = 46,375$ lbs. per sq. in.

25. Stresses in the Jacket and the Tube, System in Action
 $P_0 = 66,316$ lbs. per sq. in., and at Rest. — (See Figs. 8 and 9.) — Since the shrinkage and shrinkage pressure between the jacket and wire envelope, and the pressure on the exterior of the tube, system at rest, are the same as when the wire envelope was wrapped under a constant tension of 51,566 lbs. per sq. in., the stresses in the tube and jacket both in action and at rest will be the same as determined for that case.

STRESSES IN THE WIRE ENVELOPE, SYSTEM IN ACTION

$$P_0 = 66,316 \text{ lbs. per sq. in.}$$

(See figures 8 and 9.)

26. The tangential stress in the wire envelope, system in action, has a constant value of 71,578 lbs. per sq. in. The radial stress S_{pwa} in the wire envelope may be obtained by substitution in equation (75) of the proper values of S_{twa} , P_2 , R_2 , and r .

Outer Surface of the Wire Envelope. $r = R_2 = 13.15$, $S_{twa} = 71,578$, $P_2 = 8281$.

The radial stress is

$$-\frac{4}{3} \left(71,578 + \frac{2}{3} 8281 \right) + 71,578 = -31,220 \text{ lbs. per sq. in.}$$

compression. (81)

Inner Surface of the Wire Envelope. $r = R_1 = 9$. — The radial stress is

$$-\frac{4}{3} \left(71,578 + \frac{2}{3} 8281 \right) \left(\frac{13.15}{9} \right)^{\frac{3}{2}} + 71,578 = -60,792 \text{ lb. per sq. in.}$$

compression. (82)

STRESS IN THE WIRE ENVELOPE, SYSTEM AT REST.

(See figures 8 and 9.)

27. The tangential and radial stresses at any radius r of the wire envelope due to the action of P_0 on the gun considered as a single cylinder may be obtained from equations (43) and (44), respectively, by the substitution therein of the proper values of r . Subtracting the results thus obtained from the corresponding stresses in action, we have

Outer Surface of the Wire Envelope. $r = R_2 = 13.15$. — The tangential stress is

$$+71,578 - \left\{ 5526 + \frac{[6.55400]}{(13.15)^2} \right\} = +45,344 \text{ lbs. per sq. in.}$$

tension. (83)

The radial stress is

$$-31,220 - \left\{ 5526 - \frac{[6.55400]}{(13.15)^2} \right\} = -16,038 \text{ lbs. per sq. in.}$$

compression. (84)

Inner Surface of the Wire Envelope. $r = R_1 = 9$. — The tangential stress is

$$+71,578 - \left\{ 5526 + \frac{[6.55400]}{(9)^2} \right\} = +21,843 \text{ lbs. per sq. in.} \\ \text{tension.} \quad (85)$$

The radial stress is

$$-60,792 - \left\{ 5526 - \frac{[6.55400]}{(9)^2} \right\} = -22,109 \text{ lbs. per sq. in.} \\ \text{compression.} \quad (86)$$

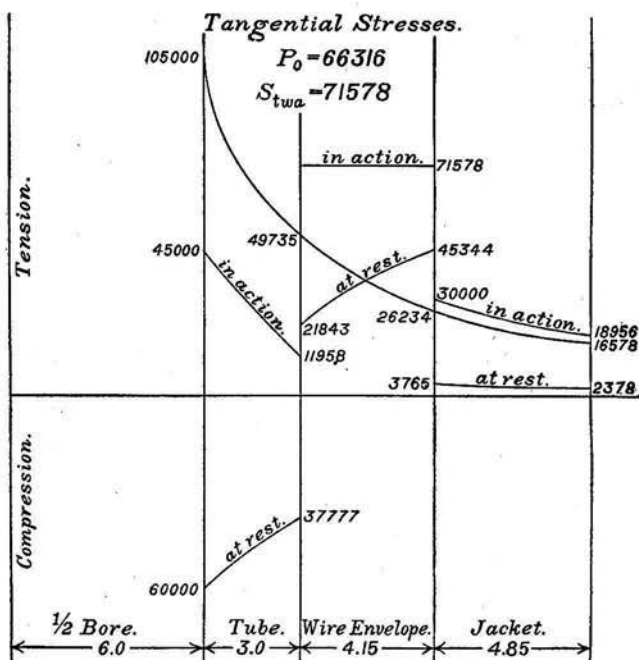


Fig. 8.

The tangential stresses, system in action when $P_0 = 66,316$ lbs. per sq. in., and system at rest, are shown graphically in Fig. 8; and the radial stresses in Fig. 9.

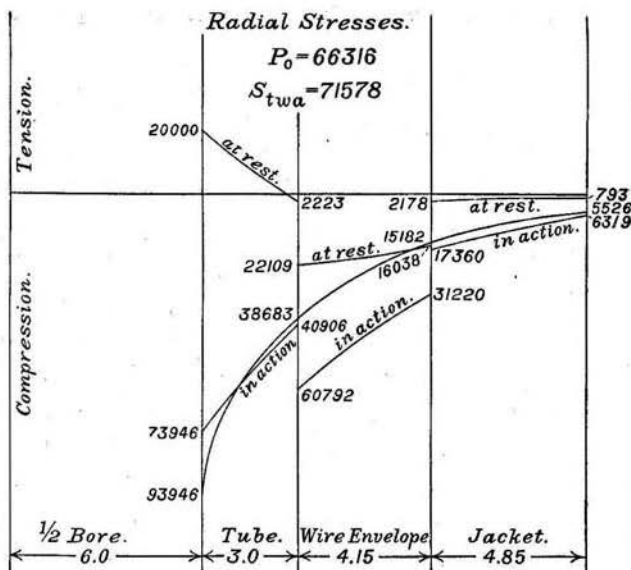


Fig. 9.

STRESSES IN ACTION, $P_0 = 42,000$ LBS. PER SQ. IN.

(See figures 10 and 11.)

28. The stresses in action, $P_0 = 42,000$ lbs. per sq. in., in the tube and jacket are the same as were obtained when the wire was wrapped under constant tension and are given by equations (55) to (58), inclusive, and (63) to (66), inclusive. The stresses in the wire envelope are obtained by adding to the stresses, system at rest, those due to the interior pressure $P_0 = 42,000$ lbs. per sq. in. acting on the gun considered as a single cylinder. The latter stresses have already been determined and used in equations (59) to (62), inclusive, relating to the stresses in action in the wire envelope, wrapped at constant tension, due to $P_0 = 42,000$ lbs. per sq. in.

Inner Surface of the Wire Envelope. — The tangential stress is

$$+21,843 + 31,499 = +53,342 \text{ lbs. per sq. in. tension. (87)}$$

The radial stress is

$$-22,109 - 24,500 = -46,609 \text{ lbs. per sq. in. compression. (88)}$$

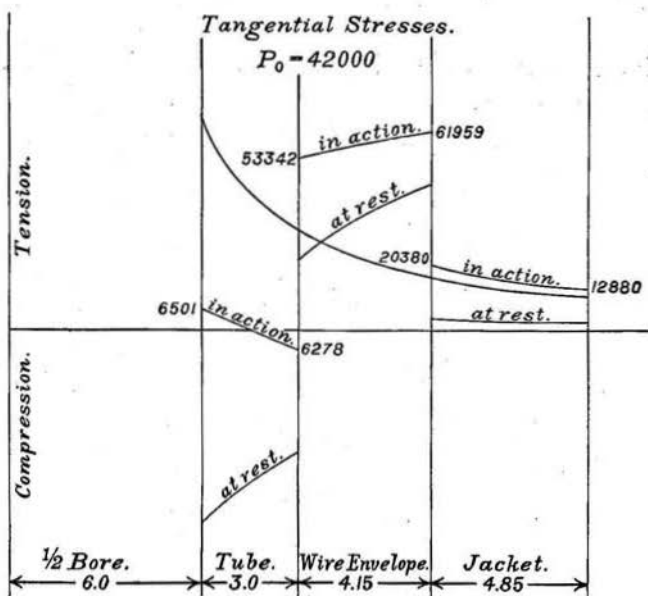


Fig. 10.

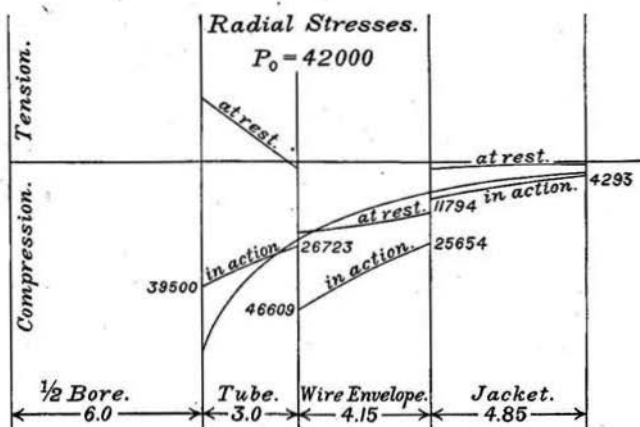


Fig. 11.

Outer Surface of the Wire Envelope. — The tangential stress is

$$+ 45,344 + 16,615 = +61,959 \text{ lbs. per sq. in. tension. (89)}$$

The radial stress is

$$-16,038 - 9616 = -25,654 \text{ lbs. per sq. in. compression. (90)}$$

Figs. 10 and 11 show graphically the tangential and radial stresses, respectively, system in action, when $P_0 = 42,000$ lbs. per sq. in.

29. Problem. — Find for a 6-inch wire-wrapped gun whose parts have the same dimensions and elastic limits as shown in Fig. 7 and whose jacket is assembled with the same shrinkage, .007 inch, but on the tube of which the wire is wrapped with varying tension so as to obtain a constant tangential stress in the wire envelope, system in action under the required powder pressure:

- (a) the constant tangential stress in the wire envelope, system in action, that will give to the gun the same tangential elastic strength as when the wire was wrapped under a constant tension of 50,000 lbs. per sq. in.;
- (b) the tension of wrapping at the inner and outer surfaces of the wire envelope;
- (c) the tangential and radial stresses at the inner and outer surfaces of each part, system in action under the maximum powder pressure which the gun can withstand without the tangential stress on any part exceeding the elastic limit in tension or compression; and
- (d) the tangential and radial stresses at the inner and outer surfaces of each part, system at rest.

CHAPTER II.

DETERMINATION OF THE FORCES BROUGHT UPON THE PRINCIPAL PARTS OF THE 3-INCH FIELD CARRIAGE BY THE DISCHARGE OF THE GUN.

30. Stability. — Total Resistance Opposed to Recoil of Gun.

— This carriage is so designed that when the gun is fired at 0 degrees elevation the carriage will not move to the rear and the wheels will not rise from the ground. Under these conditions the carriage is said to be *stable*. To prevent the carriage from moving to the rear when the gun is fired, there is provided at the end of the trail a spade of such an area that the horizontal resistance which the ground can exert against it is greater than the sum of the horizontal components of the forces brought upon the carriage by the discharge of the gun; and to prevent the wheels from rising from the ground the moment of the weight of the system, gun and carriage, around the point of support of the trail on the ground is made greater than the sum of the moments about that point of the forces brought upon the carriage by the discharge of the gun. The forces brought upon the carriage by the discharge of the gun will depend on the resistance opposed to the recoil of the gun. If no resistance is thus opposed, no force will be brought upon the carriage by the discharge of the gun except the slight friction between the gun and the carriage due to the weight of the gun and its movement in recoil. The weight of the gun acts on the carriage at all times. To limit the recoil of the gun a resistance must, however, be imposed. This resistance is imposed through the hydraulic brake at a distance of 7.156 inches below the axis of the gun. The action line of the resistance being below the axis of the gun, it will tend to rotate the gun around its center of mass, assumed to be in that axis. This rotation will be prevented by the clips on the gun engaging with those on the cradle (see Fig. 15), the clip on the cradle exerting a downward force on the forward end of the front clip of the

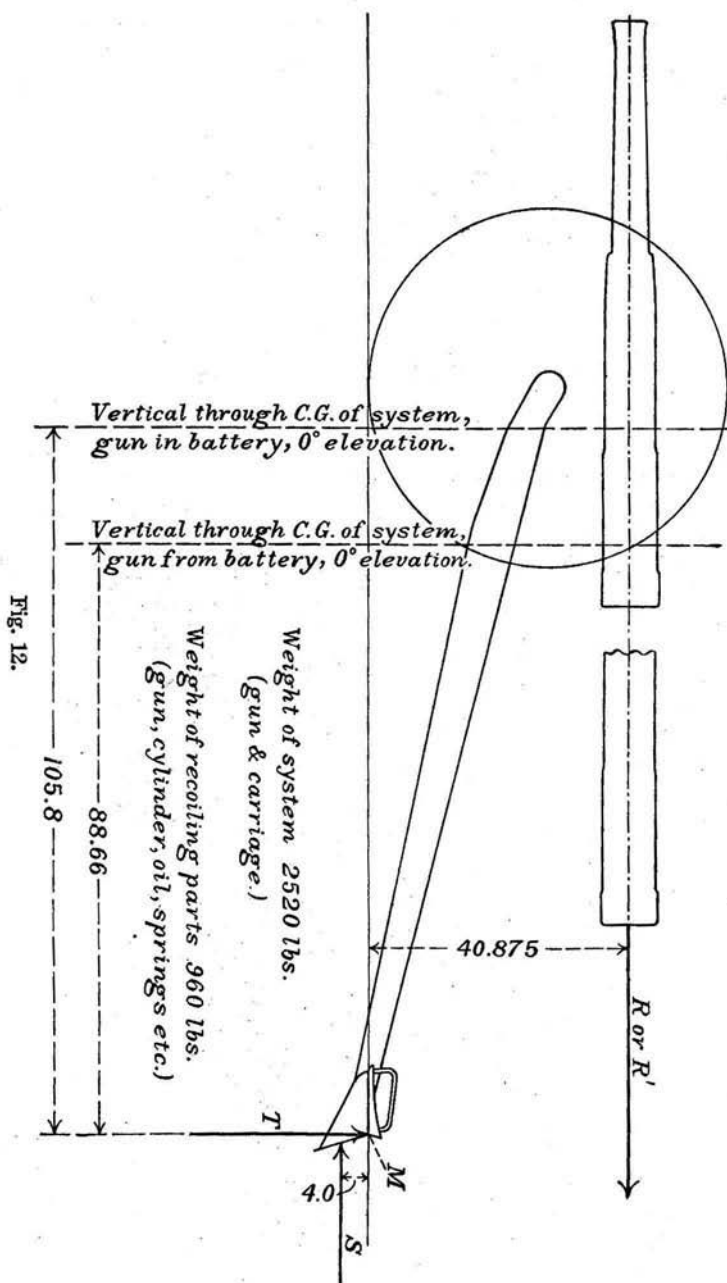
gun and an upward force on the clip of the gun at the rear end of the cradle. These forces will produce friction as the gun recoils, the action line of the friction being parallel to the axis of the gun and along the contact surfaces of the clips. The friction also opposes the recoil of the gun. The direction of the motion of the gun due to the action of the powder gases is in prolongation of its axis, and since this direction is not changed by the forces exerted on the gun by the carriage, it follows that *the resultant of all the latter forces must be a force whose action line coincides with the axis of the gun.* This resultant force is the total resistance which opposes the recoil of the gun.

31. Resultant of Forces Exerted by the Gun on the Carriage.

— Since the forces exerted by the carriage on the gun are likewise exerted by the gun on the carriage, but in opposite directions, it follows that the resultant of all the forces exerted by the gun on the carriage when the gun is fired is *a force in the prolongation of the axis of the gun equal and opposite in direction to the resistance which opposes the recoil of the gun.* As the carriage remains in equilibrium when the gun is fired the forces exerted upon it by the gun develop others between the carriage and the ground which oppose and neutralize the effect of the former so far as motion of the carriage is concerned. These forces develop others between the various parts of the carriage which, while they do not cause motion of the parts and, therefore, do not disturb their equilibrium, cause stresses in the materials of which the parts are made, which stresses must not exceed safe limits in order that the parts of the carriage may not be distorted or broken. The resultant of the forces exerted by the gun on the carriage, which is equal and opposite in direction to the resistance opposed to the recoil of the gun, is represented in Fig. 12 by R .

32. Limiting Value of the Resistance R Compatible with Stability of the Carriage. — General Expression for this Resistance.

(See Fig. 12.) — Let it be required to determine the greatest value R' which the force R may have without causing the wheels of the carriage to rise from the ground when the gun is fired at 0° elevation. When R has this value the wheels are just about to rise and consequently the pressure between them and the ground is zero. The carriage is prevented from moving to the rear under the action of the force R' by the parallel force S exerted by



the ground on the spade, the center of pressure of the ground, and therefore the point of application of the force S , being at a distance of 4 inches below the surface.

Since the pressure between the wheels and the ground is zero the only force exerted upward on the carriage by the ground to counteract the force of gravity on the system will be the vertical force T assumed to act at the rear point of contact of the float with the ground, the point about which moments will be taken to determine the conditions of stability. The forces acting on the carriage are, therefore, R' acting in prolongation of the axis of the gun; the weight, 2520 lbs., of the system, gun and carriage, acting vertically through its center of mass; and the forces S and T at the spade; as shown in Fig. 12. As the carriage is assumed to be in equilibrium under these forces, which can, because of their symmetrical distribution, be considered as contained in the central plane through the axis of the gun, the sum of the components of the forces in the vertical and horizontal directions must be zero and the sum of the moments of the forces about any point in their plane must be zero.

Whence

$$2520 - T = 0 \text{ or } T = 2520 \text{ lbs.} \quad (1)$$

$$S - R' = 0 \text{ or } S = R' \quad (2)$$

and taking moments about the point M and denoting by l the lever arm of the weight of the system with respect to this point,

$$R' \times 40.875 + S \times 4 - 2520 \times l = 0$$

or since $S = R'$ from equation (2)

$$R' = \frac{2520 \times l}{44.875} \quad (3)$$

which is the general expression for the greatest value of the resistance R that can be opposed to the recoil of the gun when it is fired at 0° elevation without causing the wheels to rise from the ground.

Limiting Value of the Resistance Corresponding to $l = 105.8$ ins. — Since the value of l will diminish as the gun recoils, R' will have a maximum value when the gun is in battery and a minimum value when the gun is at its extreme limit of recoil; and it will

diminish uniformly with the distance traveled by the gun in recoil from the maximum to the minimum value. The value of l when the gun is in battery is, from Fig. 12, 105.8 ins. and the corresponding value of R' , which we will call R'_1 , is

$$R'_1 = \frac{2520 \times 105.8}{44.875} = 5941 \text{ lbs.} \quad (4)$$

Limiting Value of the Resistance Corresponding to $l = 88.66$ ins. — Limiting Value of the Resistance Corresponding to a Length of Recoil of b ft. — It will be assumed for the present that the length of recoil of the gun on the carriage is not known. Since the value of R' diminishes uniformly with the distance traveled by the gun in recoil its value R'_2 corresponding to any distance b in feet recoiled by the gun can be determined as follows:

Assume any distance traveled by the gun in recoil, as 3.75 ft., and compute the corresponding position of the center of gravity of the system and the value of l . This has already been done and the value of l from Fig. 12 is 88.66 ins. The value of R'_2 when the gun has recoiled 3.75 ft. is, therefore,

$$R'_2 = \frac{2520 \times 88.66}{44.875} = 4979 \text{ lbs.} \quad (5)$$

The difference between R'_1 and R'_2 corresponding to a distance of 3.75 ft. recoiled by the gun is 962 lbs., and, therefore, the difference for a distance of b ft. recoiled by the gun is

$$\frac{962}{3.75} b = 256.54 b \text{ lbs.}$$

The value of R'_2 , then, corresponding to a distance of b ft. recoiled by the gun, is

$$R'_2 = R'_1 - 256.54 b = (5941 - 256.54 b) \text{ lbs.} \quad (6)$$

If we set off on a perpendicular the value of R'_1 to any scale from equation (4), and, on a second perpendicular at a horizontal distance from the first representing to any scale a distance of 3.75 ft., the value of R'_2 from equation (5), and join the ends of the perpendiculars by a right line, the ordinates of this line will represent the limiting values of R corresponding to any distance passed over by the gun in recoil which values must not be exceeded

if the wheels are not to rise from the ground when the gun is fired at an elevation of zero degrees. This line is shown in Fig. 13 by the full line.

If the resistance opposed to recoil is to be constant it may have any value less than R'_2 , but in order to meet all conditions that may arise it is the practice to make it at all points of recoil substantially less than R' , its limiting value.

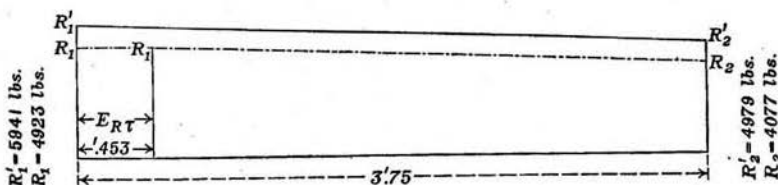


Fig. 13.

Method of Varying the Resistance Opposed to Recoil Followed in the Design of this Carriage. — In the design of the carriage by the Ordnance Department the resistance is made constant during the time the powder gases are acting on the gun, and after they cease to act it is made to decrease with the further distance passed over in recoil as shown by the dot and dash line in Fig. 13, the rate of decrease being the same as that of R' , its limiting value. This arrangement not only increases the margin of stability of the carriage during the early stages of recoil, as compared with a constant margin of stability, but facilitates the calculation of the value of the resistance corresponding to a given length of recoil of the gun or of the length of recoil corresponding to any assumed value of the resistance.

33. Relation Between the Varying Values of the Actual Resistance and the Length of Recoil of the Gun on the Carriage. — **Determination of the Values of R_1 , R_2 , and b .** — Let V_f be the maximum velocity of free recoil of the gun and recoiling parts determined as described in article 160, page 275, Lissak's Ordnance and Gunnery, τ the time corresponding to the attainment of V_f in free recoil, and E_f the corresponding space that would have been passed over by the gun in free recoil during the time τ ; τ and E_f being determined as described in article 163, page 279, Lissak's Ordnance and Gunnery.

Denoting by R_1 the actual resistance to recoil during the time τ when the powder gases are acting on the gun, and making it constant during this time, the velocity of restrained recoil at the end of the time τ will be

$$V_{rr} = V_f - \frac{R_1}{M_r} \tau \quad (7)$$

M_r being the mass of the gun and the other recoiling parts, cylinder, oil, counter-recoil springs, etc.; and R_1/M_r being the retardation of these parts produced by the resistance R_1 .

The space passed over by the gun in restrained recoil during the time τ will be

$$E_{rr} = E_f - \frac{R_1}{2 M_r} \tau^2 \quad (8)$$

which is shown in Fig. 13 as the space over which the resistance to the recoil of the gun is constant.

Calling R_2 the value of the resistance when the gun is at its extreme limit of recoil and b the total length of recoil of the gun on the carriage in feet, we may write

$$\frac{R_1 + R_2}{2} \left\{ b - E_{rr} \right\} = \frac{M_r V_{rr}^2}{2} \quad (9)$$

which expresses the fact that the work of the mean resistance $(R_1 + R_2)/2$ over the path $b - E_{rr}$ is equal to the kinetic energy of the recoiling mass at the end of the time τ .

Since the value of the resistance after the space E_{rr} has been passed over by the gun in recoil is to diminish in such manner that the line representing its diminishing values, Fig. 13, is to be parallel to the line representing the diminishing values of the limiting resistance R' , we may write the following value for R_2 corresponding to a total distance b recoiled by the gun [see deduction of equation (6)].

$$R_2 = R_1 - 256.54 (b - E_{rr}) \quad (10)$$

Substituting this value of R_2 in equation (9) and the values of E_{rr} and V_{rr} from equations (8) and (7), respectively,

$$\begin{aligned} \{ R_1 - 128.27 [b - E_f + (R_1/2 M_r) \tau^2] \} \{ b - E_f + (R_1/2 M_r) \tau^2 \} \\ = [M_r/2] [V_f - (R_1/M_r) \tau]^2 \end{aligned} \quad (11)$$

In equation (11) E_f , V_f , τ , and M_r are known so that only R_1 and b are unknown, either of which may be assumed and the other determined. Knowing R_1 and b , the value of R_2 may be obtained from equation (10) by substituting therein the value of R_1 and of E_{rr} from equation (8). If b is assumed the resulting values of R_1 and R_2 must be less than the corresponding values of the limiting resistances R'_1 and R'_2 ; and if R_1 is assumed b must not be inconveniently great and the value of R_2 must be less than that of R'_2 . In the ordinary case a safe value for R_1 would be assumed and the corresponding values of b and R_2 calculated. If these values were satisfactory they would be accepted, if not another value for R_1 would be assumed and b and R_2 recalculated; or if the design of the carriage was such that acceptable values of R_1 , R_2 , and b could not be obtained that would satisfy equation (11) the design would have to be changed.

34. Values of the Actual Resistance to Recoil of the 3-inch Field Carriage. — Since in the case of the 3-inch field carriage the length of recoil is 45 ins., equal to 3.75 ft., a value of $b = 3.75$ will be assumed and the corresponding values of R_1 and R_2 calculated from equations (11) and (10). For this carriage

$$V_f = 34.52 \text{ f.s.}; E_f = .48 \text{ ft.}; \tau = .018 \text{ secs.}; M_r = \frac{960}{32.2} = 29.813.$$

Substituting these values and $b = 3.75$ ft. in equation (11) and solving for R_1 we obtain

$$R_1 = 4923 \text{ lbs.} \quad (12)$$

Substituting this value in equation (7)

$$V_{rr} = V_f - \frac{R_1}{M_r} \tau = 34.52 - \frac{4923 \times .018}{29.813} = 31.55 \text{ f.s.} \quad (13)$$

From equation (8)

$$E_{rr} = E_f - \frac{R_1}{2 M_r} \tau^2 = .48 - \frac{4923 \times (.018)^2}{2 \times 29.813} = .453 \text{ ft.} \quad (14)$$

From equation (10)

$$R_2 = R_1 - 256.54 (b - E_{rr}) = 4923 - 256.54 (3.75 - .453) = 4077 \text{ lbs.} \quad (15)$$

The actual resistance opposed to the recoil of the gun, therefore, has a value of 4923 lbs. while the gun is recoiling over a distance of .453 ft. and then decreases uniformly to a value of 4077 lbs. at the end of recoil, the length of recoil being 3.75 ft. or 45 ins. After the distance of .453 ft. has been traveled by the gun in recoil, the equation giving the resistance at any point is as follows:

$$R_x = 4923 - 256.54 (x - .453) \quad (16)$$

in which x is the distance in feet, measured from the origin of movement, over which the gun has recoiled.

Margin of Stability. — Referring to Fig. 13, the difference, called the margin of stability, between the limiting value of the resistance that may be opposed to the recoil of the gun without causing the wheels to rise from the ground, and the actual resistance employed is 1018 lbs. at the beginning of recoil, which decreases to 902 lbs. when the powder gases cease to act on the gun, at which time the gun has recoiled a distance of .453 ft. Thereafter, and until recoil has ceased at a distance of 3.75 ft. = 45 ins., the margin of stability remains constant and equal to 902 lbs.

35. Velocity of Restrained Recoil as a Function of Space. — For substitution in the formula which gives the areas of the throttling orifices it is necessary to determine the velocity of restrained recoil as a function of the distance recoiled. Until the gun has recoiled a distance of .453 ft. the resistance is constant and the velocity of restrained recoil as a function of space must be obtained in the manner described in article 166, pages 283 and 284, Lissak's Ordnance and Gunnery. After the gun has recoiled a distance of .453 ft. the velocity of restrained recoil corresponding to any distance x measured from the origin of movement is obtained by equating the work remaining to be done by the resistance with the energy of the recoiling mass. Thus, calling R_x the value of the resistance at the point x , and V_{rx} the corresponding value of the velocity of restrained recoil,

$$\frac{R_x + R_2}{2} (b - x) = \frac{M_r V_{rx}^2}{2}$$

or
$$\frac{R_x + 4077}{2} (3.75 - x) = 14.907 V_{rx}^2 \quad (17)$$

from which the value of V_{rx} corresponding to any point x is obtained. The value of R_x for substitution in equation (17) is obtained from equation (16).

FORCES ON THE CARRIAGE CONSIDERED AS ONE PIECE, GUN AT EXTREME LIMIT OF RECOIL AND AT 15° ELEVATION.

36. The resistance opposed to the recoil of the gun on the carriage having been determined, the forces on the carriage considered as one piece, and also on the various important parts of the carriage may now be determined. In these calculations the gun will be assumed to be at the end of its travel in recoil after having been fired at 15° elevation. For a complete solution of the problem, however, the position of the gun in battery should be considered and other conditions of elevation also assumed. The parts should then be proportioned to resist the maximum stresses brought upon them under any of the assumed conditions. The method of calculating the forces in the case now under consideration is, however, exactly like that to be followed in any of the other cases mentioned.

The gun and carriage are shown diagrammatically in Fig. 14, the gun being at 15° elevation and at its extreme limit of recoil. In this position the resistance opposed to its recoil by the carriage is 4077 lbs., equation (15), which is equal and opposite in direction to the resultant force exerted by the gun on the carriage. The latter force is represented in position and direction by R_2 , Fig. 14. The horizontal component of this force tends to move the carriage to the rear, but the movement is prevented by the horizontal force S exerted by the ground on the spade, its action line (center of pressure) being at a distance of four inches below the surface of the ground. The vertical component of the force R_2 and the weight of the system tend to produce vertical motion downward, but this is prevented by the upward pressure of the ground at L and at T , there being a pressure between the wheels and the ground in this case not only because the resistance R_2 is less than the limiting resistance, but also because the gun is now elevated above zero degrees. The forces in this case and in all cases to be discussed hereafter can, because of the symmetrical disposition of the parts, be considered as co-planar, and they will hereafter be so considered without further reference thereto.

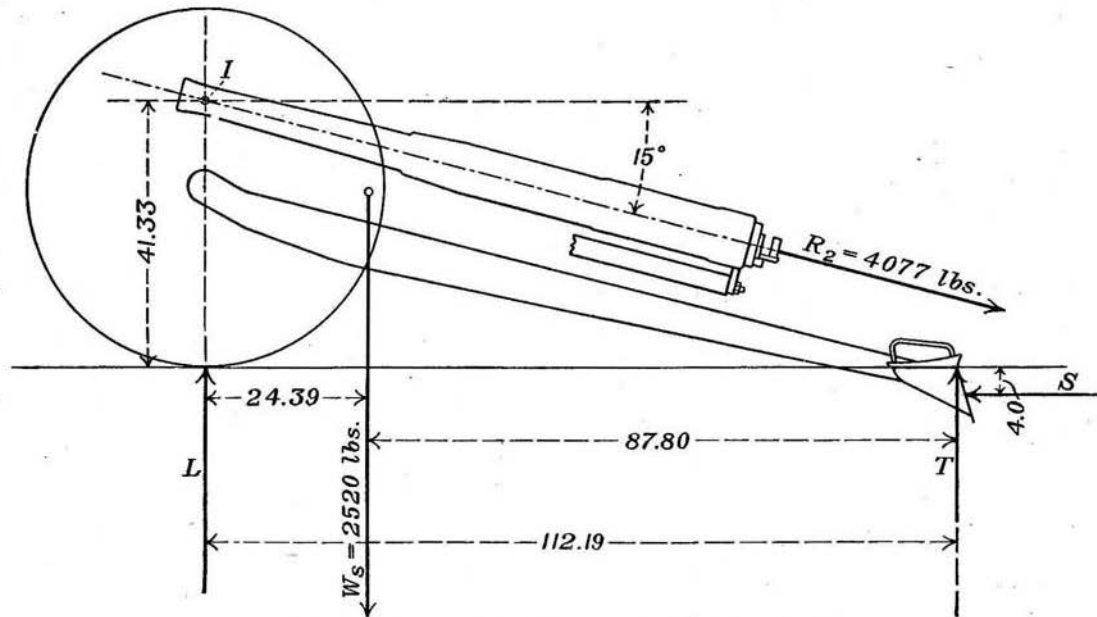


Fig. 14. — Forces on the Carriage considered as one Piece.

Since the carriage is in equilibrium, the sum of the vertical components of the forces must be zero as must the sum of the horizontal components, and also the sum of the moments of the forces about any point in their plane. Whence, considering vertical forces acting downward and horizontal forces acting in a direction opposite to that of recoil as positive,

$$S - 4077 \cos 15^\circ = 0 \quad (18)$$

and

$$4077 \sin 15^\circ + 2520 - L - T = 0 \quad (19)$$

Taking moments about I , the intersection of the action lines of the forces R_2 and L , and considering moments that tend to produce clockwise rotation as positive,

$$2520 \times 24.39 + S \times 45.33 - T \times 112.19 = 0 \quad (20)$$

From equation (18)

$$S = 4077 \cos 15^\circ = 3938 \text{ lbs.} \quad (21)$$

From equation (20)

$$T = 2139 \text{ lbs.} \quad (22)$$

From equation (19)

$$L = 1436 + \text{lbs.} \quad (23)$$

FORCES ON THE MOST IMPORTANT PARTS OF THE CARRIAGE, GUN AT EXTREME LIMIT OF RECOIL AND AT 15° ELEVATION.

37. Forces on the Gun. — Consider first the gun alone, Fig. 15. The resistance opposed to its recoil is 4077 lbs. and is the resultant of a number of forces acting between the carriage and the gun. The force P is the resistance opposed to the recoil of the gun by the pressure in the recoil cylinder, its action line being parallel to the axis of the gun and at a distance below it of 7.156 ins. As it is not exerted through the center of mass of the gun it tends to rotate it about that center. The weight of the gun, $W_g = 835$ lbs., is also a force acting on it as shown, but as it acts through the center of mass it has no tendency to produce rotation. The rotation of the gun under the action of the force P is prevented by the engagement of the clips on the gun with those of the cradle between which are developed the forces A acting downward on the gun at the front end of the forward

gun clip and B acting upward at the rear end of the clip on the cradle. The forces A and B produce friction between the contact surfaces of the clips which is equal to $f \times (A + B)$, f being the coefficient of friction assumed equal to .15. This friction, which

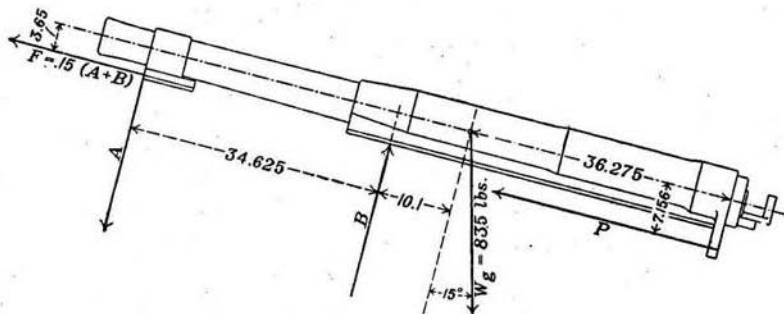


Fig. 15.

will be called F , acts along the contact surfaces of the clips in a direction parallel to the axis of the gun and opposite to that of recoil. Its action line may be taken at a distance of 3.65 ins. below the axis of the gun as shown in Fig. 15, and since it also tends to produce rotation of the gun around its center of mass, it causes additional pressure at A and B to counteract this tendency.

As the resultant of all these forces is a resistance of 4077 lbs. acting along the axis of the gun in a direction opposite to that of recoil, we may write three equations stating

- that the sum of the components of all the forces parallel to the axis of the gun is equal to 4077 lbs.;
- that the sum of the components of all the forces in a direction perpendicular to that axis is zero, since the resistance has no component in that direction; and
- that the sum of the moments of the forces about the center of mass is zero, since the resistance passes through that center and, therefore, has no moment with respect to it.

The action lines and location of all the forces are shown in Fig. 15. Forces acting to oppose the recoil of the gun and those acting downward at right angles to the axis of the gun are con-

sidered positive. Those acting in opposite directions are considered negative. Moments that tend to produce clockwise rotation are considered positive and those tending to produce counter-clockwise rotation, negative. Therefore,

$$P - W_c \sin 15^\circ + .15 (A + B) = 4077 \quad (24)$$

$$A - B + W_c \cos 15^\circ = 0 \quad (25)$$

$$P \times 7.156 + .15 (A + B) \times 3.65 + B \times 10.1 - A \times 44.725 = 0 \quad (26)$$

Substituting in equations (24) and (26) the value of B obtained by solving equation (25) for B and replacing $W_c \cos 15^\circ$ by its value $835 \cos 15^\circ = 807$ lbs. and $W_c \sin 15^\circ$ by its value of 216 lbs. and reducing, we have

$$P + .3 A = 4172 \quad (27)$$

$$P \times 7.156 - 33.53 A = -8593 \quad (28)$$

Multiplying both members of equation (27) by 7.156 and subtracting from equation (28)

$$-35.6768 A = -38,448 \quad (29)$$

$$\text{or} \quad A = 1078 \text{ lbs.} \quad (30)$$

$$\text{From equation (25)} \quad B = 1885 \text{ lbs.} \quad (31)$$

$$\text{From equation (27)} \quad P = 4172 - .3 \times 1078 = 3849 \text{ lbs.} \quad (32)$$

$$F = .15 (A + B) = .15 (1078 + 1885) = 444 \text{ lbs.} \quad (33)$$

38. Forces on the Cradle. — The forces acting between the cradle and the gun when the latter is fired act on the cradle in a direction opposite to that in which they act on the gun. They are shown in position, direction, and magnitude in Fig. 16. $W_c = 409$ lbs., the weight of the cradle, acts on it vertically through its center of mass. The forces P , F , and $W_c \sin 15^\circ$ tend to move the cradle in a direction parallel to that of the recoil of the gun, but are prevented from doing so by the pintle of the cradle bearing against the pintle socket of the rocker, giving rise to the force C between the contact surfaces which acts in the opposite direction but parallel to the force P . Rotation of the cradle about its center of mass under the action of the forces A , B , C , and F is prevented by the engagement of the clips of the pintle on the cradle with those of the pintle socket of the rocker and by the pressure

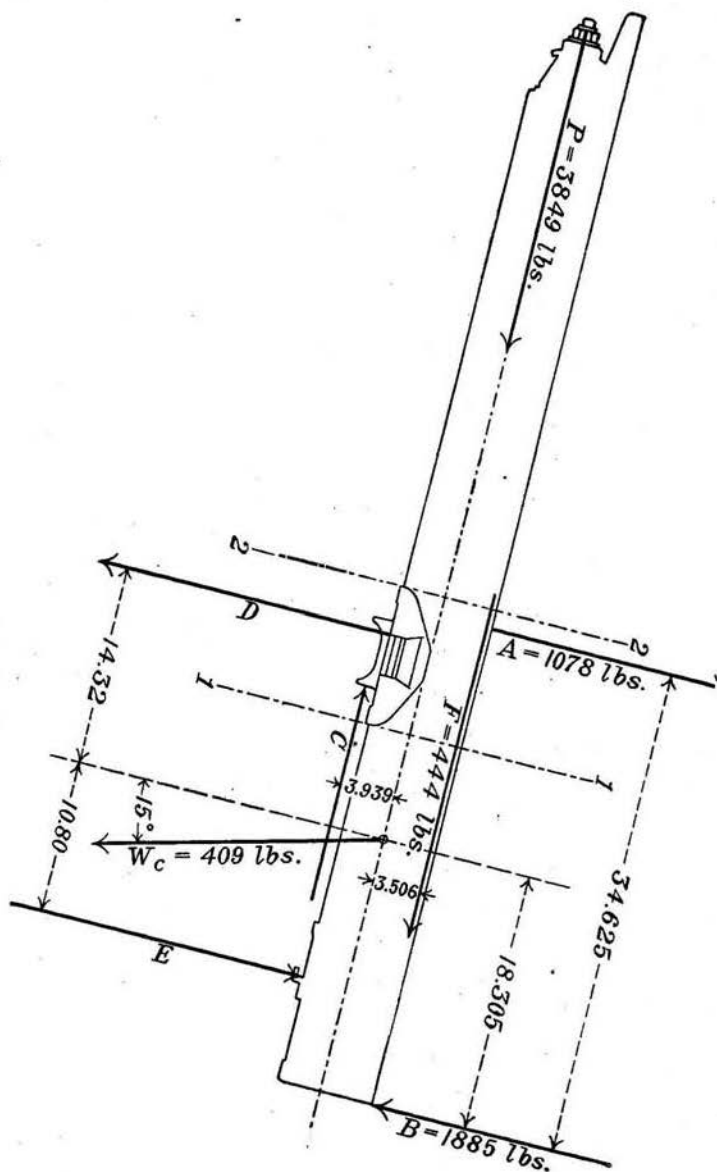


Fig. 16. — Forces on the Cradle.

between the surfaces of the rear cradle clip and the part of the rocker above the connection for the elevating screw, giving rise to a force D acting downward on the cradle at the clips of the pintle and a force E acting upward against the rear cradle clip, both forces being perpendicular to the axis of the cradle. The location of these forces is shown in Fig. 16.

As the cradle is in equilibrium under the forces acting upon it, the sum of all the components of the forces in a direction parallel to the axis of the cradle must be zero as well as the sum of all the components of the forces in a direction perpendicular to that axis. The sum of the moments of the forces about any point in their plane must also be zero.

$$\text{Therefore, } C - 444 - 3849 - 409 \sin 15^\circ = 0 \quad (34)$$

$$1885 + 409 \cos 15^\circ + D - E - 1078 = 0 \quad (35)$$

and taking moments about the center of mass of the cradle,

$$1078 \times 16.32 + 1885 \times 18.305 + 444 \times 3.506 + C \times 3.939 \\ - D \times 14.32 - E \times 10.80 = 0 \quad (36)$$

$$\text{From equation (34) } C = 4399 \text{ lbs.} \quad (37)$$

Multiplying both members of equation (35) by 10.80 and subtracting from equation (36)

$$25.12 D = 58,001. \quad (38)$$

$$\text{Whence } D = 2309 \text{ lbs.} \quad (39)$$

$$\text{From equation (35) } E = 3511 \text{ lbs.} \quad (40)$$

39. Forces on the Rocker. — The forces acting between the cradle and the rocker act on the latter in a direction opposite to that in which they act on the cradle. They are shown in position, direction, and magnitude in Fig. 17. $W_r = 56$ lbs., the weight of the rocker, acts on it vertically through its center of mass. Movement of the rocker under the action of these forces is prevented by its engagement on the axle and by the support afforded by the elevating screw, giving rise to a force between the axle and the rocker and one between the elevating screw and the rocker. The direction of the force exerted by the axle on the rocker is not known but it must be normal to the surfaces in contact and must, therefore, intersect the axis of the

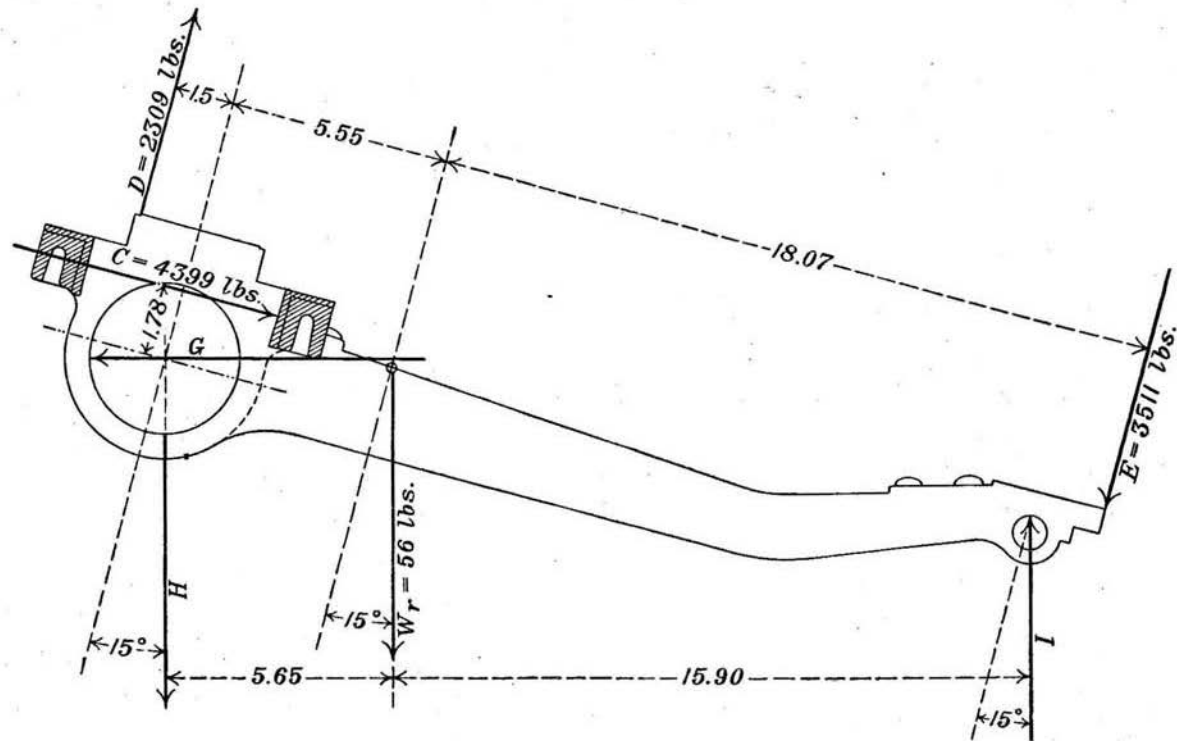


Fig. 17. — Forces on the Rocker.

axle. This force may, however, be resolved into two components, one horizontal and the other vertical, and when the intensities of the components are determined the direction of the resultant can be obtained if desired. These components are shown in Fig. 17, designated as G and H , respectively. Since the elevating screw is pivoted both to the rocker and the trail and is, therefore, free to rotate about its connections thereto, the action line of the force I , Fig. 17, exerted by it on the rocker must coincide with the axis of the screw, which is vertical when the gun is at an angle of elevation of 15° .

As the rocker is in equilibrium under the forces acting on it, we may write three equations of equilibrium as follows:

$$G - 4399 \cos 15^\circ - 2309 \sin 15^\circ + 3511 \sin 15^\circ = 0 \quad (41)$$

$$3511 \cos 15^\circ - I + 56 + H + 4399 \sin 15^\circ - 2309 \cos 15^\circ = 0 \quad (42)$$

Taking moments around the point of intersection of the forces G and H ,

$$2309 \times 1.5 + 4399 \times 1.78 + 3511 \times 23.62 - I \times 21.55 + 56 \times 5.65 = 0 \quad (43)$$

$$\text{From equation (41)} \quad G = 3938 \text{ lbs.} \quad (44)$$

$$\text{From equation (43)} \quad I = 4387 \text{ lbs.} \quad (45)$$

$$\text{From equation (42)} \quad H = 2031 \text{ lbs.} \quad (46)$$

40. Forces on the Axle, Wheels, Flasks, and Other Parts Below the Rocker, Considered as One Piece.—The forces acting between the rocker and the parts below it, act on the parts below in a direction opposite to that in which they act on the rocker. They are shown in position, direction, and magnitude in Fig. 18. $W_f = 1220$ lbs., the weight of all the parts below the rocker, acts on them through their center of mass. All movement of the parts of the carriage below the rocker under the action of these forces is prevented by the horizontal and vertical reactions of the ground which give rise to a vertical force L acting upward on the wheel, another T acting upward on the float of the spade, and a horizontal force S acting against the spade in a direction opposed to recoil. T is assumed to act on the float at its rear point of contact with the ground. The point of application of S is at the center of pressure of the earth on the spade, a distance of

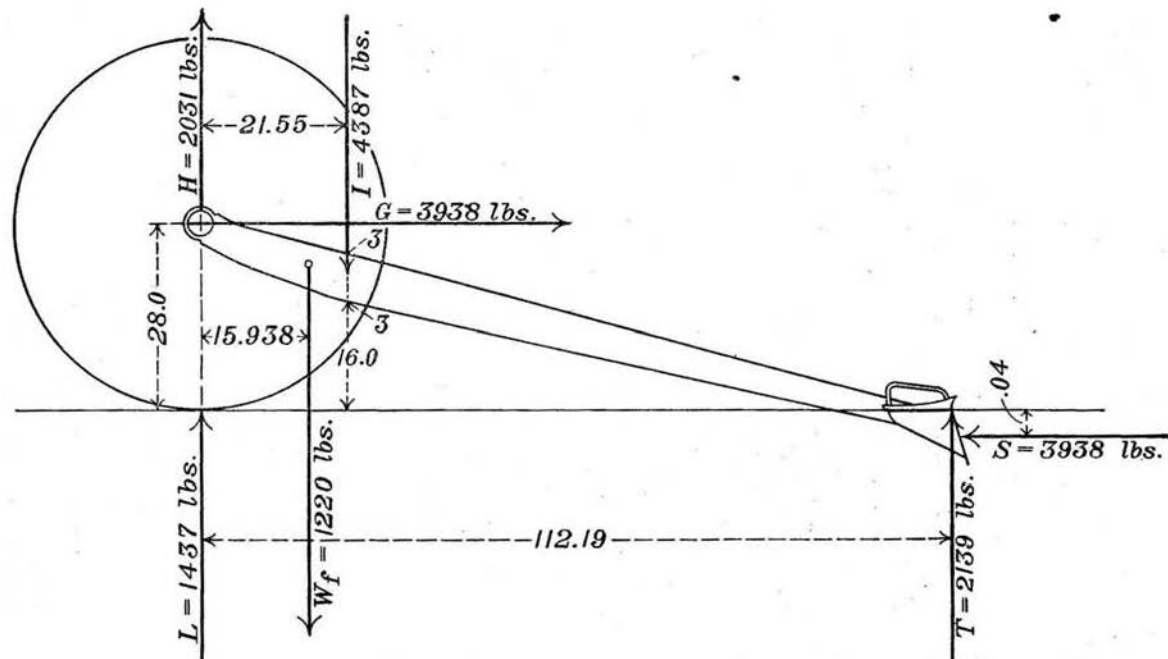


Fig. 18. — Forces on the Axle, Wheels, Flasks, and other Parts below the Rocker, considered as one Piece

four inches below the surface of the ground. The forces L , T , and S are shown in position, direction, and magnitude in Fig. 18.

Writing the ordinary equations of equilibrium

$$4387 - T + 1220 - L - 2031 = 0 \quad (47)$$

$$S - 3938 = 0 \quad (48)$$

Taking moments about the center of the axle, the point of intersection of the forces G , H , and L ,

$$4387 \times 21.55 + S \times 32.0 - T \times 112.19 + 1220 \times 15.938 = 0 \quad (49)$$

$$\text{From equation (48)} \quad S = 3938 \text{ lbs.} \quad (50)$$

$$\text{From equation (49)} \quad T = 2139 \text{ lbs.} \quad (51)$$

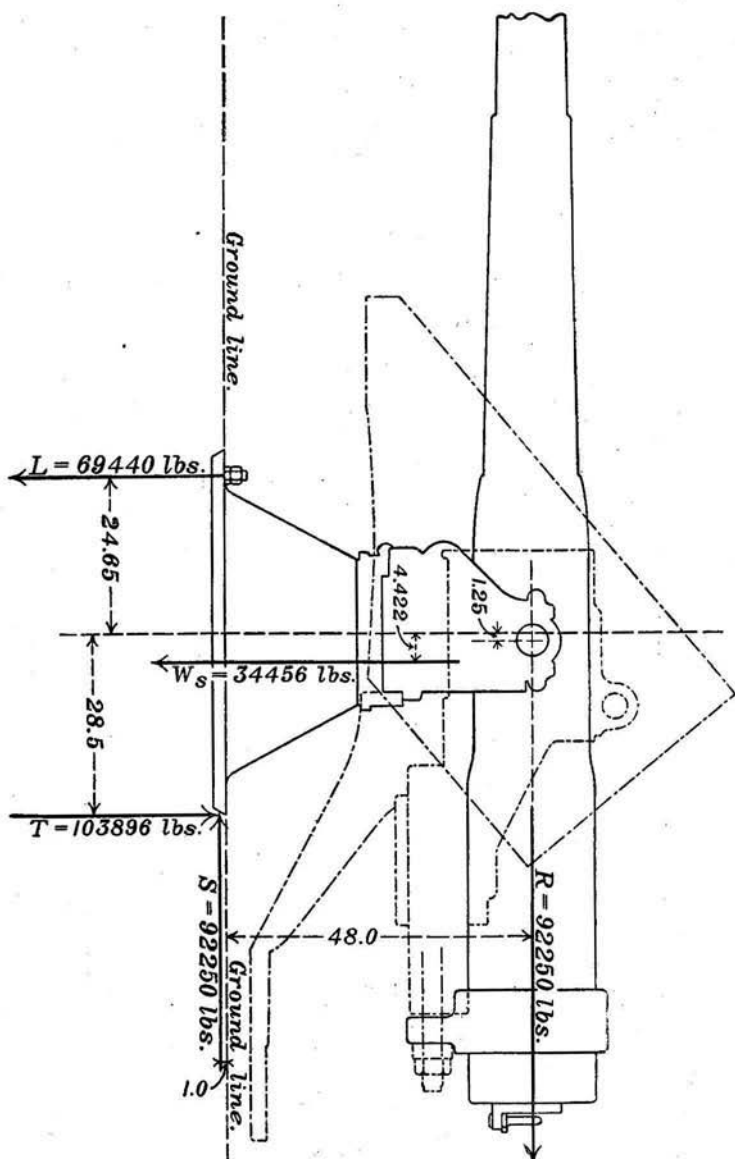
$$\text{From equation (47)} \quad L = 1437 \text{ lbs.} \quad (52)$$

Comparing the values of S , T , and L given in equations (50), (51), and (52) with those given in equations (21), (22), and (23) deduced by considering the carriage as one piece, it will be seen that the values of S and T deduced by the two methods are identical and the values of L differ by less than one pound, which proves the correctness of the values of all the forces determined above. The very slight difference in the values of L deduced by the two methods is due to not carrying out the values of the forces on the various parts beyond the decimal point.

PROBLEMS.

41. Figs. 19 to 24, inclusive, indicate diagrammatically the location, direction, and magnitude of the forces on the 5-inch barbette carriage, model 1903, when the gun has been fired at 0° elevation and is at its extreme limit of recoil. This carriage is in all respects similar to the 6-inch barbette carriage described in article 195, pages 335 to 337, Lissak's Ordnance and Gunnery. It was designed to present a resistance of 92,250 lbs. to the recoil of the gun.

Fig. 19. — Forces on the Carriage considered as one Piece



1. Assuming a resistance of 92,250 lbs. to the recoil of the gun, determine the values of the forces S , T , and L shown in Fig. 19. Explain why these forces are developed at the points indicated.

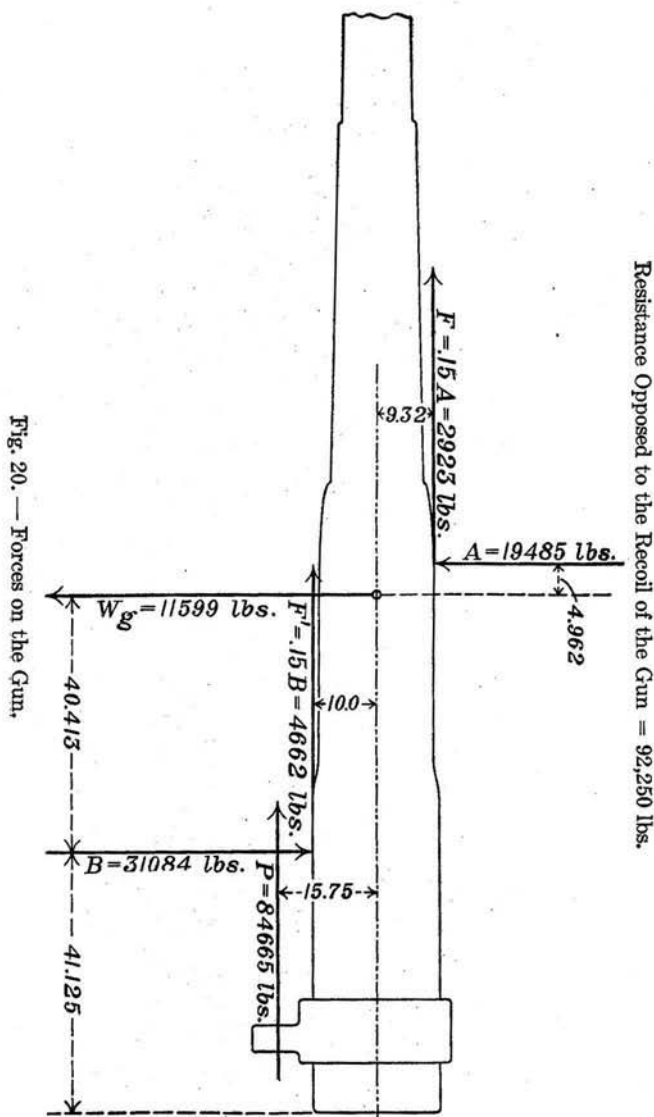
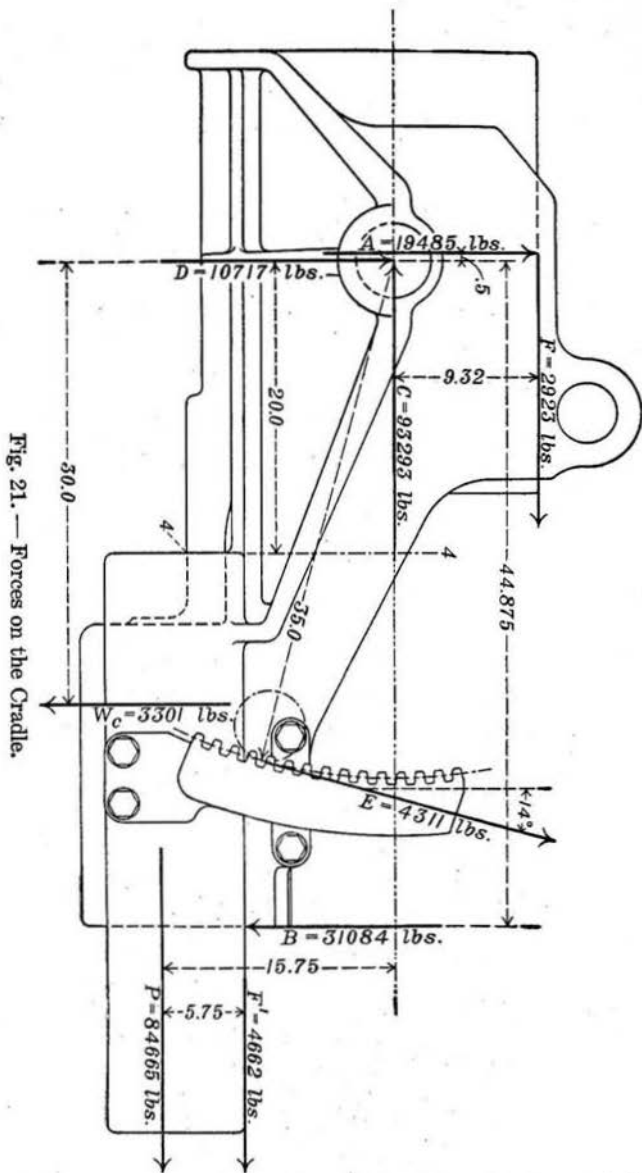


Fig. 20. — Forces on the Gun.

2. Assume the same resistance to the recoil of the gun as in problem 1, and determine the values of the forces A , B , P , F , and F' shown in Fig. 20. Explain why these forces are developed at the points indicated.



3. Assume the values of the forces A, B, P, F, F', and W_c, and determine the values of the forces C, D, and E shown in Fig. 21. Explain why these forces are developed at the points indicated.

The Shield and Brackets are attached to the Pivot Yoke and are considered as forming one Piece with it.

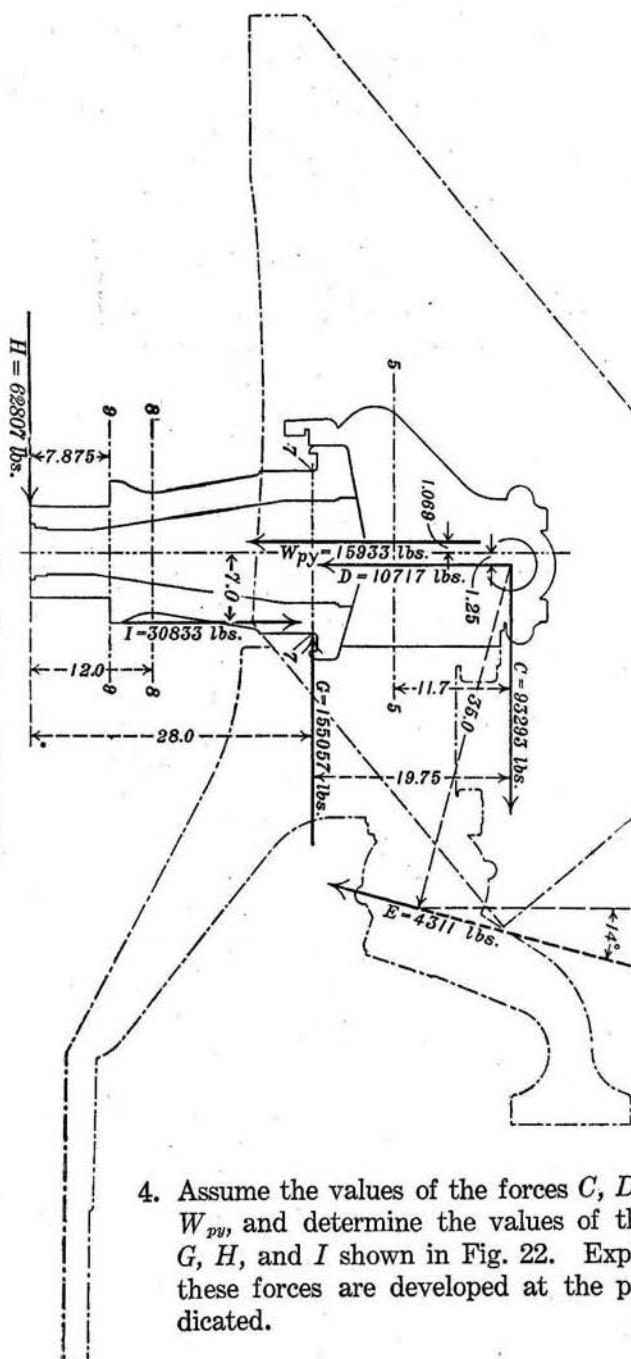


Fig. 22. — Forces on the Pivot Yoke.

4. Assume the values of the forces C , D , E , and W_{py} , and determine the values of the forces G , H , and I shown in Fig. 22. Explain why these forces are developed at the points indicated.

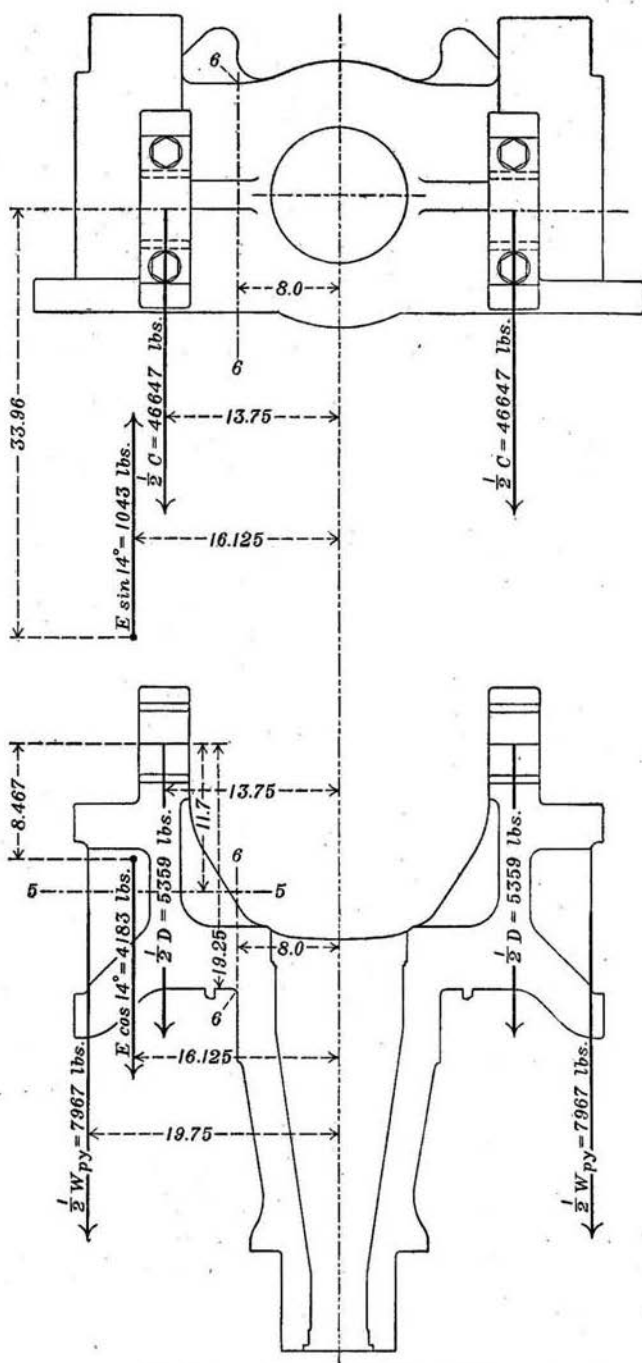


Fig. 23. — Forces on the Pivot Yoke.

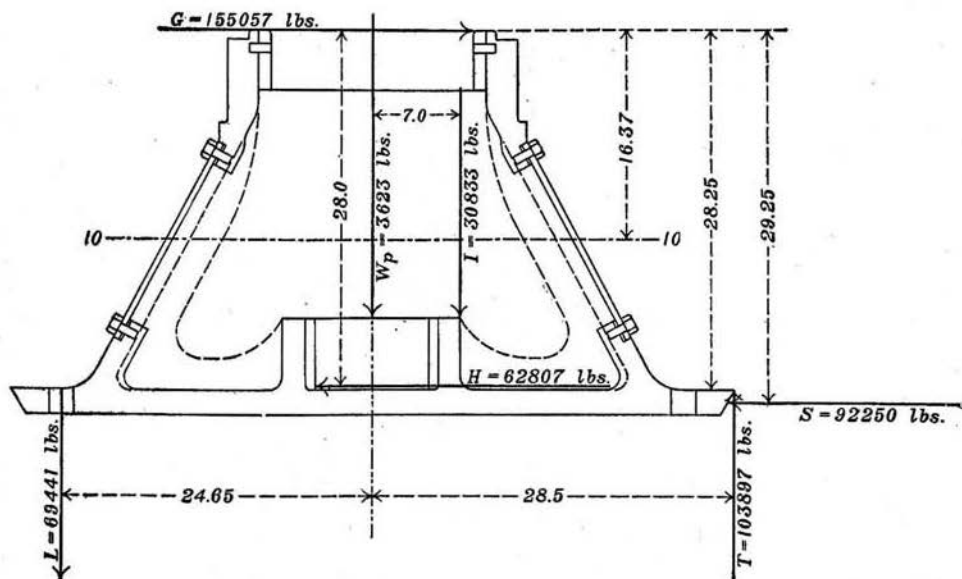


Fig. 24. — Forces on the Pedestal.

5. Assume the values of the forces G , H , I , and W_p , and determine the values of the forces S , T , and L shown in Fig. 24. Explain why these forces are developed at the points indicated.

CHAPTER III.

DETERMINATION OF THE FORCES BROUGHT UPON THE PRINCIPAL PARTS OF A DISAPPEARING GUN CARRIAGE BY THE DISCHARGE OF THE GUN.

42. The 6-inch disappearing carriage model of 1905 M1 is chosen to illustrate the subject; and the forces brought upon the gun, gun levers, top carriage, and elevating arm will be determined.

The first step is to obtain the curve showing the velocity of free recoil of the gun as a function of time.

Velocity of the Projectile in the Bore. — Fig. 25 shows the velocity of the projectile in the bore as a function of the travel of the projectile in inches.

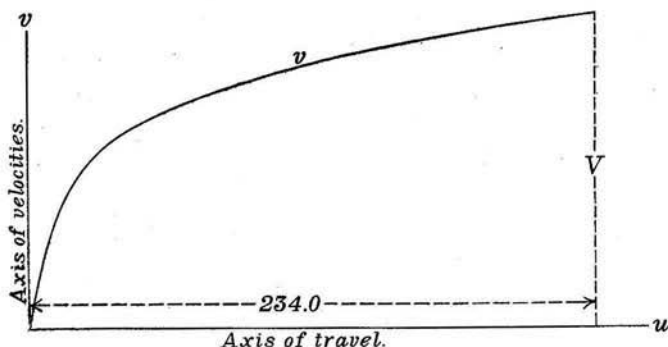


Fig. 25.

The ordinates of this curve corresponding to various values of the travel of the projectile calculated by the formulas of interior ballistics are given in Table 1.

TABLE 1.

Travel of projectile in inches.	Velocity in bore, feet per second.	Travel of projectile in inches.	Velocity in bore, feet per second.
2	223.24	54	1776.61
4	411.60	60	1829.30
6	577.41	72	1924.05
8	714.57	84	2004.32
10	835.79	96	2076.75
12	940.95	108	2142.55
15	1074.25	120	2202.73
18	1184.05	132	2259.86
21	1275.70	144	2309.20
24	1353.46	156	2356.67
27	1419.15	168	2400.89
30	1477.17	180	2442.23
33	1527.95	192	2481.01
36	1573.20	204	2517.49
42	1651.56	216	2551.89
48	1718.15	234	2600.02

Curve of Reciprocals. — **Velocity and Travel of Gun in Free Recoil while the Projectile is in the Bore.** — Fig. 26 shows the reciprocals of the velocities of the projectile in the bore as a function of the travel of the projectile in inches.

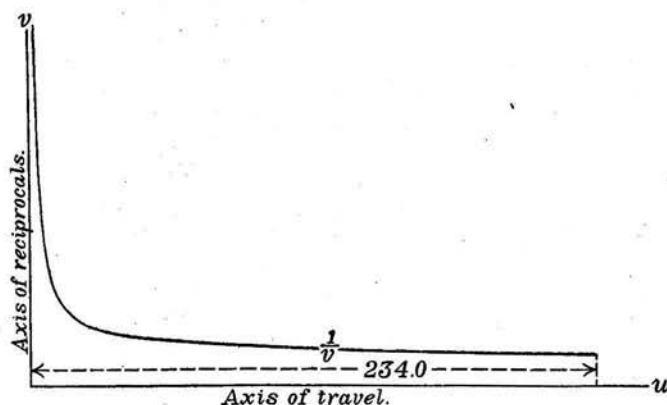


Fig. 26.

Table 2 gives the values of the reciprocals of the velocities of the projectile in the bore corresponding to various values of the travel of the projectile, the corresponding times obtained by

measuring the areas under the curve of Fig. 26 up to the corresponding limiting ordinates, the corresponding values of the velocities of free recoil of the gun, and the corresponding distances traveled by it in free recoil. The velocity of free recoil of the gun while the projectile is in the bore is obtained from equation (2), page 275, Lissak's Ordnance and Gunnery, which is as follows:

$$v_f = \frac{w + \frac{1}{2} \tilde{w}}{W} v$$

in which w , the weight of the projectile, is 106 lbs.; $\tilde{w}$, the weight of the powder charge, is 30 lbs.; W , the weight of the gun, is 12,764 lbs., and v_f is the velocity of free recoil of the gun corre-

TABLE 2.

Travel of projectile in inches.	Reciprocal of velocity in bore.	Time in seconds.	Velocity of free recoil of gun, feet per second.	Length of free recoil in feet.
2	.0044796	.0014932	2.116	.00158
4	.0024296	.0020187	3.902	.00316
6	.0017359	.0023557	5.474	.00474
8	.0013995	.0026157	6.774	.00632
10	.0011965	.0028308	7.923	.00790
12	.0010628	.0030184	8.920	.00948
15	.0009309	.0032665	10.181	.01185
18	.0008446	.0034879	11.225	.01422
21	.0007839	.0036912	12.094	.01659
24	.0007389	.0038814	12.831	.01896
27	.0007047	.0040617	13.454	.02133
30	.0006770	.0042343	14.003	.02370
33	.0006545	.0044007	14.485	.02607
36	.0006356	.0045619	14.914	.02844
42	.0006055	.0048720	15.657	.03318
48	.0005820	.0051687	16.288	.03792
54	.0005629	.0054548	16.842	.04266
60	.0005467	.0057196	17.342	.04740
72	.0005194	.0062595	18.240	.05688
84	.0004989	.0067616	19.001	.06636
96	.0004815	.0072517	19.688	.07584
108	.0004667	.0077257	20.312	.08532
120	.0004540	.0081860	20.882	.09480
132	.0004425	.0086342	21.424	.10428
144	.0004331	.0090712	21.892	.11376
156	.0004243	.0094994	22.341	.12324
168	.0004161	.0099198	22.760	.13272
180	.0004095	.010333	23.152	.14220
192	.0004031	.010739	23.520	.15168
204	.0003972	.011139	23.866	.16116
216	.0003919	.011534	24.192	.17064
234	.0003846	.012116	24.648	.18486

$\tau = .0606$ second.

$E = 1.6786$ feet.

sponding to a velocity v of the projectile. The distance traveled by the gun in free recoil while the projectile is in the bore is given by the formula

$$u_f = \frac{w + \frac{1}{2} \tilde{\omega} u}{W} \frac{12}{12}$$

in which w , $\tilde{\omega}$, and W have the same meanings as before, u is the travel of the projectile in inches, and u_f is the distance in feet traveled by the gun in free recoil.

Curve of Free Recoil. — From the data given in the third and fourth columns of table 2 the curve showing the velocity of free recoil of the gun as a function of time may be plotted up to the time when the projectile leaves the bore of the gun. The remainder of this curve up to the time when the maximum velocity of free recoil is reached and the powder gases cease to act on the gun is obtained as outlined in paragraph 163, page 278, Lissak's Ordnance and Gunnery. The maximum velocity of free recoil computed from equation (4), page 275, Lissak's Ordnance and Gunnery, is

$$V_f = (wV + 4700\tilde{\omega})/W = 32.64 \text{ f.s.}$$

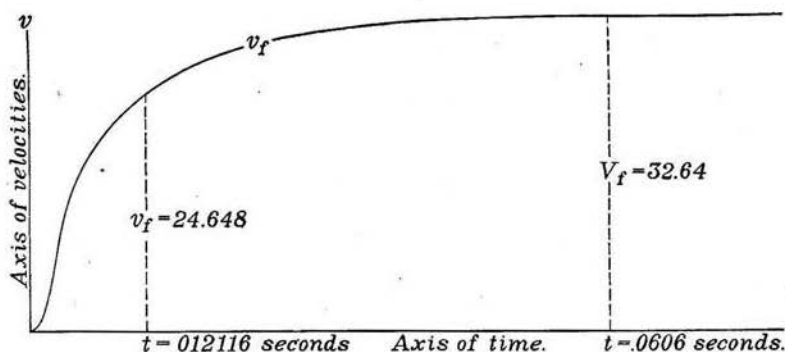


Fig. 27.

With this value as an ordinate to the proper scale a line is drawn parallel to the axis of t and the curve obtained from table 2 is continued with its general rate of change in curvature until it becomes tangent to this line. Fig. 27 shows the curve so obtained, representing the velocity of free recoil of the gun as a function of time.

From Fig. 27 the time when the maximum velocity of free recoil is reached is .0606 secs. This time is designated by τ .

To obtain to a close degree of approximation the distance traveled by the gun in free recoil after the projectile has left the bore and until the powder gases have ceased to act, the velocities of free recoil corresponding to .016, .020, .028, .038, and .048 secs. are measured from the curve shown in Fig. 27. We also have .012116 and .0606 as the times when the projectile left the bore and when the powder gases ceased to act, respectively. The velocities corresponding to these times in sequence are 24.648, 27.103, 28.764, 30.715, 31.881, 32.442, and 32.64 f. s., respectively. Assuming that between any two of these times the velocity increases uniformly, the space passed over by the gun in free recoil is equal to the product of the difference in time by the mean velocity during the interval.

For the first interval

$$\frac{24.648 + 27.103}{2} \times (.016 - .012116) = .10050 \text{ ft.}$$

For the second interval

$$\frac{27.103 + 28.764}{2} \times (.020 - .016) = .11173 \text{ ft.}$$

and similarly the spaces passed over by the gun during each of the succeeding intervals are found to be .237196, .31298, .321615, and .41001 ft., respectively. The sum of these spaces, 1.49374 ft., is the distance traveled by the gun in free recoil after the projectile has left the muzzle and until the powder gases have ceased to act. Adding to this distance .18486 ft., the space over which the gun traveled in free recoil while the projectile was in the bore, we have 1.6786 ft. as the total distance traveled by the gun in free recoil up to the time when the powder gases ceased to act on it. This distance is designated by E .

When a planimeter is available the distances recoiled may be more accurately obtained by measuring the areas under the curve, Fig. 27. The use of data given by the curve of free recoil will appear later.

43. Length and Lettering of Parts. — Reference to Coordinate Axes. — The parts upon which the forces are to be determined are shown in Fig. 28.

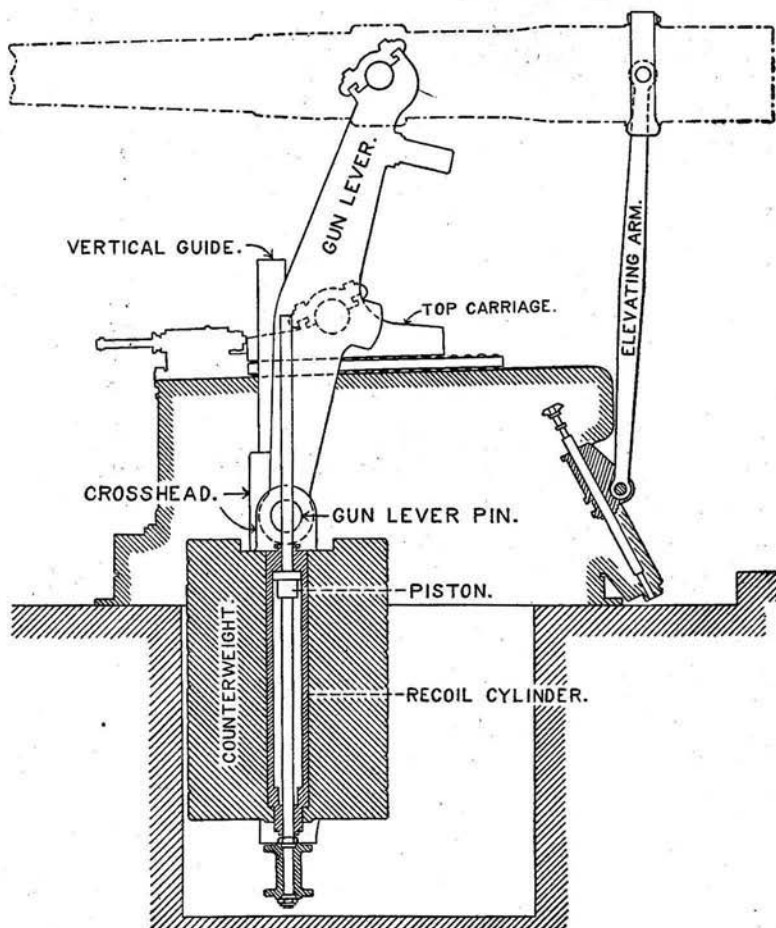


Fig. 28.

Fig. 29 is a diagram showing the lengths of the parts under consideration and their positions with respect to each other and to the axes of coordinates.

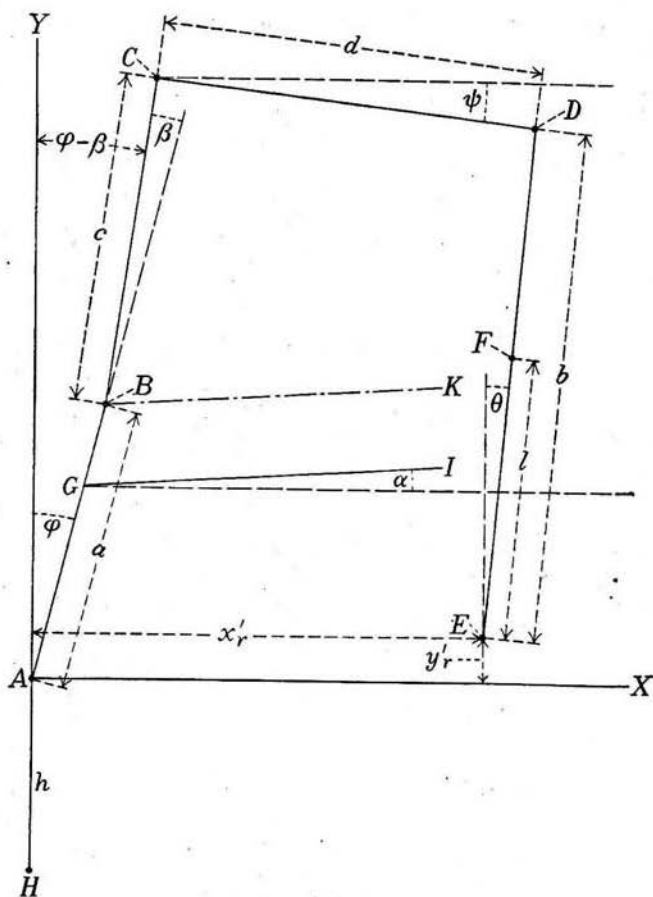


Fig. 29.

The origin of coordinates is assumed at A , the projection of the axis of the gun-lever pin when the gun is in battery. The forces are considered as co-planar and the axis of X will be taken as horizontal and the axis of Y as vertical. ABC is the median line of the gun lever. a is the length of the part between the axis of the gun-lever pin and the axis of the gun-lever axle, and ϕ is

the angle which this part makes with the vertical; c is the length of the part between the axes of the gun-lever axle and the gun trunnions, and $\phi - \beta$ is the angle which this part makes with the vertical. β is a constant angle. CD represents the axis of the gun, the points C and D representing the axes of the gun trunnions and elevating-band trunnions, respectively; d is the distance between these points; and ψ is the angle between the axis of the gun and the horizontal. At the instant of firing ψ is equal to the angle of elevation. ED represents the elevating arm pivoted to a fixed part (the elevating slide of the rear transom) of the carriage at E and to the trunnions of the elevating band at D ; and θ is the angle which the elevating arm makes with the vertical. b is the length of the elevating arm and l the length of the part from the pivot E to the center of gravity F of the arm. x' , and y' , are the coordinates of E . The gun-lever axle is carried in bearings at B in the top carriage, which during recoil slides along the roller path GI inclined at an angle α with the horizontal. The path of the axis of the gun-lever axle during recoil is indicated by the dot and dash line BK . H is the center of mass of the counterweight directly under the origin of coordinates, and h is its distance below that point.

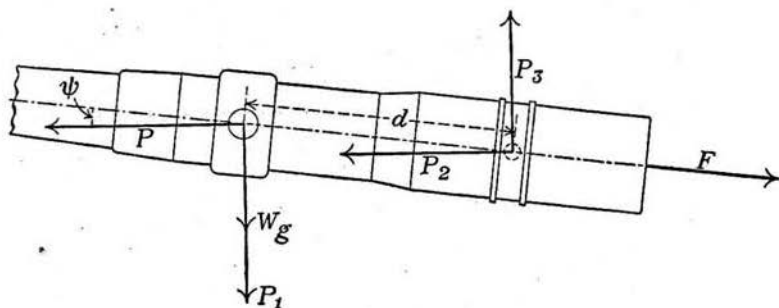


Fig. 30.

44. Forces on the Gun. — Fig. 30 represents the gun and the forces acting on it at the instant when the maximum powder pressure occurs. The velocity corresponding to the point of inflection in Fig. 27 is 10 f. s. which is therefore the velocity of free recoil at the instant of maximum powder pressure. From Table 2 the corresponding distance traveled by the gun in free

recoil is .01151 ft. = .1381 ins. The actual distance recoiled by the gun will be somewhat less than this, so that for all practical purposes it will be sufficiently accurate to consider the coordinates of the various points of the system and the dimensions of the angles at the instant when the maximum powder pressure occurs as having the same values as they had when the gun was about to be fired.

At the time of maximum powder pressure the gases will exert a force F on the gun in the direction of its axis. The weight of the gun W_g is a force acting upon it through its center of mass which is assumed to be in the axis of the gun trunnions. The gun levers and the elevating arm restrain the free movement of the gun, retarding it in the direction of the axis of X and giving it an acceleration in the direction of the axis of Y . The forces exerted on the gun by these parts must be normal to the surfaces in contact and must, therefore, intersect the axes of the gun trunnions and elevating-band trunnions, respectively, but as their direction is not known the force which the gun levers exert on the gun is resolved into horizontal and vertical components P and P_1 , respectively, and the force exerted on the gun by the elevating arm is resolved into a horizontal component P_2 and a vertical component P_3 .

Since the forces may be considered as co-planar the algebraic sum of their horizontal components must equal the product of the mass of the gun by its horizontal acceleration and similarly for the vertical components. The algebraic sum of the moments of the forces with respect to the center of mass must also equal the moment of inertia of the gun, taken with respect to the axis through its center of mass perpendicular to the plane of the forces, multiplied by the angular acceleration about that axis. We may then write the following equations:

$$F \cos \psi - P - P_2 = M_g d^2 x_g / dt^2 \quad (1)$$

$$-F \sin \psi - W_g - P_1 + P_3 = M_g d^2 y_g / dt^2 \quad (2)$$

$$P_2 d \sin \psi - P_3 d \cos \psi = \Sigma m r_g^2 \times d^2 \psi / dt^2 \quad (3)$$

In the above equations the subscript g indicates that the symbols under which it is placed refer to the gun. The subscript a will be similarly used for the gun levers and the subscripts c , e , and w for the top carriage, elevating arm, and counterweight, respectively.

45. Forces on the Gun Levers. — Fig. 31 represents the gun levers and the forces acting thereon.

The top circle represents the bearings in the gun levers for the gun trunnions to which the upper ends of the levers are attached,

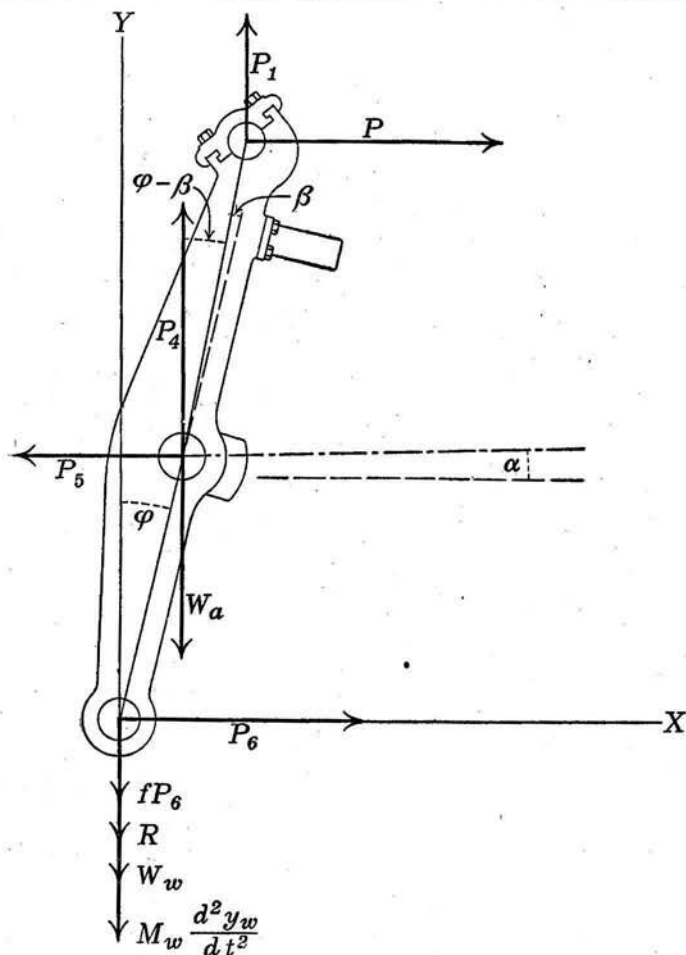


Fig. 31.

the middle circle the trunnions of the gun-lever axle carried in bearings in the top carriage, and the bottom circle the bearings for the gun-lever pins to which the lower ends of the levers are attached and to which are also fastened the counterweight and

the hydraulic recoil cylinder. The gun-lever pins, the lower ends of the gun levers, the counterweight, and the recoil cylinder are constrained by the construction of the carriage to move only in a vertical direction.

The forces P and P_1 which the gun levers exert on the gun are exerted in the opposite directions as shown in Fig. 31 by the gun on the gun levers. The top carriage resists the translation of the gun levers and the force exerted by it upon them must be normal to the surfaces in contact and must, therefore, intersect the axis of the gun-lever axle trunnions. As the direction of this force is not known it will be resolved into horizontal and vertical components P_5 and P_4 as shown in the figure. The weight of the gun levers W_a acts at the center of mass as shown. The lower ends of the levers tend to move to the front around the gun-lever axle under the action of the forces brought upon them by the discharge of the gun, but are prevented from doing so by the construction of the carriage which brings a force P_6 to bear upon them as shown. The force P_6 also produces a force of friction fP_6 between the vertical guides of the carriage and the cross-head, which is attached to the gun-lever pins and forms a part of the counterweight. This friction is due to the upward movement of the lower part of the levers, counterweight, etc. f is the coefficient of sliding friction. The force R , which is the constant resistance to recoil of the hydraulic cylinder, acts vertically on the lower ends of the gun levers through the cross-head and gun-lever pins as does the weight W_w of the counterweight. The counterweight must also be given an acceleration in the direction of the axis of Y during recoil and the vertical force on the lower ends of the gun levers necessary for this purpose is measured by the product of the mass of the counterweight by its acceleration. This force is represented by

$$M_w d^2 y_w / dt^2.$$

Placing the algebraic sum of the components of the various forces acting on the gun levers in the direction of each of the two axes equal, respectively, to the product of the mass of the gun levers and its acceleration in each of those directions, we have

$$P - P_5 + P_6 = M_a d^2 x_a / dt^2 \quad (4)$$

$$P_1 + P_4 - W_a - fP_6 - R - W_w - M_w d^2 y_w / dt^2 = M_a d^2 y_a / dt^2 \quad (5)$$

Placing the algebraic sum of the moments of the forces about the center of mass of the gun levers equal to the moment of inertia of the gun levers, taken with respect to an axis through their center of mass perpendicular to the plane of the forces, multiplied by the angular acceleration about that axis; and representing the coordinates of the centers of mass of the gun, gun levers, and gun-lever pins by $x_g, y_g; x_a, y_a$; and y_p , respectively, ($x_p = 0$) we have, since the center of mass of the gun levers is in the axis of their trunnions,

$$P(y_g - y_a) - P_1(x_g - x_a) - P_6(y_a - y_p) - [fP_6 + R + W_w + M_w d^2 y_w / dt^2] x_a = \Sigma m r_a^2 d^2 \phi / dt^2 \quad (6)$$

46. Forces on the Top Carriage. — Fig. 32 represents the top carriage and the forces acting thereon.

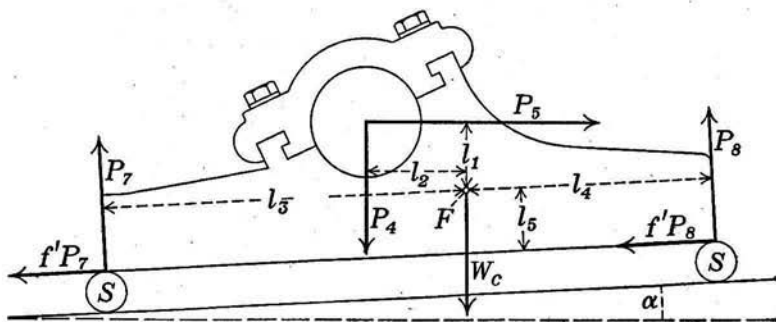


Fig. 32.

The horizontal and vertical forces P_5 and P_4 which the top carriage exerts on the gun levers are exerted in the opposite directions as shown in Fig. 32 by the gun levers on the top carriage. The top carriage rests on rollers S , and for the purpose of the determination of the forces it will be assumed that it bears on the front and rear rollers only. These rollers exert forces P_7 and P_8 on the top carriage in a direction normal to the surface of its roller path, that is, upward but making an angle α with the vertical. These forces when the top carriage moves to the rear give rise to the forces of friction $f'P_7$ and $f'P_8$ parallel to the surface of the roller path and passing through the points of contact of the top carriage with the rollers. f' is the coefficient of rolling friction. The weight W_c of the top carriage acts on it

through its center of mass F as shown. The lever arms of the forces P_5 , P_4 , P_7 , P_8 , $f'P_7$, and $f'P_8$ with respect to the center of mass are l_1 , l_2 , l_3 , l_4 , and l_5 , respectively, the lever arms of $f'P_7$ and $f'P_8$ being equal.

Placing the algebraic sum of the components of the forces in the direction of each of the two axes equal, respectively, to the product of the mass of the top carriage and its acceleration in

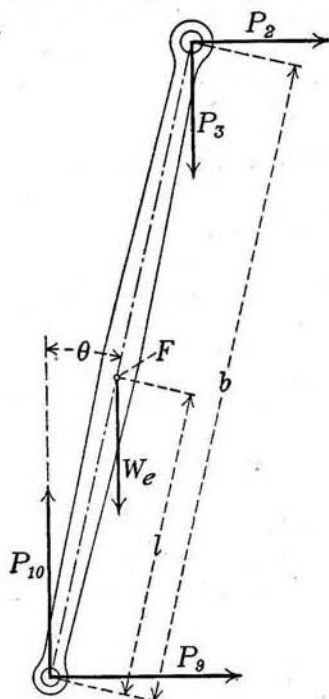


Fig. 33.

each of those directions, and the algebraic sum of the moments of the forces about the center of mass equal to zero, since there is no rotation of the top carriage, we have the following equations:

$$P_5 - P_7 \sin \alpha - P_8 \sin \alpha - f'P_7 \cos \alpha - f'P_8 \cos \alpha = M_c d^2 x_c / dt^2 \quad (7)$$

$$P_7 \cos \alpha + P_8 \cos \alpha - P_4 - f'P_7 \sin \alpha - f'P_8 \sin \alpha - W_c = M_c d^2 y_c / dt^2 \quad (8)$$

$$P_5 l_1 - P_4 l_2 + P_7 l_3 - P_8 l_4 + f'P_7 l_5 + f'P_8 l_5 = 0 \quad (9)$$

47. Forces on the Elevating Arm.—Fig. 33 represents the elevating arm and the forces acting thereon.

The horizontal and vertical forces P_2 and P_3 , which the elevating arm exerts on the gun, are exerted in the opposite directions as shown in the figure by the gun on the elevating arm. A pin through the lower end of the elevating arm rotates in bearings in the elevating slide which, when the gun is being elevated or depressed, slides in inclined guideways in the rear transom of the carriage. At other times the elevating slide is stationary. Motion of translation of the pin forming the lower part of the elevating arm under the action of the weight of the arm and the forces P_2 and P_3 is prevented by a force exerted on the pin by the elevating slide. This force is normal to the surfaces in contact and, therefore, intersects the axis of the pin, but as its direction is not known it must be resolved into unknown horizontal and vertical components P_9 and P_{10} as shown. The weight of the elevating arm acts through its center of mass F .

Placing the algebraic sum of the components of these forces in the direction of each of the two axes equal, respectively, to the product of the mass of the elevating arm and its acceleration in each of those directions; and the algebraic sum of the moments of the forces about the center of mass equal to the moment of inertia of the elevating arm, taken with respect to the axis through its center of mass perpendicular to the plane of the forces, multiplied by the angular acceleration about that axis, we have

$$P_2 + P_9 = M_e d^2 x_e / dt^2 \quad (10)$$

$$P_{10} - P_3 - W_e = M_e d^2 y_e / dt^2 \quad (11)$$

$$P_2 (b - l) \cos \theta + P_3 (b - l) \sin \theta - P_9 l \cos \theta + P_{10} l \sin \theta = \Sigma m r_e^2 d^2 \theta / dt^2 \quad (12)$$

48. Reduction of the Number of Unknown Quantities in Equations (1) to (12).— In these equations F , the force of the powder gases, and all fixed dimensions are known. For any assumed position of the gun in recoil the angles ϕ , $\phi - \beta$, ψ , and θ as well as the coordinates of the centers of mass and other important points, are known from the construction of the carriage; and further it is sufficiently accurate for all practical purposes to assume that their values when the maximum powder pressure occurs are the same as they were immediately before the gun was fired. The angles α and β are constant. The weights, masses, and moments of inertia of the various parts are also known from

the construction of the carriage. There are, therefore, in the twelve equations twenty-four unknown quantities of which twelve, including the constant resistance of the recoil cylinder, are forces and twelve are accelerations, linear and angular.

There are, however, certain definite relations, true at all times, between the centers of mass and other important points of the various parts of the carriage. These relations may be expressed in terms of the trigonometrical functions of the variable angles ϕ , $\phi - \beta$, ψ , and θ ; and from them by differentiation may be obtained similar relations between the various velocities, linear and angular; and by differentiating again, similar relations between the accelerations, linear and angular. Having obtained these relations between the accelerations, all of them may be expressed in terms of $d^2\phi/dt^2$, and the trigonometrical functions of the variable angles which are known for any assumed position of the gun in recoil, thus reducing the unknown quantities in equations (1) to (12), inclusive, to one angular acceleration and twelve forces. A further reduction of the number of unknown quantities to twelve, thereby permitting the solution of the equations and the determination of the values of the various forces, may be made by obtaining independently the value of R , the constant resistance of the recoil cylinder.

49. Method of Determining R .—After the powder gases have ceased to act on the gun, the work of the resistance R over the remainder of its path during the time the system is coming to rest plus the work done on the system against the force of gravity during this time is equal to the sum of the kinetic energies of translation and rotation possessed by the system at the beginning of the interval of time considered. Letting x'' and y'' with the proper subscripts represent the coordinates of the centers of mass of the various parts of the system in the recoiled position, and x and y with the proper subscripts the coordinates at any time during recoil, we may write the following equation:

$$\left. \begin{aligned} R(y''_w - y_w) + W_w(y''_w - y_w) + W_a(y''_a - y_a) + W_c(y''_c - y_c) \\ - W_g(y_g - y''_g) - W_e(y_e - y''_e) = (M_g/2)(dx_g/dt)^2 \\ + (M_g/2)(dy_g/dt)^2 + (\Sigma mr_g^2/2)(d\psi/dt)^2 + (M_a/2)(dx_a/dt)^2 + \\ (M_a/2)(dy_a/dt)^2 + (\Sigma mr_a^2/2)(d\phi/dt)^2 + (M_c/2)(dx_c/dt)^2 \\ + (M_c/2)(dy_c/dt)^2 + (M_e/2)(dx_e/dt)^2 + (M_e/2)(dy_e/dt)^2 \\ + (\Sigma mr_e^2/2)(d\theta/dt)^2 + (M_w/2)(dy_w/dt)^2 \end{aligned} \right\} (13)$$

In stating equation (13) the resistances of friction have been omitted as they depend largely on the pressures between the parts which are unknown. As a result the value of R deduced from equation (13), while sufficiently exact, will be somewhat larger than required to bring the system to rest at the desired point.

The variable coordinates, y_w , y_a , etc., in equation (13) may be expressed in terms of the trigonometrical functions of the variable angles, and the differentials may be expressed in terms of these functions and $d\phi/dt$. The variable angles may also be expressed in terms of ϕ . This having been done it is then only necessary, in order to solve equation (13) for R , to select some value for ϕ which occurs after the powder gases have ceased to act on the gun, and to determine the corresponding value of $d\phi/dt$. The data given by the curve of free recoil of the gun will enable the required values of ϕ and $d\phi/dt$ to be chosen with sufficient accuracy, and the value of ϕ which we shall select is that corresponding to the instant the powder gases cease to act on the gun.

50. Value of ϕ and of the Velocity of Restrained Recoil of the Gun at the Instant the Powder Gases Cease to Act on the Gun. — From Table 2 it is seen that the distance traveled by the gun in free recoil up to the time when the projectile leaves the bore is .18486 ft. and the corresponding velocity of free recoil is 24.648 f. s. The value of x_g when the gun is in battery is $x'_g = 1.9628$ ft. and, assuming

$$x_g - x'_g = .18486$$

whence

$$x_g = .1849 + 1.9628 = 2.1477,$$

the corresponding value of ϕ in free recoil from equation (14) below is approximately $14^\circ 9'.4$. Also the distance traveled by the gun in free recoil up to the time τ when the powder gases cease to act upon it is 1.6786 ft., the corresponding maximum velocity of free recoil is 32.64 f. s. and, assuming

$$x_g = 1.6786 + x'_g,$$

the corresponding value of ϕ in free recoil is approximately $23^\circ 37'.8$.

A number of foreign ordnance engineers of prominence assume, in solving problems of a nature similar to this, that the maximum

velocity of restrained recoil is instantly acquired and equal to the maximum velocity of free recoil; but the experience of the Ordnance Department, U. S. Army, has shown that a closer approximation than this, and one sufficiently exact for all practical purposes, is to assume that due to the restraint of the recoil brake the velocity of the gun in restrained recoil at the instant the powder gases cease to act on it is, for carriages of this type, about eight-tenths of the maximum velocity of free recoil, and that the value of the angle ϕ at the same instant is equal to its value when the gun is in battery plus about eight-tenths of its increase in value in free recoil up to the time τ .^{*} The value of ϕ when the gun is in battery being 13° , its value when the powder gases cease to act will be assumed as 21° . The assumed values of ϕ and v_r lie between those obtained above for free recoil at the instant the projectile leaves the bore and at the instant the powder gases cease to act on the gun, respectively.

51. Determination of the Values of the Coordinates of the Centers of Mass in Terms of the Trigonometrical Functions of the Angles ϕ , $\phi - \beta$, ψ , θ , and the Constant Angle α . — Letting x' and y' with the proper subscripts represent the coordinates of the centers of mass of the various parts when the gun is in battery, and x'_r and y'_r the coordinates of the axis of the pin connecting the lower end of the elevating arm to the elevating slide, we may write the following equations from the relations shown in Fig. 29:

$$x_g = a \sin \phi + c \sin (\phi - \beta) \quad (14)$$

$$x_a = a \sin \phi \quad (15)$$

$$x_c = a \sin \phi + x'_c - x'_a \quad (16)$$

$$x_e = x'_r + l \sin \theta \quad (17)$$

$$x_e = x_g + d \cos \psi - (b - l) \sin \theta = a \sin \phi + c \sin (\phi - \beta) + d \cos \psi - (b - l) \sin \theta \quad (18)$$

$$y_g = y'_a + (x_a - x'_a) \tan \alpha + c \cos (\phi - \beta) = y'_a + a \sin \phi \tan \alpha - x'_a \tan \alpha + c \cos (\phi - \beta) \quad (19)$$

^{*} A more rigorous method of determining the velocity of the gun in restrained recoil and the value of ϕ at the instant the powder gases cease to act on the gun is used by the gun-carriage division of the Ordnance Department. It is too extensive for discussion here, but the results obtained by it have shown that the approximate method outlined above is sufficiently accurate for all practical purposes.

$$y_a = y'_a + (x_a - x'_a) \tan \alpha = y'_a + a \sin \phi \tan \alpha - x'_a \tan \alpha \quad (20)$$

$$y_c = y'_a + a \sin \phi \tan \alpha - x'_a \tan \alpha - (y'_a - y'_c) \quad (21)$$

$$y_p = y'_a + a \sin \phi \tan \alpha - x'_a \tan \alpha - a \cos \phi \quad (21\frac{1}{2})$$

$$y_w = y'_a + a \sin \phi \tan \alpha - x'_a \tan \alpha - a \cos \phi - h \quad (22)$$

$$y_e = y'_r + l \cos \theta \quad (23)$$

$$y_e = y_o - d \sin \psi - (b - l) \cos \theta = y'_a + a \sin \phi \tan \alpha - x'_a \tan \alpha + c \cos (\phi - \beta) - d \sin \psi - (b - l) \cos \theta \quad (24)$$

52. Determination of the Values of ψ and θ when the Value of ϕ is Known or Assumed. — Equating the values of x_e from equations (17) and (18) we have

$$b \sin \theta = a \sin \phi + c \sin (\phi - \beta) + d \cos \psi - x'_r,$$

and representing $a \sin \phi + c \sin (\phi - \beta) - x'_r$ by Z ,

$$b \sin \theta = d \cos \psi + Z \quad (25)$$

Equating the values of y_e from equations (23) and (24) we have

$$b \cos \theta = y'_a + a \sin \phi \tan \alpha - x'_a \tan \alpha + c \cos (\phi - \beta) - d \sin \psi - y'_r,$$

and representing

$$y'_a + a \sin \phi \tan \alpha - x'_a \tan \alpha + c \cos (\phi - \beta) - y'_r \text{ by } Y, \\ b \cos \theta = -d \sin \psi + Y \quad (26)$$

Squaring equations (25) and (26) and adding we have

$$b^2 = d^2 + 2 dZ \cos \psi - 2 dY \sin \psi + Z^2 + Y^2$$

which may be put in the form

$$Z \cos \psi - Y \sin \psi = (b^2 - d^2 - Z^2 - Y^2) / 2d$$

Dividing by $\sqrt{Z^2 + Y^2}$

$$\frac{Z}{\sqrt{Z^2 + Y^2}} \cos \psi - \frac{Y}{\sqrt{Z^2 + Y^2}} \sin \psi = \frac{b^2 - d^2 - Z^2 - Y^2}{2d \sqrt{Z^2 + Y^2}} \quad (26\frac{1}{2})$$

Let

$$A = \tan^{-1} Z/Y$$

whence

$$\sin A = \frac{Z}{\sqrt{Z^2 + Y^2}}$$

and

$$\cos A = \frac{Y}{\sqrt{Z^2 + Y^2}}$$

Replacing in equation (26½)

$$\frac{Z}{\sqrt{Z^2 + Y^2}} \text{ by } \sin A \text{ and } \frac{Y}{\sqrt{Z^2 + Y^2}} \text{ by } \cos A$$

we have

$$\sin A \cos \psi - \cos A \sin \psi = \sin (A - \psi) = \frac{b^2 - d^2 - Z^2 - Y^2}{2d\sqrt{Z^2 + Y^2}} \quad (27)$$

in which $A = \tan^{-1} Z/Y$.

For any value of ϕ the corresponding value of ψ can be obtained from equation (27), and having ϕ and ψ the corresponding value of θ can be obtained from equation (26).

53. Determination of the Velocities in Terms of the Trigonometrical Functions of the Angles, and of $d\phi/dt$.—Differentiating both sides of equations (14) to (24), inclusive, excepting equation (21½), and dividing by dt we have

$$dx_o/dt = a \cos \phi (d\phi/dt) + c \cos (\phi - \beta) (d\phi/dt) \quad (28)$$

$$dx_a/dt = a \cos \phi (d\phi/dt) \quad (29)$$

$$dx_c/dt = a \cos \phi (d\phi/dt) \quad (30)$$

$$dx_e/dt = l \cos \theta (d\theta/dt) \quad (31)$$

$$dx_e/dt = a \cos \phi (d\phi/dt) + c \cos (\phi - \beta) (d\phi/dt) - d \sin \psi (d\psi/dt) - (b - l) \cos \theta (d\theta/dt) \quad (32)$$

$$dy_o/dt = a \tan \alpha \cos \phi (d\phi/dt) - c \sin (\phi - \beta) (d\phi/dt) \quad (33)$$

$$dy_a/dt = a \tan \alpha \cos \phi (d\phi/dt) \quad (34)$$

$$dy_c/dt = a \tan \alpha \cos \phi (d\phi/dt) \quad (35)$$

$$dy_w/dt = a \tan \alpha \cos \phi (d\phi/dt) + a \sin \phi (d\phi/dt) \quad (36)$$

$$dy_e/dt = -l \sin \theta (d\theta/dt) \quad (37)$$

$$dy_e/dt = a \tan \alpha \cos \phi (d\phi/dt) - c \sin (\phi - \beta) (d\phi/dt) - d \cos \psi (d\psi/dt) + (b - l) \sin \theta (d\theta/dt) \quad (38)$$

Equating the values of dx_e/dt given by equations (31) and (32) we have

$$b \cos \theta (d\theta/dt) = a \cos \phi (d\phi/dt) + c \cos (\phi - \beta) (d\phi/dt) - d \sin \psi (d\psi/dt) \quad (39)$$

and equating the values of dy_e/dt given by equations (37) and (38) we have

$$-b \sin \theta (d\theta/dt) = a \tan \alpha \cos \phi (d\phi/dt) - c \sin (\phi - \beta) (d\phi/dt) - d \cos \psi (d\psi/dt) \quad (40)$$

Multiplying equation (39) by $\sin \theta$ and equation (40) by $\cos \theta$ and adding we have

$$\begin{aligned} & \sin \theta a \cos \phi (d\phi/dt) + \sin \theta c \cos (\phi - \beta) (d\phi/dt) \\ & + \cos \theta a \tan \alpha \cos \phi (d\phi/dt) - \cos \theta c \sin (\phi - \beta) (d\phi/dt) \\ & = \sin \theta d \sin \psi (d\psi/dt) + \cos \theta d \cos \psi (d\psi/dt) \end{aligned}$$

or

$$\frac{d\psi}{dt} = \left\{ \frac{\sin \theta \{a \cos \phi + c \cos (\phi - \beta)\}}{d (\sin \theta \sin \psi + \cos \theta \cos \psi)} + \frac{\cos \theta \{a \tan \alpha \cos \phi - c \sin (\phi - \beta)\}}{d (\sin \theta \sin \psi + \cos \theta \cos \psi)} \right\} \frac{d\phi}{dt} \quad (41)$$

Dividing equation (39) by $b \cos \theta$ we have

$$\frac{d\theta}{dt} = \frac{a \cos \phi + c \cos (\phi - \beta)}{b \cos \theta} \frac{d\phi}{dt} - \frac{d \sin \psi}{b \cos \theta} \frac{d\psi}{dt}$$

or replacing $d\psi/dt$ by its value in terms of $d\phi/dt$ from equation (41) and representing the coefficient of $d\phi/dt$ in that equation by X

$$\frac{d\theta}{dt} = \left\{ \frac{a \cos \phi + c \cos (\phi - \beta)}{b \cos \theta} - \frac{d \sin \psi}{b \cos \theta} X \right\} \frac{d\phi}{dt} \quad (42)$$

For any value of $d\phi/dt$ the corresponding values of $d\psi/dt$ and $d\theta/dt$ can be obtained from equations (41) and (42), respectively, when the angle ϕ is known. The values of all the linear velocities can then be obtained from equations (28) to (37), inclusive.

54. Determination of the Accelerations in Terms of the Trigonometrical Functions of the Angles, and of $d^2\phi/dt^2$. — Differentiating both sides of equations (28) to (38), inclusive, and dividing by dt we obtain

$$d^2x/dt^2 = \{a \cos \phi + c \cos (\phi - \beta)\} (d^2\phi/dt^2) - \{a \sin \phi + c \sin (\phi - \beta)\} (d\phi/dt)^2 \quad (43)$$

$$d^2x_a/dt^2 = a \cos \phi (d^2\phi/dt^2) - a \sin \phi (d\phi/dt)^2 \quad (44)$$

$$d^2x_c/dt^2 = a \cos \phi (d^2\phi/dt^2) - a \sin \phi (d\phi/dt)^2 \quad (45)$$

$$d^2x_e/dt^2 = l \cos \theta (d^2\theta/dt^2) - l \sin \theta (d\theta/dt)^2 \quad (46)$$

$$\left. \begin{aligned} d^2x_e/dt^2 &= \{a \cos \phi + c \cos (\phi - \beta)\} (d^2\phi/dt^2) \\ &\quad - d \sin \psi (d^2\psi/dt^2) - (b - l) \cos \theta (d^2\theta/dt^2)^2 \\ &\quad - \{a \sin \phi + c \sin (\phi - \beta)\} (d\phi/dt)^2 \\ &\quad - d \cos \psi (d\psi/dt)^2 + (b - l) \sin \theta (d\theta/dt)^2 \end{aligned} \right\} \quad (47)$$

$$d^2y_e/dt^2 = \{a \tan \alpha \cos \phi - c \sin (\phi - \beta)\} (d^2\phi/dt^2) - \{a \tan \alpha \sin \phi + c \cos (\phi - \beta)\} (d\phi/dt)^2 \quad (48)$$

$$d^2y_a/dt^2 = a \tan \alpha \cos \phi (d^2\phi/dt^2) - a \tan \alpha \sin \phi (d\phi/dt)^2 \quad (49)$$

$$d^2y_c/dt^2 = a \tan \alpha \cos \phi (d^2\phi/dt^2) - a \tan \alpha \sin \phi (d\phi/dt)^2 \quad (50)$$

$$d^2y_w/dt^2 = \{a \tan \alpha \cos \phi + a \sin \phi\} (d^2\phi/dt^2) - \{a \tan \alpha \sin \phi - a \cos \phi\} (d\phi/dt)^2 \quad (51)$$

$$d^2y_s/dt^2 = -l \sin \theta (d^2\theta/dt^2) - l \cos \theta (d\theta/dt)^2 \quad (52)$$

$$\left. \begin{aligned} d^2y_e/dt^2 &= \{a \tan \alpha \cos \phi - c \sin (\phi - \beta)\} (d^2\phi/dt^2) \\ &\quad - d \cos \psi (d^2\psi/dt^2) + (b - l) \sin \theta (d^2\theta/dt^2) \\ &\quad - \{a \tan \alpha \sin \phi + c \cos (\phi - \beta)\} (d\phi/dt)^2 \\ &\quad + d \sin \psi (d\psi/dt)^2 + (b - l) \cos \theta (d\theta/dt)^2 \end{aligned} \right\} \quad (53)$$

Equating the values of d^2x_e/dt^2 from equations (46) and (47) we have

$$\left. \begin{aligned} &b \cos \theta (d^2\theta/dt^2) - b \sin \theta (d\theta/dt)^2 \\ &= \{a \cos \phi + c \cos (\phi - \beta)\} (d^2\phi/dt^2) - d \sin \psi (d^2\psi/dt^2) \\ &\quad - \{a \sin \phi + c \sin (\phi - \beta)\} (d\phi/dt)^2 - d \cos \psi (d\psi/dt)^2 \end{aligned} \right\} \quad (54)$$

and equating the values of d^2y_e/dt^2 from equations (52) and (53)

$$\left. \begin{aligned} &-b \sin \theta (d^2\theta/dt^2) - b \cos \theta (d\theta/dt)^2 = \\ &\{a \tan \alpha \cos \phi - c \sin (\phi - \beta)\} (d^2\phi/dt^2) - d \cos \psi (d^2\psi/dt^2) \\ &- \{a \tan \alpha \sin \phi + c \cos (\phi - \beta)\} (d\phi/dt)^2 + d \sin \psi (d\psi/dt)^2 \end{aligned} \right\} \quad (55)$$

Multiplying equation (54) by $\sin \theta$ and equation (55) by $\cos \theta$ and adding we have

$$\left. \begin{aligned} -b(d\theta/dt)^2 &= [\sin \theta \{a \cos \phi + c \cos (\phi - \beta)\} \\ &\quad + \cos \theta \{a \tan \alpha \cos \phi - c \sin (\phi - \beta)\}] (d^2\phi/dt^2) \\ &\quad - [d \sin \theta \sin \psi + d \cos \theta \cos \psi] (d^2\psi/dt^2) \\ &\quad - [\sin \theta \{a \sin \phi + c \sin (\phi - \beta)\} + \cos \theta \{a \tan \alpha \sin \phi \\ &\quad + c \cos (\phi - \beta)\}] (d\phi/dt)^2 - [d \sin \theta \cos \psi \\ &\quad - d \cos \theta \sin \psi] (d\psi/dt)^2 \end{aligned} \right\}$$

or

$$\frac{d^2\psi}{dt^2} = \left\{ \begin{aligned} & \frac{\sin \theta \{a \cos \phi + c \cos (\phi - \beta)\}}{d \sin \theta \sin \psi + d \cos \theta \cos \psi} \frac{d^2\phi}{dt^2} \\ & + \frac{\cos \theta \{a \tan \alpha \cos \phi - c \sin (\phi - \beta)\}}{d \sin \theta \sin \psi + d \cos \theta \cos \psi} \frac{d^2\phi}{dt^2} \\ & - \frac{\sin \theta \{a \sin \phi + c \sin (\phi - \beta)\}}{d \sin \theta \sin \psi + d \cos \theta \cos \psi} \left(\frac{d\phi}{dt}\right)^2 \\ & - \frac{\cos \theta \{a \tan \alpha \sin \phi + c \cos (\phi - \beta)\}}{d \sin \theta \sin \psi + d \cos \theta \cos \psi} \left(\frac{d\phi}{dt}\right)^2 \\ & - \frac{\sin \theta \cos \psi - \cos \theta \sin \psi}{\sin \theta \sin \psi + \cos \theta \cos \psi} \left(\frac{d\psi}{dt}\right)^2 \\ & + \frac{b}{d \sin \theta \sin \psi + d \cos \theta \cos \psi} \left(\frac{d\theta}{dt}\right)^2 \end{aligned} \right\} \quad (56)^*$$

Dividing equation (54) by $b \cos \theta$ and solving for $d^2\theta/dt^2$ we have

$$\frac{d^2\theta}{dt^2} = \left\{ \begin{aligned} & \frac{a \cos \phi + c \cos (\phi - \beta)}{b \cos \theta} \frac{d^2\phi}{dt^2} \\ & - \frac{d \sin \psi}{b \cos \theta} \frac{d^2\psi}{dt^2} \\ & - \frac{a \sin \phi + c \sin (\phi - \beta)}{b \cos \theta} \left(\frac{d\phi}{dt}\right)^2 \\ & - \frac{d \cos \psi}{b \cos \theta} \left(\frac{d\psi}{dt}\right)^2 + \tan \theta \left(\frac{d\theta}{dt}\right)^2 \end{aligned} \right\}$$

or representing the coefficients of

$$d^2\phi/dt^2, \quad (d\phi/dt)^2, \quad (d\psi/dt)^2, \quad \text{and} \quad (d\theta/dt)^2$$

in equation (56) by $X, X', X'',$ and X''' , respectively, we have

$$\frac{d^2\theta}{dt^2} = \left\{ \begin{aligned} & \frac{a \cos \phi + c \cos (\phi - \beta) - d \sin \psi X}{b \cos \theta} \frac{d^2\phi}{dt^2} \\ & - \frac{a \sin \phi + c \sin (\phi - \beta) - d \sin \psi X'}{b \cos \theta} \left(\frac{d\phi}{dt}\right)^2 \\ & - \frac{d (\cos \psi - \sin \psi X'')}{b \cos \theta} \left(\frac{d\psi}{dt}\right)^2 \\ & - \frac{d \sin \psi X''' - b \sin \theta}{b \cos \theta} \left(\frac{d\theta}{dt}\right)^2 \end{aligned} \right\} \quad (57)$$

* When this equation is used to determine the value of $d^2\psi/dt^2$ the numerical work can be simplified somewhat by noting that $\sin \theta \sin \psi + \cos \theta \cos \psi = \cos (\theta - \psi)$ and $\sin \theta \cos \psi - \cos \theta \sin \psi = \sin (\theta - \psi)$.

For any value of $d^2\phi/dt^2$ the corresponding values of $d^2\psi/dt^2$ and $d^2\theta/dt^2$ can be obtained from equations (56) and (57), respectively, when the angle ϕ and the angular velocities are known. The values of the linear accelerations can then be obtained from equations (43) to (53), inclusive.

55. Determination of the Value of R from Equation (13) (Gun fired at 0° Elevation).—The following data are known from the construction of the gun and carriage:

TABLE 3.

$\alpha = 1^\circ 20'$	$y'_r = 0.5563$ ft.	$W_g = 12764$ lbs.
$\phi' = 13^\circ$	$y'_g = 9.2942$ "	$M_g = 396.88$
$\phi'' = 88^\circ$	$y''_g = 4.6612$ "	$W_a = 6040.8$ lbs.
$\beta = 2^\circ$	$y'_a = 4.2224$ "	$M_a = 187.84$
$(\phi' - \beta) = 11^\circ$	$y''_a = 4.3008$ "	$W_c = 2528$ lbs.
$(\phi'' - \beta) = 86^\circ$	$y'_c = 3.7571$ "	$M_c = 78.61$
$\theta' = 3^\circ 58'$	$y''_c = 3.8355$ "	$W_e = 707$ lbs.
$\psi' =$ angle of elevation	$y'_e = 4.6548$ "	$M_e = 21.984$
$\psi'' = 5^\circ$	$y''_e = 2.2571$ "	$W_w = 19797$ lbs.
$x'_g = 1.9628$ ft.	$y'_p = 0.0$ "	$M_w = 615.58$
$x''_g = 9.4857$ "	$y'_w = -3.0$ "	$\Sigma mr^2_g = 14967$
$x'_a = 0.975$ "	$y''_w = 1.1492$ "	$\Sigma mr^2_a = 1250.4$
$x''_a = 4.3317$ "	$a = 4.3333$ "	$\Sigma mr^2_e = 155.08$
$x'_c = 1.6401$ "	$b = 8.75$ "	$l_1 = 0.4653$ ft.
$x''_c = 4.9968$ "	$c = 5.1667$ "	$l_2 = 0.6651$ "
$x'_e = 7.2145$ "	$d = 5.5$ "	$l_3 = 2.4875$ "
$x''_e = 10.579$ "	$l = 4.1042$ "	$l_4 = 1.6792$ "
$x'_r = 7.0002$ "	$h = 3.0$ "	$l_5 = 0.4255$ "

In the computation of R it is customary to assume that the gun is fired at zero degrees elevation, whence $\psi = 0$, and this assumption will be made here. No difficulty arises, however, in computing R for any other given elevation of the gun when fired, the method to be pursued in that case being exactly similar to that which follows.

For the reasons given in article 50, page 78, the value of ϕ at the instant the powder gases cease to act on the gun will be taken as 21° and the corresponding velocity of the gun in restrained recoil as

$$\sqrt{(dx_g/dt)^2 + (dy_g/dt)^2} = .8 V_f = 26.112 \text{ f. s.},$$

V_f being the maximum velocity of free recoil equal to 32.64 f. s.

With $\phi = 21^\circ$ the values of ψ and θ , computed from equations (27) and (26), respectively, are

$$\psi = -7'.3 \quad \theta = 11^\circ 26'.3$$

With $\phi = 21^\circ$ and $\theta = 11^\circ 26'.3$ the values of y_w , y_a , y_c , y_g , and y_s , computed from equations (22), (20), (21), (19), and (23), respectively, are

$$\begin{aligned} y_w &= -2.8078 \text{ ft.} & y_g &= 9.1226 \text{ ft.} \\ y_a &= 4.2374 \text{ ft.} & y_s &= 4.5790 \text{ ft.} \\ y_c &= 3.7721 \text{ ft.} \end{aligned}$$

With $\sqrt{(dx_g/dt)^2 + (dy_g/dt)^2} = 26.112 \text{ f. s.}$ and $\phi = 21^\circ$, the value of $d\phi/dt$, computed from equations (28) and (33), is

$$d\phi/dt = 2.8788 \text{ radians per sec.}$$

With $d\phi/dt = 2.8788 \text{ radians per sec.}$, $\phi = 21^\circ$, $\psi = -7'.3$, and $\theta = 11^\circ 26'.3$, the values of $d\psi/dt$, $d\theta/dt$, dx_g/dt , dy_g/dt , dx_a/dt , dy_a/dt , dx_c/dt , dy_c/dt , dx_e/dt , dy_e/dt , and dy_w/dt , computed from equations (41), (42), (28), (33), (29), (34), (30), (35), (31), (37), and (36), respectively, are as follows:

$$\begin{aligned} d\psi/dt &= .1147 \text{ radians per sec.} & dx_c/dt &= 11.645 \text{ f. s.} \\ d\theta/dt &= 2.9978 \text{ radians per sec.} & dy_c/dt &= 0.2708 \text{ f. s.} \\ dx_g/dt &= 25.709 \text{ f. s.} & dx_e/dt &= 12.059 \text{ f. s.} \\ dy_g/dt &= -4.5715 \text{ f. s.} & dy_e/dt &= -2.4401 \text{ f. s.} \\ dx_a/dt &= 11.645 \text{ f. s.} & dy_w/dt &= 4.741 \text{ f. s.} \\ dy_a/dt &= 0.2708 \text{ f. s.} \end{aligned}$$

Substituting in equation (13) the values of the coordinates and the velocities just determined, together with the values of the coordinates when the gun is from battery and the values of the masses and moments of inertia from Table 3, and solving for R , we obtain

$$R = 37312 \text{ lbs.}$$

56. Determination of the Values of the Forces from Equations (1) to (12), Inclusive. Gun Fired at 0° Elevation. — The first step in the solution of equations (1) to (12), inclusive, is to determine the values of all the accelerations in terms of $d^2\phi/dt^2$. To do this it is again necessary to obtain the values of $d\phi/dt$,

$d\psi/dt$, and $d\theta/dt$ this time, however, corresponding to the position of the parts at the instant of maximum pressure of the powder gases. As stated in article 44, page 71, it will be sufficient to consider that the values of the coordinates and the angles are at this instant the same as just before the gun was fired. The value of $\sqrt{(dx_o/dt)^2 + (dy_o/dt)^2}$ will again be taken as eight-tenths of the velocity of free recoil which at this time, as previously determined, is 10 f. s.

With $\sqrt{(dx_o/dt)^2 + (dy_o/dt)^2} = .8 \times 10 = 8 \text{ f. s.}$

the values of $d\phi/dt$, $d\psi/dt$, and $d\theta/dt$ computed from equations (28), (33), (41), and (42), respectively, are

$$d\phi/dt = .8569 \text{ radians per sec.}$$

$$d\psi/dt = -.04256 \text{ radians per sec.}$$

$$d\theta/dt = .9123 \text{ radians per sec.}$$

With these values of $d\phi/dt$, $d\psi/dt$, and $d\theta/dt$ and the known values of the angles, the values of all the accelerations in terms of $d^2\phi/dt^2$ may be obtained as follows:

$$\text{From equation (43) } d^2x_o/dt^2 = 9.2937 (d^2\phi/dt^2) - 1.44 \quad (43')$$

$$\text{From equation (44) } d^2x_a/dt^2 = 4.2219 (d^2\phi/dt^2) - .716 \quad (44')$$

$$\text{From equation (45) } d^2x_c/dt^2 = 4.2219 (d^2\phi/dt^2) - .716 \quad (45')$$

$$\text{From equation (48) } d^2y_o/dt^2 = -.887585 (d^2\phi/dt^2) - 3.741 \quad (48')$$

$$\text{From equation (49) } d^2y_a/dt^2 = .098265 (d^2\phi/dt^2) - .0167 \quad (49')$$

$$\text{From equation (50) } d^2y_c/dt^2 = .098265 (d^2\phi/dt^2) - .0167 \quad (50')$$

$$\text{From equation (51) } d^2y_w/dt^2 = 1.07299 (d^2\phi/dt^2) + 3.083 \quad (51')$$

$$\text{From equation (56) } d^2\psi/dt^2 = -.04421 (d^2\phi/dt^2) + .62915 \quad (56')$$

$$\text{From equation (57) } d^2\theta/dt^2 = 1.0647 (d^2\phi/dt^2) - .1083 \quad (57')$$

$$\text{From equation (46) } d^2x_e/dt^2 = 4.3592 (d^2\phi/dt^2) - .2363 \quad (46')$$

$$\text{From equation (52) } d^2y_e/dt^2 = -.30228 (d^2\phi/dt^2) - 3.408 \quad (52')$$

By substituting these expressions for the accelerations and the value of $R = 37312 \text{ lbs.}$ in equations (1) to (12), inclusive, the number of unknown quantities therein will be reduced to twelve and the equations can be readily solved.

Rewriting the equations with these substitutions and replacing the symbols of the constants therein by their values from

Table 3 we have, since the value of ψ in the case under consideration is 0 and the value of F corresponding to a powder pressure of 36000 lbs. per sq. in. is

$$36000 \times \pi (6)^2 / 4 = 1017860 \text{ lbs.},$$

$$1017860 - P - P_2 = 396.88 [9.2937 (d^2\phi/dt^2) - 1.44] \quad (1')$$

$$-12764 - P_1 + P_3 = 396.88 [-.887585 (d^2\phi/dt^2) - 3.741] \quad (2')$$

$$-5.5 P_3 = 14967 [-.04421 (d^2\phi/dt^2) + .62915] \quad (3')$$

$$P - P_5 + P_6 = 187.84 [4.2219 (d^2\phi/dt^2) - .716] \quad (4')$$

$$\left. \begin{aligned} P_1 + P_4 - 6040.8 - .15 P_6 - 37312 - 19797 \\ - 615.58 [1.07299 (d^2\phi/dt^2) + 3.083] \\ = 187.84 [.098265 (d^2\phi/dt^2) - .0167] \end{aligned} \right\} \quad (5')$$

$$\left. \begin{aligned} 5.0718 P - .9878 P_1 - 4.2224 P_6 - .975 [.15 P_6 \\ + 37312 + 19797 + 615.58 \{1.07299 (d^2\phi/dt^2) \\ + 3.083\}] = 1250.4 (d^2\phi/dt^2) \end{aligned} \right\} \quad (6')$$

$$\left. \begin{aligned} P_5 = .02327 P_7 - .02327 P_8 - .005 \times .99972 P_7 \\ - .005 \times .99972 P_8 = 78.61 [4.2219 (d^2\phi/dt^2) - .716] \end{aligned} \right\} \quad (7')$$

$$\left. \begin{aligned} .99972 P_7 + .99972 P_8 - P_4 - .005 \times .0237 P_7 \\ - .005 \times .0237 P_8 - 2528 \\ = 78.61 [.098265 (d^2\phi/dt^2) - .0167] \end{aligned} \right\} \quad (8')$$

$$.4653 P_5 - .6651 P_4 + 2.4875 P_7 - 1.6792 P_8 + .4255 \\ \times .005 P_7 + .4255 \times .005 P_8 = 0 \quad (9')$$

$$P_2 + P_9 = 21.984 [4.3592 (d^2\phi/dt^2) - .2363] \quad (10')$$

$$P_{10} - P_3 - 707 = 21.984 [-.30228 (d^2\phi/dt^2) - 3.408] \quad (11')$$

$$4.6347 P_2 + .3214 P_3 - 4.0944 P_9 + .2839 P_{10} \\ = 155.08 [1.0647 (d^2\phi/dt^2) - .1083] \quad (12')$$

SOLUTION OF EQUATIONS (1') TO (12'), INCLUSIVE.

$$\text{From equation (3')} \quad P_3 = 120.3 (d^2\phi/dt^2) - 1712 \quad (3'')$$

Substituting this value of P_3 in equation (2')

$$P_1 = 472.57 (d^2\phi/dt^2) - 12991.3 \quad (2'')$$

Substituting the value of P_3 in equation (11')

$$P_{10} = 113.66 (d^2\phi/dt^2) - 1079.92 \quad (11'')$$

Substituting in equation (12') the values of P_3 from equation (3''), P_{10} from equation (11''), and P_9 from equation (10'),

$$P_2 = 55.739 (d^2\phi/dt^2) + 93.80 \quad (12'')$$

$$\text{From equation (1')} \quad P = 1018337.7 - 3744.24 (d^2\phi/dt^2) \quad (1''')$$

$$\text{From equation (6')} \quad P_6 = 1171851 - 4888.27 (d^2\phi/dt^2) \quad (6''')$$

$$\text{From equation (4')} \quad P_5 = 2190323.19 - 9425.41 (d^2\phi/dt^2) \quad (4''')$$

$$\text{From equation (5')} \quad P_4 = 253813.38 - 26.828 (d^2\phi/dt^2) \quad (5''')$$

In equation (7') the coefficient of P_7 and P_8 when the two terms containing P_7 and the two containing P_8 are consolidated is .02827, and in equation (8') when these terms are consolidated the coefficient is .9996. Therefore, by multiplying equation (7') by $\frac{.9996}{.02827}$ and adding the result to equation (8'), P_7 and P_8 are eliminated and

$$35.359 P_5 - P_4 - 2528 = 35.359 \times 78.61 [4.2219 (d^2\phi/dt^2) - .716] + 78.61 [.098265 (d^2\phi/dt^2) - .0167] \quad (7''')$$

Substituting for P_5 and P_4 in equation (7''') their values from equations (4''') and (5''')

$$344493 (d^2\phi/dt^2) = 77193650 \quad (7''''')$$

$$\text{Whence} \quad d^2\phi/dt^2 = 224.08 \text{ radians per sec. per sec.} \quad (7''''')$$

With the value of $d^2\phi/dt^2 = 224.08$ we obtain

$$\text{From equation (1''')} \quad P = 179338 \text{ lbs.} \quad (1''''')$$

$$\text{From equation (2''')} \quad P_1 = 92902 \text{ lbs.} \quad (2''''')$$

$$\text{From equation (12''')} \quad P_2 = 12584 \text{ lbs.} \quad (12''''')$$

$$\text{From equation (3''')} \quad P_3 = 25245 \text{ lbs.} \quad (3''''')$$

$$\text{From equation (5''')} \quad P_4 = 135748 \text{ lbs.} \quad (5''''')$$

$$\text{From equation (4''')} \quad P_5 = 78223 \text{ lbs.} \quad (4''''')$$

$$\text{From equation (6''')} \quad P_6 = 76476 \text{ lbs.} \quad (6''''')$$

$$fP_6 = .15 P_6 = 11471 \text{ lbs.} \quad (6''''')$$

From equation (51') and the value of $M_w = 615.58$ from Table 3,

$$M_w (d^2y_w/dt^2) = 149907 \text{ lbs.} \quad (51''')$$

The total vertical force acting on the lower ends of the gun levers through the gun-lever pins is

$$fP_6 + R + W_w + M_w d^2 y_w / dt^2 = 11471 + 37312 + 19797 \\ + 149907 = 218487 \text{ lbs.} \quad (51''')$$

To obtain the values of P_7 and P_8 replace P_4 and P_5 in equation (9') by their values from equations (5''') and (4'''), respectively, and solve for P_8 in terms of P_7 . Then substitute this value of P_8 and the numerical values of P_4 and $d^2 \phi / dt^2$ in equation (8') and solve for P_7 . The value of P_8 then follows from that of P_7 .

$$\text{From equation (9')} \quad P_8 = 1.4845 P_7 - 32127 \quad (9'')$$

$$\text{From equation (8')} \quad P_7 = 69304 \text{ lbs.} \quad (8'')$$

$$\text{From equation (9'')} \quad P_8 = 70755 \text{ lbs.} \quad (9''')$$

From equation (10') after substituting therein the values of P_2 and $d^2 \phi / dt^2$

$$P_9 = 8885 \text{ lbs.} \quad (10'')$$

From equation (11'') after substituting therein the value of $d^2 \phi / dt^2$

$$P_{10} = 24386 \text{ lbs.} \quad (11''')$$

57. Computation of the Values of the Forces when the Gun is Fired at Any Angle of Elevation. — When Fired at Extreme Angles of Elevation and Depression. — The determination of the forces acting on the carriage has been made under the supposition that the gun was fired at zero degrees elevation. The method used is, however, equally applicable to a case with any other angle of elevation. In the latter case the force of the powder gases will have a vertical as well as a horizontal component, but the value of each component will be known and no additional unknown quantities will be introduced into the equations. x'_r and y'_r , the coordinates of the axis of the pin at the lower end of the elevating arm, will vary with the elevation given to the gun, but their values corresponding to any given elevation can be readily obtained from the drawings of the carriage.

In order that the maximum forces on each part of the gun carriage may be determined for use in so proportioning the various parts that the stresses in them will not be dangerously great, the forces should be computed not only for an elevation

of the gun of zero degrees but also for the extreme angles of elevation and depression for which the carriage is designed. For the same reason it is often necessary, particularly in the case of barbette and mobile artillery carriages, to compute the forces corresponding to different positions of the gun in recoil, generally its positions in and from battery.

58. Maximum Values of the Forces on a Disappearing Carriage during Recoil. — Velocity and Acceleration Curves of the Recoiling Parts. — In the case of a disappearing carriage of the service type the peculiar constraint of the moving parts during recoil is such that the maximum accelerations of some of the parts occur somewhat later than the instant of maximum powder pressure. However, by plotting the velocity curves of the various parts from the relations established in equations (14) to (42), inclusive, it is seen that the maximum accelerations of all parts occur at times sufficiently near that of the maximum powder pressure to warrant the assumption that the values of the forces computed for that instant vary but little from their maximum values.

In plotting the velocity curves referred to, the velocity of recoil of the top carriage is first measured by a Sebert velocimeter and may then be plotted both as a function of time and of space. For any measured velocity the corresponding coordinates x_c and y_c of the center of mass of the top carriage are known and from them may be obtained by either of equations (16) or (21) the corresponding value of ϕ . The corresponding values of the coordinates of the centers of mass of the other moving parts, and the values of the angles ψ and θ , can then be obtained from equations (14) to (27), inclusive. The corresponding velocities, linear and angular, of the other moving parts can now be determined from the measured velocity of the top carriage and equations (28) to (42), inclusive. As the velocity of the top carriage is measured both as a function of time and of space, the velocity curves of the other moving parts may also be plotted as functions either of time or of space. The accelerations are obtained by measuring the angles which the tangents to the velocity curves as a function of time make with the axis of time. The tangents of these angles give the required accelerations, which may then be plotted either as functions of time or of space.

Having plotted an acceleration curve of one moving part, the acceleration curves of the other moving parts may, if desired, be plotted from the data given by the plotted acceleration curve and its corresponding velocity curves as a function of time and of space, and the relations established in equations (14) to (57), inclusive.

If it is desired to determine the acceleration curves of the various moving parts before the carriage is constructed, it may be done in the manner to be described later in article 66.

59. Computation of the Values of the Forces at Any Instant While the Projectile is in the Bore. — If we wish to compute the values of the forces on the parts of the carriage at any time while the projectile is in the bore other than the time of maximum powder pressure, the method to be followed is similar to that described for the instant of maximum powder pressure. For example, suppose it is desired to compute the values of the forces at the instant the projectile leaves the bore. The value of R , the constant resistance of the recoil cylinder, will be equal to 37312 lbs. as before. The value of the force F at this instant can be obtained from the curve of pressures while the projectile is in the bore plotted by the methods of interior ballistics. The velocity of the gun in restrained recoil can be determined very closely as before from a consideration of the data given by Table 2, and the value of ϕ may be obtained from those data and equation (14). The values of the angles ψ and θ and of the coordinates of the centers of mass at the instant the projectile leaves the bore can then be obtained from equations (14) to (27), inclusive. The value of $d\phi/dt$ can be found from the velocity of the gun in restrained recoil selected after consideration of Table 2, and equations (28) and (33). The values of $d\psi/dt$ and $d\theta/dt$ then follow from equations (41) and (42), respectively, and the values of the accelerations in terms of $d^2\phi/dt^2$ from equations (43) to (57), inclusive.

By substituting these data together with the values of the constants from Table 3 in equations (1) to (12), inclusive, they may be readily solved as before giving the value of $\frac{d^2\phi}{dt^2}$ and the values of the forces P to P_{10} at the instant the projectile leaves the bore.

60. Computation of the Values of the Forces at any Instant after the Powder Gases have Ceased to Act on the Gun. — After the powder gases have ceased to act on the gun the angular velocity $d\phi/dt$ corresponding to any assumed value of ϕ can be obtained from equation (13), or more readily from equation (13'') as explained later in article 64. The values of ψ , θ , $d\psi/dt$, $d\theta/dt$, of the coordinates of the centers of mass, and of the accelerations in terms of $d^2\phi/dt^2$, can then be determined in the manner already explained. The value of R is 37312 lbs. as before.

Substituting these data and the values of the constants from Table 3 in equations (1) to (12), inclusive, and making $F = 0$ since the powder gases have ceased to act on the gun, the equations may be solved giving the values of $d^2\phi/dt^2$ and the forces P to P_{10} corresponding to the assumed value of ϕ . A negative value for $d^2\phi/dt^2$ obtained in the solution of equations (1) to (12) indicates that the rotation of the gun levers is being retarded.

61. Centripetal Accelerations. — It will be noted that each expression for the value of the different accelerations in terms of the angular acceleration $d^2\phi/dt^2$, equations (43) to (57), inclusive, contains at least one term in which the square of an angular velocity occurs. This is due to the rotary motion of the parts and the consequent centripetal accelerations along the radii equal to $V^2/\rho = \rho\omega^2$ in which ρ is the radial distance of the center of mass of the body from the center about which it is rotating, V is the linear velocity of the center of mass, and ω the angular velocity of the rotating body. The components of the centripetal accelerations in the directions of the axes of X and Y must enter in the expressions for the various linear accelerations in those directions, and due to the relations between the variable angles the expressions for the angular accelerations $d^2\theta/dt^2$ and $d^2\psi/dt^2$ will also contain terms involving the squares of the angular velocities.

As a general rule, however, when forces whose duration is very short, like those of the powder gases in a gun, are considered, the accelerations produced by them will be relatively very great as compared with the velocities because of the very short period of time during which the forces act. Referring to the expressions for the various accelerations given on pages 87 and 88, the numerical terms represent the values of the terms into which the squares of the angular velocities entered; and as the angular

acceleration $d^2\phi/dt^2$ has been determined to be 224.08 radians per sec. per sec., it follows that the terms containing the squares of the angular velocities are of relatively little importance as regards their effect on the computed values of the forces. On this account and because of the labor involved in considering them, it is customary to neglect such terms in computing the forces brought upon a gun carriage when the gun is fired. Had this been done in the case under consideration, it would not have been necessary to determine any of the angular velocities of the parts at the instant of maximum powder pressure, and the last terms of the acceleration equations on pages 87 and 88 would not appear.

62. Effect of the Movement of the Parts on the Intensities of the Forces. — It is well to consider the effect of permitting movement of the parts of the gun carriage when the gun is fired. As a result of such movement the total force brought upon the upper ends of the gun levers is but 201970 lbs., this being the square root of the sum of the squares of its horizontal and vertical components P and P_1 . The total force on the upper end of the elevating arm is 28208 lbs., its horizontal and vertical components being P_2 and P_3 , respectively. Were the gun levers rigidly fixed in position the whole force of the powder gases, 1017860 lbs., would be brought against their upper ends; but when the parts of the carriage are permitted to move, as in this case, by far the larger part of this force is absorbed in giving acceleration to the gun without causing any stress in the carriage parts.

If it were not for the translation of the center of mass of the gun levers to the rear on the top carriage during recoil of the gun, the gun and counterweight would have about the same acceleration. Comparing the mass of the counterweight, 615.58, with that of the gun, 396.88, it will be seen that if the top carriage were not permitted to move to the rear, more than half of the force exerted by the powder gases would have to be transmitted through the gun levers to the counterweight in order to give it about the acceleration of the gun. The design of the carriage in this case would of course have to be changed for the counterweight would have to move in the arc of a circle instead of in a vertical line as at present. The construction of the carriage, which permits the center of mass of the gun levers to translate to the rear

on the top carriage, materially reduces the acceleration of the counterweight and permits the use of much lighter gun levers than would otherwise be possible. The acceleration of the gun in the horizontal direction at the instant of maximum powder pressure obtained from equation (43') by making $d^2\phi/dt^2 = 224.08$ radians per sec. per sec. is 2078.42 ft. per sec. per sec., and its acceleration in the vertical direction obtained from equation (48') is 202.37 ft. per sec. per sec. The acceleration of the counterweight occurs only in the vertical direction and its value at the instant of maximum powder pressure obtained from equation (51') is only 243.21 ft. per sec. per sec., or about one-ninth of the horizontal acceleration of the gun.

DETERMINATION OF THE PROFILE OF THE THROTTLING GROOVES IN THE RECOIL CYLINDER.

63. Formula for the Area of Orifice. — **Experimental Modification to Allow for the Contraction of the Liquid Vein.** — The area of orifice is given by equation (20), page 287, Lissak's Ordnance and Gunnery, which is as follows:

$$a^2 = \gamma A^2 v_r^2 / 2 g P \quad (58)$$

in which a is the area of orifice in square feet, A is the effective area of the piston in square feet, v_r is the velocity of restrained recoil, P is the total pressure on the piston, g is the acceleration due to gravity, and γ is the weight of a cubic foot of the liquid in the recoil cylinder.

This expression, however, does not take into account the contraction of the liquid vein. From actual measurements of velocities of recoil and the corresponding pressures in the recoil cylinders, made at the Sandy Hook Proving Ground with a Sebert velocimeter and a special form of the ordinary indicator used for indicating the steam pressure corresponding to different positions of the piston in the cylinder of a steam-engine, it has been determined that the value of the area of orifice required to give the desired pressure on the piston is obtained by substituting for the velocity of flow of the liquid through the orifice

$$v_i = v_r A / a \quad (59)$$

given by equation (16), page 287, Lissak's Ordnance and Gunnery, a larger value

$$v_{lc} = bv_l + c = (bv_r A/a) + c \quad (60)$$

in which b and c are experimental constants that vary with the diameter of the recoil cylinder, the velocity of recoil, and the shape of the orifice.

Following the lines of the discussion on page 287, Lissak's Ordnance and Gunnery, the pressure required to produce a velocity of flow v_{lc} through an orifice is the pressure due to a column of liquid whose height is given by the equation

$$v_{lc}^2 = 2gh \quad (61)$$

Substituting for v_{lc} its value from equation (60) and solving for h , we have

$$h = [(bv_r A/a) + c]^2 / 2g \quad (62)$$

The weight of a cubic foot of the liquid being γ , the weight of a column whose area of cross-section is equal to that of the piston is $A\gamma h$. $A\gamma h$ is, therefore, P , the pressure on the piston, and multiplying both sides of equation (62) by $A\gamma$ we obtain

$$P = A\gamma h = [(bv_r A/a) + c]^2 A\gamma / 2g \quad (63)$$

Solving equation (63) for a we have

$$a = bv_r A / [\sqrt{2gP/A\gamma} - c] \quad (64)$$

which is the form of equation used in place of equation (58) when it is desired to take into account the contraction of the liquid vein. The values of b and c for the 6-inch disappearing carriage, model of 1905 M1, have been taken as 1.3 and 122, respectively. Making these substitutions in equation (64) it becomes

$$a = 1.3 v_r A / [\sqrt{2gP/A\gamma} - 122] \quad (65)$$

To express the area of orifice and the area of the piston in square inches divide a and A by 144 and $\sqrt{A}$ by $\sqrt{144}$ obtaining

$$a'' = 1.3 v_r A'' / [\sqrt{288gP/A''\gamma} - 122] \quad (66)$$

The density of oil used in the cylinder of the carriage being .85, the weight of a cubic foot of the liquid is

$$\gamma = .85 \times 62.5 = 53.125 \text{ lbs.}$$

The total resistance of the recoil cylinder is 37312 lbs. of which about 1100 lbs. is the friction in the stuffing-box against

the piston-rod. The total pressure of the liquid against the piston is, therefore,

$$P = 37312 - 1100 = 36212 \text{ lbs.}$$

The diameter of the piston is 7.23 ins. and that of the piston-rod is 3.25 ins., so that the effective area of the piston is

$$A'' = \pi \{ (7.23)^2 - (3.25)^2 \} / 4 = 32.76 \text{ sq. ins.}$$

Substituting these values of γ , P , and A in equation (66) and taking $g = 32.2$, we have

$$a'' = \frac{1.3 \times 32.76 v_r}{\left(\frac{288 \times 32.2 \times 36212}{32.76 \times 53.125} \right)^{\frac{1}{2}} - 122} = .1342 v_r \quad (67)$$

The piston-rod of this carriage is stationary, and since the recoil cylinder is attached to the cross-head to which the counterweight is also attached, the cylinder has the same movement as the counterweight, and we may write

$$a'' = .1342 dy_w / dt \quad (68)$$

64. Profile of the Throttling Grooves.—The areas of the throttling grooves are calculated to correspond to the setting of the throttling valve-stem in the 4th notch which gives an opening through the valve of .17 sq. in. In addition the piston has a clearance in the cylinder of .02 in. on the diameter which gives a constant area for the escape of the liquid around the piston equal to

$$\pi [(7.25/2)^2 - (7.23/2)^2] = .2275 \text{ sq. in.}$$

Both of these areas must be subtracted from a'' to get the area of the throttling grooves. There are two such grooves and they have a uniform width of 1.25 ins., so that their depth d at any point corresponding to a velocity dy_w/dt of the counterweight will be

$$d = \frac{a'' - .17 - .2275}{2 \times 1.25} = \frac{.1342 (dy_w/dt) - .3975}{2.50}$$

$$\text{or} \quad d = .05369 (dy_w/dt) - .159 \quad (69)$$

The profile of each throttling groove will be a curve whose ordinates are the values of d given by equation (69) for various values of dy_w/dt , and whose abscissas are the values of y_w corresponding to the same values of dy_w/dt .

The value of y_w for any assumed value of ϕ is given by equation (22). The value of $d\phi/dt$ for any assumed value of ϕ equal to or greater than 21° , the value of ϕ corresponding to the instant when the powder gases cease to act on the gun, can be obtained from equation (13), and with this value of $d\phi/dt$ the corresponding value of dy_w/dt can be obtained from equation (36). Substituting in equation (13) the values of the linear and angular velocities in terms of $d\phi/dt$ from equations (28) to (42), inclusive, solving for $d\phi/dt$, and representing, for the sake of simplicity of expression, the coefficients of $d\phi/dt$ in equations (41) and (42) by X and U , respectively, we obtain

$$\frac{d\phi}{dt} = \frac{\left[R(y''_w - y_w) + W_w(y''_w - y_w) + W_a(y''_a - y_a) + W_c(y''_c - y_c) - W_g(y_g - y'_g) - W_e(y_e - y'_e) \right]^{\frac{1}{2}}}{\left[\frac{1}{2} M_g [a \cos \phi + c \cos (\phi - \beta)]^2 + \frac{1}{2} M [a \tan \alpha \cos \phi]^2 - c \sin (\phi - \beta)]^2 + \frac{1}{2} \Sigma m r_g^2 X^2 + \frac{1}{2} M_a [a \cos \phi]^2 + \frac{1}{2} M_a [a \tan \alpha \cos \phi]^2 + \frac{1}{2} \Sigma m r_a^2 + \frac{1}{2} M_c [a \cos \phi]^2 + \frac{1}{2} M_c [a \tan \alpha \cos \phi]^2 + \frac{1}{2} M_e [l \cos \theta U]^2 + \frac{1}{2} M_e [l \sin \theta U]^2 + \frac{1}{2} \Sigma m r_e^2 U^2 + \frac{1}{2} M_w [a \tan \alpha \cos \phi + a \sin \phi]^2 \right]^{\frac{1}{2}}} \quad (13')$$

in which

$$X = \frac{\sin \theta [a \cos \phi + c \cos (\phi - \beta)] + \cos \theta [a \tan \alpha \cos \phi - c \sin (\phi - \beta)]}{d [\sin \theta \sin \psi + \cos \theta \cos \psi]}$$

$$\text{and} \quad U = \frac{a \cos \phi + c \cos (\phi - \beta) - d \sin \psi X}{b \cos \theta}$$

Substituting in equation (13') the known values of the resistance R , of the masses, moments of inertia, and the ordinates of the centers of mass when the parts are in the recoiled position, it becomes

$$\frac{d\phi}{dt} = \frac{\left[333719 - 57109 y_w - 6041 y_a - 2528 y_c - 12764 y_g - 707 y_e \right]^{\frac{1}{2}}}{\left[198.44 [a \cos \phi + c \cos (\phi - \beta)]^2 + 198.44 [a \tan \alpha \cos \phi]^2 - c \sin (\phi - \beta)]^2 + 7483.5 X^2 + 93.92 [a \cos \phi]^2 + 93.92 [a \tan \alpha \cos \phi]^2 + 625.2 + 39.305 [a \cos \phi]^2 + 39.305 [a \tan \alpha \cos \phi]^2 + 10.992 [l \cos \theta U]^2 + 10.992 [l \sin \theta U]^2 + 77.54 U^2 + 307.79 [a \tan \alpha \cos \phi + a \sin \phi]^2 \right]^{\frac{1}{2}}} \quad (13'')$$

The first step in the solution of equation (13'') to determine the value of $d\phi/dt$ corresponding to any assumed value of ϕ , is to calculate ψ and θ from equations (27) and (26), respectively, and from them and the assumed value of ϕ , the corresponding values of X and U . With ϕ , ψ , θ , X , and U known, the value of the denominator of equation (13'') may be determined. The values of y_w , y_a , y_c , y_d , and y_e may then be obtained from equations (22), (20), (21), (19), and (23), respectively, and the value of the numerator of equation (13'') determined. The value of $d\phi/dt$ follows. In this manner the values of $d\phi/dt$ for $\phi = 30^\circ, 40^\circ, 50^\circ, 60^\circ, 70^\circ, 80^\circ$, and 85° have been calculated. These values, together with the value of $d\phi/dt$ for 21° already determined, and the corresponding values of ψ , θ , y_w , dy_w/dt , and d , equation (69), are shown in the following table, in which are also shown for comparison with dy_w/dt the corresponding velocities v_c of the top carriage determined by obtaining dx_a/dt from equation (29) and dividing it by $\cos \alpha$.

TABLE 4.

ϕ	ψ	θ	y_w inches.	$d\phi/dt$ radians per sec.	dy_w/dt (v_w) ft. per sec.	d inches.	v_c ft. per sec.
21°	$-0^\circ 7'.3$	$11^\circ 26'.3$	2.31	2.8788	4.741	0.096	11.645
30°	$+0^\circ 43'.8$	$20^\circ 41'.5$	5.97	2.8295	6.378	0.183	10.621
40°	$+2^\circ 37'.5$	$30^\circ 38'.8$	11.34	2.7213	7.790	0.259	9.036
50°	$+5^\circ 4'.2$	$40^\circ 6'.7$	17.90	2.5915	8.770	0.312	7.220
60°	$+7^\circ 25'.5$	$48^\circ 50'.7$	25.44	2.3927	9.100	0.330	5.126
70°	$+8^\circ 50'.5$	$56^\circ 30'.3$	33.75	2.0592	8.456	0.295	3.053
80°	$+8^\circ 6'.7$	$62^\circ 34'$	42.56	1.4214	6.091	0.168	1.070
85°	$+6^\circ 34'.4$	$64^\circ 40'$	47.07	0.7326	3.1692	0.111	0.2767
88°	$+5^\circ$	$65^\circ 31'.4$	49.79	0	0	0	0

As the profile of the throttling groove is a smooth curve the number of points given in Table 4 will ordinarily be sufficient to enable it to be plotted from the abscissa $y_w = 2.31$ ins. to the

* For convenience in discussing the profile of the throttling groove, y_w will hereafter be referred to the center of mass of the counterweight when the gun is in battery instead of to the center of coordinates heretofore assumed. It will also be expressed in inches. Its values under these conditions are obtained by adding the value of $h = 3$ ft. to the values of y_w obtained from equation (22) and multiplying the results by twelve to reduce feet to inches.

end, although after plotting it from the table it may be found desirable to calculate one or two more points on the curve to determine with certainty the location of its maximum ordinate and the point at which it ends. On account of the subtractive term in equation (69) the throttling groove does not commence at the point where y_w is zero and does not extend to the point where y_w is 49.79 ins. From equation (69), d will be equal to zero when dy_w/dt is equal to $.159/.05369 = 2.961$ f. s. Until dy_w/dt attains this value the groove will not commence and it will cease as soon as dy_w/dt decreases to this value after having reached its maximum. The point where the groove ends becomes apparent at once when its profile is plotted to a large scale with a sufficient number of calculated points. The point where it commences is determined as described in the next paragraph.

Equations (13), (13'), and (13'') are not applicable to values of ϕ less than 21° for then the powder gases are still acting on the gun. $d\phi/dt$ can not, therefore, be obtained from equation (13'') for values of y_w less than 2.31 ins. Owing to the rapidly changing force exerted by the powder gases on the gun and to the impracticability of deducing a simple equation expressing the value of the force as a function of time or space, it is not practicable to obtain by analytical methods reliable values for dy_w/dt while the powder gases are acting, although these values may be obtained approximately from the curve of free recoil of the gun as a function of space. In the case of those barbette and mobile artillery carriages in which the resistance to recoil is constant and applied in the direction of the axis of the gun, the curve showing the velocity of restrained recoil may be accurately determined by graphical methods as described in Par. 166, pages 283 and 284, Lissak's *Ordnance and Gunnery*. The general form of the profile of the throttling grooves for such carriages is well known and it is of the same character beyond the point where the powder gases cease to act as the profile of the throttling groove of the disappearing carriage. It is a reasonable assumption, therefore, that the part of the profile of the throttling groove which corresponds to the position of the parts while the powder gases are acting is also similar in the two cases. For values of y_w less than 2.31 ins., therefore, the profile of the throttling groove of the 6-inch disappearing carriage, model of 1905 MI, is obtained by prolonging

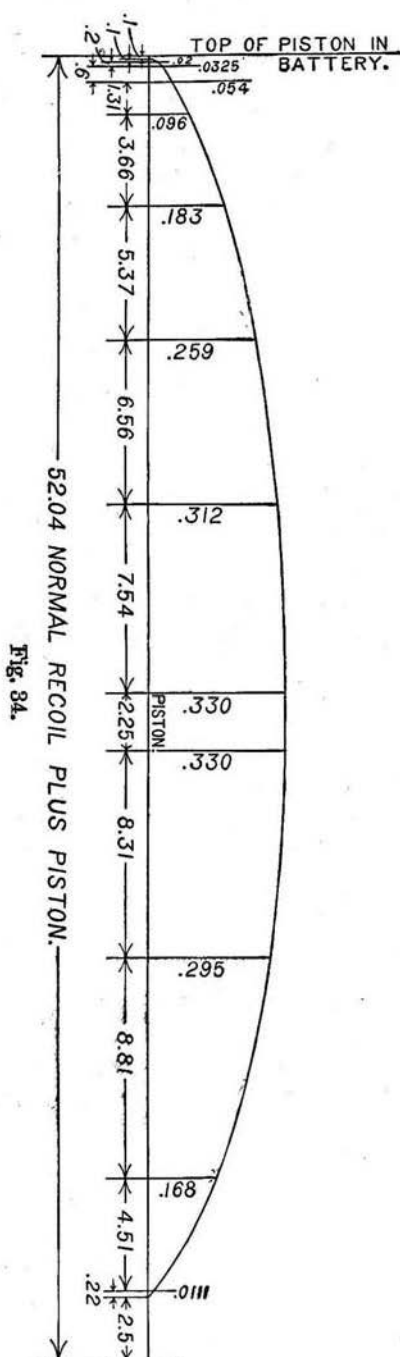
the curve already plotted back to the origin and giving to it the ordinary form.

Each new type or model of carriage is tested at the Sandy Hook Proving Ground, and any necessary changes in its design and in the areas of the throttling orifices are determined there. These changes are then applied to all carriages of that model before they are issued to the service.

Fig. 34 shows the complete profile of the throttling groove for this carriage as it would be furnished to the shops for cutting the grooves in the recoil cylinder.

It will be observed on examining this figure that above the word "PISTON" the curve is straight for a distance of 2.25 ins., its ordinate there being the maximum ordinate. This arrangement is necessary because the piston is 2.25 ins. long and the smallest opening between it and the wall at the bottom of the groove, or the controlling orifice, is at the upper edge of the piston until the maximum ordinate of the groove is reached during recoil. On account of the length of the cylindrical piston the maximum orifice could not be obtained if the straight part of the profile of the groove did not exist, for before the upper edge of the piston reached the maximum ordinate the lower edge would have passed it and the opening at the lower edge would have become the controlling orifice. The straight part of the profile permits the orifice at the upper edge of the piston to be the controlling orifice until it reaches the maximum ordinate, after which the orifice at the lower edge of the piston becomes the controlling orifice until recoil ends.

65. Velocities of the Counterweight and the Top Carriage Compared. — Advantages of the Recoil Cylinder in the Counterweight and of the Counter-Recoil Buffer Acting Against the Top Carriage. — The velocities of the top carriage shown in the last column of Table 4 have been calculated for comparison with those of the counterweight, and to illustrate by an example taken from a service carriage the statements made in Par. 200, page 346, Lissak's Ordnance and Gunnery; as to the reasons for changing the position of the recoil cylinders of the disappearing carriage from the top carriage to the counterweight while retaining the action of the counter-recoil buffer on the top carriage. By examination of the table it will be seen that the velocity of the top



carriage toward the end of recoil is much less than that of the counterweight, so that a recoil cylinder in the counterweight gives better control of the final movement of the gun in recoil than does one placed in the top carriage. As the loading angle and the height of the breech of the gun are greatly affected by a comparatively small change in the total amount of recoil, it is important that this variation be as small as possible.

The relations between the velocities of the counterweight and top carriage at any point are the same in counter-recoil as in recoil, so that when the gun is almost in battery during counter-recoil the velocity of the top carriage is much greater than that of the counterweight and a counter-recoil buffer acting against the top carriage gives better results than would one acting on the counterweight.

66. Velocity and Acceleration Curves of the Recoiling Parts. — Article 58 gives a description of the method of plotting the velocity and acceleration curves of the recoiling parts of a disappearing carriage based upon the measurement of the velocities of the top carriage by the Sebert velocimeter. If these curves are desired before the carriage is manufactured they can be obtained from the velocities of restrained recoil used in the calculations of the profile of the throttling grooves. For example, Table 4 gives the velocity of recoil of the counterweight as a function of space corresponding to a number of positions of the counterweight in recoil after the powder gases have ceased to act on the gun. The velocity of the counterweight corresponding to a number of its positions while the powder gases are acting may also be obtained by substituting in equation (69) the values of d scaled from the early part of the profile of the throttling groove. By plotting the reciprocals of the velocities of the counterweight as a function of space and integrating under the curve up to any ordinate, the area will represent the time corresponding to the space over which the counterweight has recoiled, these operations being the same as those for obtaining the curve of the velocity of the projectile in the bore as a function of time. Having the times corresponding to the various distances recoiled by the counterweight, its velocity of recoil may be plotted as a function of time as well as of space. The velocity of recoil of each of the other moving parts and the corresponding accelerations of all the moving parts may now be

plotted as a function either of time or space in a manner similar to that described in article 58.

In this way we can obtain, before the carriage is built, the total time of recoil, and the position, velocity, and acceleration of each of the recoiling parts of a disappearing carriage corresponding to any interval of time measured from the instant when recoil began.

67. Method of Designing a Gun Carriage. — The forces on the various parts of a gun carriage having been determined, it is then necessary to compute the stresses caused by them in the parts and to so proportion the parts that the stresses will be kept within permissible limits. The method of doing this will be explained in Chapter IV. From what has preceded it is apparent that the masses, and therefore the weights, and the distribution of the mass of each part, represented by its moment of inertia, affect to a considerable degree the forces brought upon the parts when the gun is fired; while on the other hand, the intensities of the forces govern in most cases the sizes and distribution of the mass of the parts. It is evident, therefore, that the design of a gun carriage, as of any other machine where the strength of the parts must be considered, is a matter of trial in which the actual amount of work involved is largely a matter of the skill of the designer and his experience with the type of gun carriage or machine being designed. The general outline of the carriage is first laid down on the drawing and the dimensions of the parts determined approximately by experience and judgment. The weights of the parts are then calculated from the drawing by obtaining their volumes and multiplying them by the weights of a unit volume of the various materials used. The centers of mass of the parts and their moments of inertia are calculated in accordance with the principles given in works on mechanics. (The method of calculating centers of mass and moments of inertia of parts from a drawing will be illustrated later in Chapter IV.)

The forces on the parts are next calculated and then the stresses caused by them. It will generally happen that the stresses in some of the parts are too great for safety and that in other parts they are smaller than they need be. Those parts in which the stresses are too great must be increased in size or changed in shape in such manner as to decrease the stresses to a reasonable figure. If

it is important that the weight of the carriage be kept as small as possible, as is the case with all mobile artillery carriages, those parts in which the stresses are smaller than they need be must be decreased in size and weight until the stresses are raised to the maximum permissible limits. If weight is not important and the unnecessarily large size of the parts is not objectionable otherwise, the parts may not be changed.

If the changes in the parts are so considerable as to require a redetermination of the weights, centers of mass, moments of inertia, and finally of the forces on the parts, this must be done; and the stresses on the parts due to the changed forces must again be determined. These processes are repeated until the sizes, weights, and shapes of the parts are considered satisfactory.

CHAPTER IV.

STRESSES IN PARTS OF GUN CARRIAGES.

68. Strength of Materials. — The determination of the stresses in parts of gun carriages requires a knowledge of strength of materials, but as this subject is studied by the cadets of the first class in the course of civil and military engineering the principles involved will not be deduced here. The most important facts and formulas will, however, be stated and briefly explained.

69. Stresses of Tension and Compression. — Let AB , Fig. 35, be a rod fixed at the end A . If a force T acts upon it in the direction of its axis to elongate it as shown, the force will produce

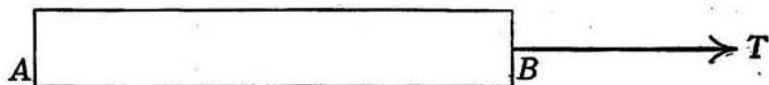


Fig. 35.

a stress of tension in the rod which will be distributed uniformly over each cross-section between B and A , the intensity of the stress per unit of area being equal to the total force divided by the area of cross-section considered.

If the direction of the force be reversed it will compress the rod producing in it a stress of compression. This stress also will be distributed uniformly over each cross-section between B and A and its intensity per unit of area will equal the total force divided by the area of cross-section considered.

If the force T be not applied at the end of the rod but at some section nearer A it will cause a stress in the sections between it and A but none in those between it and the free end B . If a number of forces act instead of one force T each will produce a stress in every section affected by it and the algebraic sum of these stresses in any section will be the total stress in the section.

In general, if a rod is in equilibrium under the action of a number of forces, all those that are parallel to its axis will cause either tension or compression in the rod. If any section of the rod be considered the tension or compression in it will be due to all of the forces acting on one side (either side) of that section; and if on either side there are no forces acting there will be no stress in the section. All forces acting toward a section produce compression therein and all those acting away from it produce tension therein.

70. Shearing Stress.—If two flat plates are laid one upon the other and fastened together by rivets any force applied to the plates to slide one along the other will be a simple shearing force as applied to the rivets. This force will be resisted by a shearing stress in the cross-sections of the rivets lying in the plane of contact of the plates. The shearing stress will be distributed uniformly over the cross-sections of the rivets and its intensity per unit of area will be equal to the total shearing force divided by the total area of cross-section.

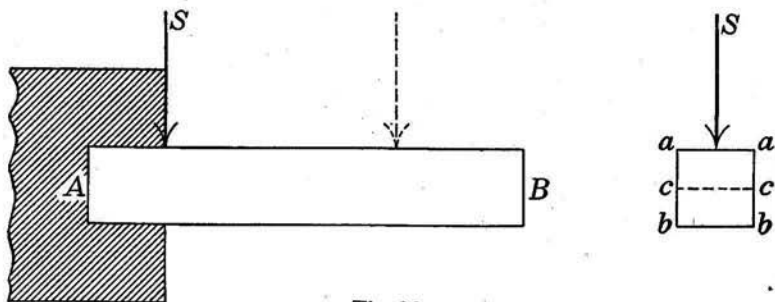


Fig. 36.

If a solid bar be considered instead of the plates the same force would tend to slide one part past the other in the same manner as if they were separate parts fastened together by rivets as before, and the stress developed between them would be one of simple shear.

Let *AB*, Fig. 36, be a block imbedded in a wall and *S* a force applied to the block through the center of the section next the wall as shown. The force will tend to slide that portion of the block projecting from the wall past the portion imbedded therein

and will produce in the section next the wall a shearing stress uniformly distributed over it, whose intensity per unit of area will equal the total force divided by the area of the section.

If the force S be not applied next the wall but at a distance from it as shown by the dotted force line it will produce a shearing stress in every section between it and the wall but none in the sections between it and the free end of the block. It will also produce a bending stress in the sections between it and the wall which will be discussed later. In this and every other case of a shearing stress caused by a force that also produces a bending stress, the shearing stress is not distributed uniformly over the section but varies from a maximum at the axis of the section (called the neutral axis) which is perpendicular to the force to zero at the points or lines of the section most distant from the neutral axis. Thus in Fig. 36 $aabb$ is the section next the wall. The shearing stress therein due to the force next the wall is uniformly distributed, but that due to the force shown by the dotted line varies from a maximum at the line cc to zero at the lines aa and bb . It is ordinarily assumed that a shearing stress, however caused, is uniformly distributed over the section since when caused by a force that also produces a bending stress it is generally much smaller than the latter.

If a number of shearing forces act on a piece, each will produce a shearing stress in every section affected by it and the algebraic sum of these stresses in any section will be the total shearing stress in the section.

In general, if a piece is in equilibrium under the action of a number of forces, all those that are perpendicular to its axis will cause shear in the piece, and the total shearing stress in any section will be due to all the forces perpendicular to the axis of the piece acting on one side (either side) of that section. If on either side there are no forces acting there will be no stress in the section.

TORSIONAL STRESS.

71. Rod Fixed at One End. — Let AB , Fig. 37, be a rod fixed at the end A^* and acted on at the free end by a force F perpendicular to its axis as shown. If this force intersected the axis of the rod it would produce in the rod only shearing and bending

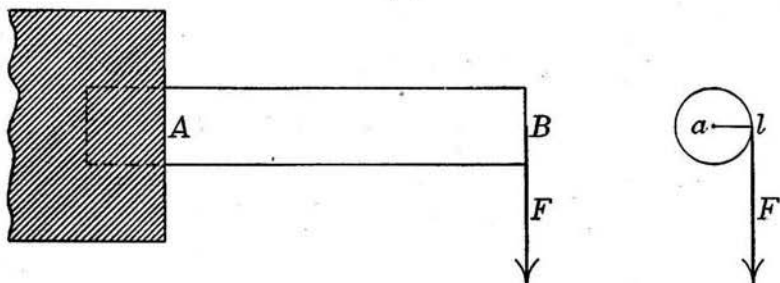


Fig. 37.

stresses as already explained, but having a lever arm with respect to the axis, it twists the rod producing in it a torsional stress also. Shearing stresses have already been considered and bending stresses will be taken up later, so that the twisting or torsional stress in the rod will alone be discussed at present.

The torsional moment of the force at any section between it and the fixed end of the rod is

$$M_t = F \times al \quad (1)$$

in which M_t is the torsional moment, F is the intensity of the force, and al is its lever arm in inches with respect to the axis of the rod. The torsional stress will be proportional to the torsional

* In order that a rod or beam may be fixed at one or both ends a portion of its length at one or both ends must be clamped in some manner that will prevent all movement of the portions clamped. The rod may be clamped by having its end or ends embedded in a wall or by any other suitable means. When the length of a rod that is fixed at one or both ends is referred to, it is to be understood as meaning the length of that portion which is not clamped; and when the fixed end of a rod fixed at one end only is referred to, it is to be understood as referring to the section of the rod at the end of the clamp next the unclamped portion of the rod. Similarly, when the ends of a rod that is fixed at both ends are referred to, it is to be understood as meaning the sections at the end of both clamps next the unclamped portion of the rod.

moment but its intensity on any elementary area of a cross-section will vary directly with the distance of that area from the axis. If a number of twisting forces act on the rod each will produce a torsional moment at every section affected by it and the algebraic sum of these moments at any section will be the total torsional moment at the section. If there is no twisting force at the free end of the rod there will be no torsional moment at any section between the free end and that section whose plane contains the twisting force nearest to the free end.

Rod Fixed at Both Ends.* — In Fig. 37 the rod is fixed at one end only; if fixed at both ends as shown in Fig. 38 the force F will produce a torsional moment on each side of the section in whose plane it is contained but the moment will not be equal to

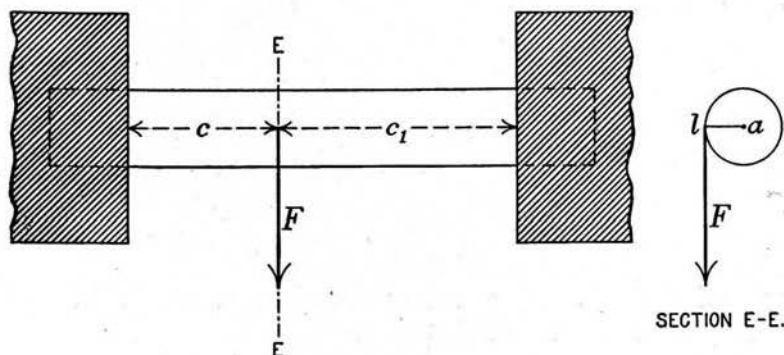


Fig. 38.

$F \times al$ and it will be different on the two sides of the section. If c and c_1 represent the distances of this section from the left and right ends, respectively, of the rod, the torsional moment at any section within the portion c will be

$$M_t = F \times al \times \frac{c_1}{c + c_1} \quad (2)^\dagger$$

and that at any section within the portion c_1 will be

$$M_t = F \times al \times \frac{c}{c + c_1} \quad (3)^\dagger$$

* See footnote, page 109.

† From Reuleaux's Constructor.

the larger of the two moments pertaining to the section whose plane contains the force.

General Case of Torsional Moments.—When a rod is in equilibrium under the forces acting on it and all the twisting forces and couples either to the left or to the right of a section are known, the torsional moment at that section is the algebraic sum of the individual moments with respect to the axis of the rod of each of the twisting forces and couples on either side of the section. The reactions between the rod and its supports must be included among the twisting forces and couples on either side of the section, and when this can be done it is unnecessary to consider whether the rod is fixed at one or both ends. If on either side of the section there are no twisting forces or couples there is no torsional moment at the section.

Intensity of Torsional Stress.—Let I_p be the polar moment of inertia of a cross-section of the rod, M_t the torsional moment at the section, and S'''_t the torsional stress per unit of area at a unit's distance from the axis; then, as shown on pages 38 and 39, Fieberger's Civil Engineering,

$$S'''_t = M_t / I_p$$

and the stress per unit of area at any distance r from the axis will be

$$rS'''_t = M_t r / I_p \quad (4)$$

The maximum stress will evidently occur at that point of the section most distant from the axis.

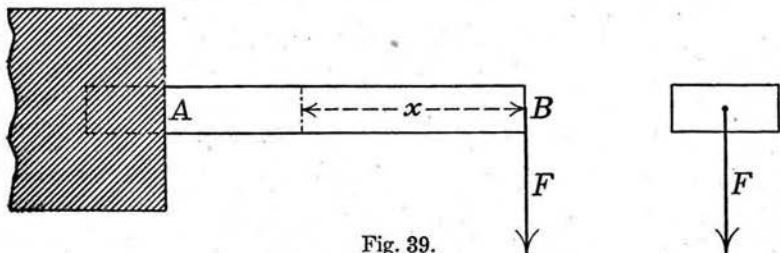


Fig. 39.

BENDING STRESS.

72. Cantilever Subjected to a Concentrated Bending Force.—Let AB , Fig. 39, be a rod or beam fixed at the end A and acted

* See footnote, page 109.

on at the free end by a force F whose action line is perpendicular to and intersects the axis of the beam. Such a force has no torsional moment at any cross-section of the beam but produces in each section a shearing and a bending stress. The bending stress only will be considered. The force F bends the beam and rotates each section about its axis perpendicular to the action line of the force, the amount of this rotation varying directly as the moment of the force with respect to the section. This moment is called the bending moment of the force and is equal to

$$M = Fx \quad (5)^*$$

in which M is the bending moment, F is the intensity of the force, and x the perpendicular distance in inches between it and the section under consideration.

The rotation of the sections causes the fibres on top to be extended producing in them a stress of tension, and those on the bottom to be compressed producing in them a stress of compression. If the force acted upward the upper fibres of the beam would be compressed and the lower fibres extended. The fibres intersecting the axes about which the sections rotate, called the neutral axes, are neither extended nor compressed, and the tension or compression in any other fibre at any section is directly proportional to the bending moment at that section and to the distance of the fibre from the neutral axis. A bending stress, therefore, is one in which the fibres on one side of the neutral axis are subjected to a tensile stress and those on the other side to a compressive stress.

In Fig. 39 the force acts at the free end of the beam and every section between it and the fixed end is subjected to a bending moment and resulting bending stress, but if the force did not act at the free end there would be no bending moment or bending stress in any section between the force and that end. If a number of bending forces act on the beam, each will produce a bending moment at every section affected by it, and the algebraic sum of all these moments at any section will be the total bending moment at the section.

* The general expressions for the bending moments given by equations (5) to (23), inclusive, are taken from Merriman's Text Book on the Mechanics of Materials.

A beam such as is shown in Fig. 39 is called a cantilever. The maximum bending moment occurs at the section at the fixed end.

Intensity of Bending Stress.—Let I be the moment of inertia of a cross-section of a beam with respect to its neutral axis, M the bending moment at the section, and S''' the stress per unit of area at a unit's distance from the neutral axis; then, as shown on pages 49 and 50, Fieberger's Civil Engineering, the stress per unit of area at any distance y from the neutral axis is

$$S'''y = My/I \quad (6)$$

Beam Fixed at Both Ends Subjected to a Concentrated Bending Force.*—If both ends of the beam are fixed as in Fig. 40 the

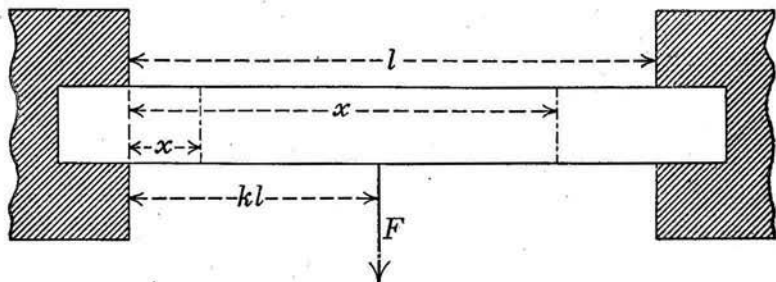


Fig. 40.

force F will produce a bending moment on each side of the section in whose plane it is contained. If l is the length of the beam in inches and kl the distance of the force from the left end, k being a fraction less than unity, the bending moment at any section on the left of the force at a distance x from the left end of the beam is

$$M = -Flk(1 - 2k + k^2) + F(1 - 3k^2 + 2k^3)x \quad (7)$$

and the bending moment at any section on the right of the force at a distance x from the left end of the beam is

$$M = -Flk(1 - 2k + k^2) + Fk(l - 3kx + 2k^2x) \quad (8)$$

If k is $\frac{1}{2}$, that is, if the force is applied at the middle of the beam,

* See footnote, page 109.

the bending moment at any section on the left of the force at a distance x from the left end is

$$M = -\frac{Fl}{8} + \frac{Fx}{2} \quad (9)$$

and at any section on the right of the force at a distance x from the left end, it is

$$M = -\frac{Fl}{8} + \frac{F}{2}(l - x) \quad (10)$$

The maximum bending moment in this case occurs at the ends of the beam and under the force, and is

$$M = \pm Fl/8 \quad (11)$$

If a number of forces parallel to F act on the beam each will cause bending moments in the various sections thereof which may be determined from equations (7) and (8), and the algebraic sum of all those moments at any section will be the total bending moment at the section.

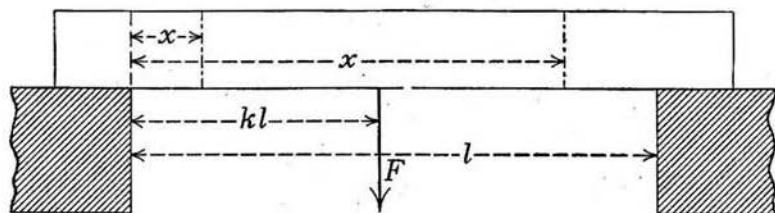


Fig. 41.

Beam Supported at Both Ends Subjected to a Concentrated Bending Force.* — If the ends of the beam are not fixed but are merely supported as shown in Fig. 41, the bending moment at any section on the left of the force at a distance x from the left support is

$$M = F(1 - k)x \quad (12)$$

and at any section on the right of the force at a distance x from the left support, it is

$$M = Fk(l - x) \quad (13)$$

in which l is the length of the beam in inches.

* The length of a beam that is supported at both ends is taken to be the distance between the supports, and the sections of the beam at the supports are referred to as the ends of the beam.

When $x = 0$ or $x = l$, $M = 0$

and, therefore, the bending moment is zero at the ends of a beam that is merely supported. The maximum bending moment occurs under the force and is

$$M = F(1 - k)kl \quad (14)$$

If the force acting at the middle of the beam k is $\frac{1}{2}$ and the bending moment at any section on the left of the force at a distance x from the left support is

$$M = Fx/2 \quad (15)$$

and at any section on the right of the force at a distance x from the left support, it is

$$M = F(l - x)/2 \quad (16)$$

The maximum bending moment occurs under the force and is

$$M = Fl/4 \quad (17)$$

If a number of forces parallel to F act on the beam, each will cause bending moments in the various sections thereof which may be determined from equations (12) and (13); and the algebraic sum of all these bending moments at any section will be the total bending moment at the section.

Cantilever Subjected to a Uniformly Distributed Bending Force. — If the force F shown in Fig. 39 were uniformly distributed over the length of the beam instead of being concentrated at the free end, the bending moment at any section at a distance x from the *free* end of the beam would be

$$M = wx^2/2 \quad (18)$$

in which w is the intensity of the force per linear inch of the beam. The maximum bending moment would occur at the fixed end of the beam and would be

$$M = wl^2/2 \quad (19)$$

l being the length of the beam in inches.

Beam Fixed at Both Ends Subjected to a Uniformly Distributed Bending Force. — If the force in Fig. 40 were uniformly distributed over the length of the beam the bending moment at any section would be

$$M = \frac{wx}{2} - \frac{wx^2}{2} - \frac{wl^2}{12} \quad (20)$$

in which x is the distance of the section from the left end of the beam, l the length of the beam in inches, and w the intensity of the force per linear inch of the beam. The maximum bending moment would occur at the ends of the beam and would be

$$M = -wl^2/12 \quad (21)$$

Beam Supported at Both Ends Subjected to a Uniformly Distributed Bending Force. — If the force in Fig. 41 were uniformly distributed over the length of the beam between the supports, the bending moment at any section would be

$$M = \frac{wlx}{2} - \frac{wx^2}{2} \quad (22)$$

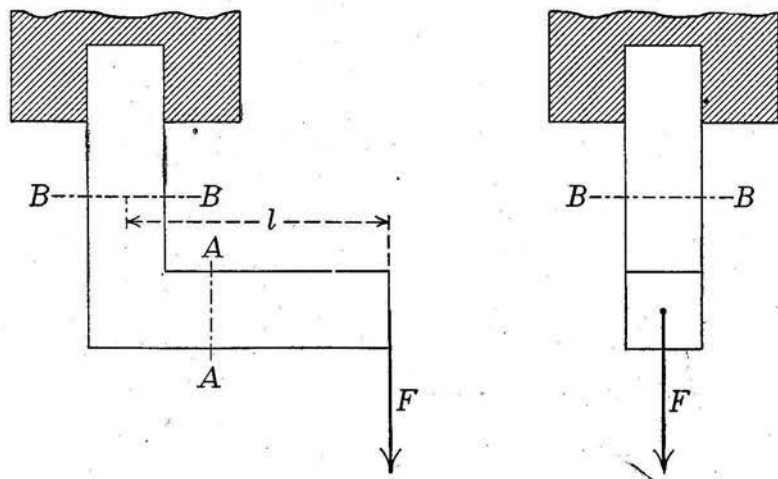


Fig. 42.

in which x , l and w have the same significance as before. The bending moment at the ends of the beam would be zero. The maximum bending moment would occur at the middle of the beam and would be

$$M = wl^2/8 \quad (23)$$

Bending Moment at a Section of a Beam Due to Any Force Having a Lever Arm with Respect to an Axis of the Section. — In the preceding discussion of bending moments the force has always been taken perpendicular to the axis of the beam and

parallel to the section; but a bending moment at a section may be caused by any force that has a lever arm with respect to one of the axes of the section. Thus in Fig. 42 the force F not only has a bending moment with respect to any section such as AA in the horizontal portion of the beam but it also has one with respect to any section such as BB in the vertical part. The bending moment with respect to the section BB is Fl where l is the perpendicular distance in inches between the action line of the force and the axis of BB perpendicular to the plane of the paper.* If a number of forces parallel to F act on the beam each having a lever arm with respect to the same axis of section BB , each will cause a bending moment at the section and the algebraic sum of all the bending moments will be the total bending moment at the section.

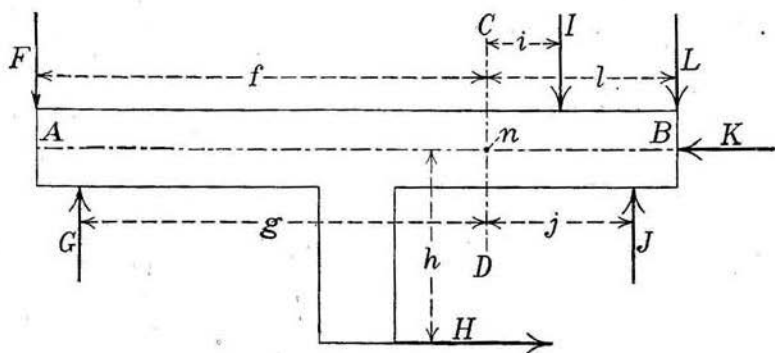


Fig. 43.

General Case of Bending Moments.—When a beam is in equilibrium under the forces acting upon it and all the bending forces either to the left or to the right of a section are known, the total bending moment at that section is obtained by taking the algebraic sum of the individual bending moments at the section due to each of the forces on its left or to each of the forces on its right. The reactions between the beam and its supports must be included among the forces considered and when this can be done it is unnecessary to consider whether the beam is a cantilever, or

* The force F also produces tension in Section BB .

one supported or fixed at both ends. If on either side of the section there are no bending forces there is no bending stress in the section. Thus in Fig. 43 AB is a beam with a projecting vertical part, which is in equilibrium under the action of the forces F, G, H, I, J, K , and L ; F, G , and H acting on the left of the section CD and the others on its right. The bending moment at section CD , considering moments that produce compression in the upper fibres as positive, is

$$M = -Ff + Gg - Hh = -Ii + Jj - Ll$$

The force K does not appear in the expression for the bending moment as it intersects the neutral axis of the section, shown projected at n , and, therefore, has no lever arm with respect to it. In the ordinary case the forces F, G, J, L , and K would be the reactions of the supports on the beam.

COMBINED STRESSES.

73.—It frequently happens in engineering structures that a piece is subjected to tension or compression, shear, bending, and torsion, or to some of these stresses, at the same time. When this is the case each stress is computed as it if were the only one existing in the piece and then all are combined in a manner to be described.

Tension or Compression and Bending.—Since a bending stress is one of tension on one side of the neutral axis of a section and compression on the other side, a stress of tension or compression in a section due to a simple tensile or compressive force is added algebraically to the stress in the section caused by a bending force. Thus if a piece subjected to a bending stress is also subjected to a simple stress of tension, the intensity of the stress in all the fibres on the tension side of the neutral axis due to the bending force will be increased by the intensity of the simple stress of tension; and the intensity of the stress in all the fibres on the compression side of the neutral axis due to the bending force will be decreased by the intensity of the simple stress of tension.

Tension or Compression and Shear.—If a piece be subjected to either tension or compression and shear the resulting stress of

tension or compression in any section due to combination with the shear is

$$S_{tc} = \frac{1}{2} S_t + \sqrt{S_s^2 + (S_t/2)^2} \quad (24)^*$$

and the resulting shearing stress due to combination with the tension or compression is

$$S_{sc} = \sqrt{S_s^2 + (S_t/2)^2} \quad (25)^*$$

in which S_{tc} and S_{sc} are the resulting stresses per unit of area of tension or compression and shear, respectively; S_t is the stress per unit of area of tension or compression computed as if the shearing stress did not exist; and S_s is the shearing stress per unit of area computed as if the stress of tension or compression did not exist. If S_t is tension S_{tc} will be tension; and if S_t is compression S_{tc} will be compression.

Bending and Shear.—Since a bending stress is one causing tension in the fibres on one side of the neutral axis and compression on the other side, the formulas for combining it with a shearing stress are the same as for combining a stress of tension or compression with one of shear. S_t in this case would represent the stress of tension or compression due to the bending force computed as if the shearing stress did not exist.

Ordinarily a bending stress is not combined with a shearing stress due to the same bending force or to a parallel bending force, because such a shearing stress varies from a maximum at the neutral axis, where the bending stress is zero, to zero at the fibre farthest from the neutral axis, where the bending stress is a maximum.

Shear and Torsion.—A torsional stress is essentially one of shear differing from the ordinary shearing stress only in that its intensity per unit of area varies with the distance of the fibre being considered from the axis of the piece under torsion. A shearing stress distributed uniformly over a cross-section may, therefore, be combined with a torsional stress by simple algebraic addition. If the particular fibre under consideration is subjected to a torsional stress which acts in a direction opposite to that of the shear, the combined stress is numerically equal to

* From Fiebigger's Civil Engineering.

the difference of the torsional and the shearing stresses computed as if each existed alone, but if at the particular fibre under consideration the stresses act in the same direction the combined stress is numerically equal to their sum.

Tension or Compression and Torsion. — Since a torsional stress is a form of shearing stress the formulas for combining it with a stress of tension or compression are the same as for combining a shearing stress therewith. S_t in this case would represent the torsional stress computed as if the stress of tension or compression did not exist.

Bending and Torsion. — The formulas for combining bending and shear apply here also; but since each stress in this case varies in intensity with the position in the section of the fibre under consideration, care must be taken to see that the stresses combined pertain to the same fibre. The maximum stresses are what it is generally desired to determine and consequently it is customary to obtain first the maximum simple stress S_t due to bending alone, which occurs in that fibre most distant from the neutral axis, and to combine it with the stress S_s due to torsion alone which occurs in the same fibre. Ordinarily the maximum simple stresses S_t and S_s occur at the same fibre but when this is not the case the maximum value of S_s must also be found and combined with the value of S_t at the same fibre to determine which of the combined stresses is the greater. It may also happen that the maximum combined stress S_{tc} or S_{sc} occurs in a fibre in which neither S_t nor S_s is a maximum. No general rule can be laid down to cover a case of this kind but consideration of the shape of the section will generally indicate where the maximum combined stress is likely to occur.

Method of Combining Stresses. — If a piece is subjected to tension or compression, bending, shear, and torsion, at the same time, the stress of tension or compression should be combined by algebraic addition with the bending stress, and the shearing stress combined with the torsional stress in the same way. The resulting stress of tension or compression should then be combined with the resulting shearing stress by equations (24) and (25).

NEUTRAL AXIS. CENTER OF GRAVITY. MOMENT OF INERTIA.

74. Neutral Axis. — When the bending force is parallel to the plane of the section the neutral axis is the straight line through the center of gravity* of the section perpendicular to the force. When the bending force is perpendicular to the plane of the section it tends to bend it about every straight line passing through its center of gravity that does not intersect the action line of the force, prolonged if necessary. Any such line, therefore, becomes the neutral axis of the section when the bending about itself is being considered.

Irregular Sections. — The positions of the centers of gravity of a number of plane figures and the moments of inertia of these figures with respect to various axes passing through their centers of gravity are given in most works on mechanics; but in many or most instances in the design of gun carriage parts the sections upon which bending and torsional stresses occur are of complex shape for which the position of the center of gravity, and the moment of inertia, must be specially determined.

Determination of the Centers of Gravity of Irregular Sections. — The position of the center of gravity of an irregular plane figure may be determined from the principle that the moment of the total area of the figure about any axis must equal the sum of the moments about that axis of the various partial areas into which the figure may be divided. The moment of an area about an axis is the product of the area by the distance between the axis and a line parallel to it passing through the center of gravity of the area. If, therefore, the sum of the moments of the partial areas about any axis is determined the quotient obtained by dividing that sum by the total area will be the distance from the axis to a parallel line passing through the center of gravity of the figure. By repeating these operations with respect to another axis the direction and position of a second line passing through the center of gravity of the figure can be determined. The inter-

* What is here called the center of gravity of a section is really its center of figure, for strictly speaking a surface has no weight. However, as it is convenient to use the term center of gravity for center of figure this practice will be continued throughout the text.

section of the two lines so determined must necessarily be the center of gravity of the figure.

When finding the center of gravity of an irregular plane figure or section in this way, the partial areas into which it is subdivided should be of such regular forms that the area and the position of the center of gravity of each are known or can readily be obtained by the ordinary rules of mensuration. The areas and the positions of the centers of gravity of a number of regular plane figures are given in Table 5, in which will also be found the moment of inertia of each figure about an axis passing through its center of gravity. The last will be needed for determining the moments of inertia of irregular plane sections as will be described later.

Example. — To illustrate the method of determining the centers of gravity of irregular sections, let it be required to determine the center of gravity of the section shown in Fig. 44.

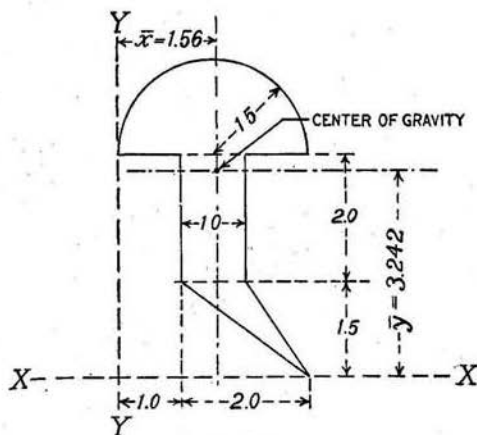
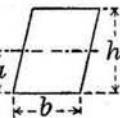
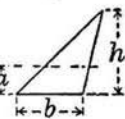
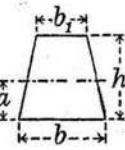
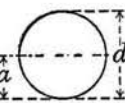
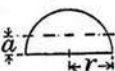
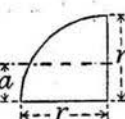
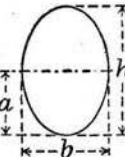
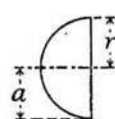


Fig. 44.

Divide the section into a semi-circle, a rectangle, and a triangle by the dotted lines as shown. Draw two axes XX and YY perpendicular to each other through the lowest and extreme left-hand points, respectively, of the section. Now if two lines passing through the center of gravity of the section be determined, one of them parallel to XX and the other to YY, their intersection will give the center of gravity desired. Taking moments

TABLE 5.

In the table the moment of inertia is given for the axis shown by the dotted line passing through the center of gravity of the figure. The distance a is measured from the axis to some prominent line or point of the figure.

Figure.	Moment of inertia.	Distance a .	Area of figure.
Rectangle 	$bd^3/12$	$d/2$	bd
Parallelogram 	$bh^3/12$	$h/2$	bh
Triangle 	$bh^3/36$	$h/3$	$bh/2$
Trapezoid 	$\frac{b^2 + 4bb_1 + b_1^2}{36(b+b_1)} h^3$	$\frac{b+2b_1}{b+b_1} \frac{h}{3}$	$\frac{b+b_1}{2} h$
Circle 	$\pi d^4/64 = .0491 d^4$	$d/2$	$\pi d^2/4 = .7854 d^2$
Semi-circle 	$.110 r^4$	$.4244 r$	$\pi r^2/2 = 1.5708 r^2$
Circular Quadrant 	$.055 r^4$	$.4244 r$	$\pi r^2/4 = .7854 r^2$
Ellipse 	$\pi bh^3/64 = .0491 bh^3$	$h/2$	$\pi bh/4 = .7854 bh$
Semi-circle 	$\pi r^4/8 = .3937 r^4$	r	$\pi r^2/2 = 1.5708 r^2$

about XX , and consulting Table 5 for the areas of the elementary figures and the locations of their centers of gravity, we have

$$\left. \begin{aligned} \frac{\pi \times (1.5)^2}{2} (.4244 \times 1.5 + 2 + 1.5) + (1 \times 2) (1 + 1.5) + \\ \left(1 \times \frac{1.5}{2} \right) \left(1.5 - \frac{1.5}{3} \right) = \left\{ \pi \times \frac{(1.5)^2}{2} + 1 \times 2 + 1 \times \frac{1.5}{2} \right\} \bar{y} \end{aligned} \right\} (26)$$

in which $\bar{y}$ is the distance between the axis XX and a line parallel thereto passing through the center of gravity of the section.

From equation (26) $\bar{y} = 3.242$ ins.

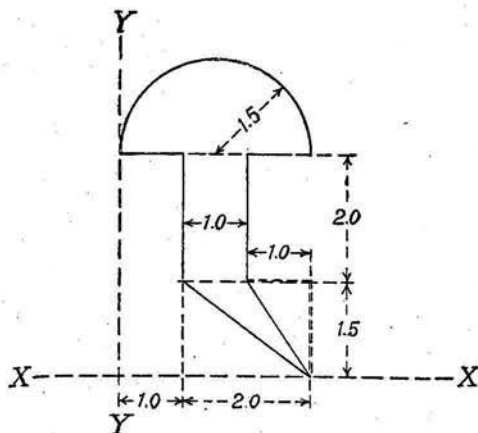


Fig. 45.

Taking moments about YY , and adding the triangle shown in double dotted lines in Fig. 45 to aid in obtaining the moment of the original triangle about this axis, we have

$$\left. \begin{aligned} \frac{\pi \times (1.5)^2}{2} \times 1.5 + (1 \times 2) (.5 + 1) + \left(1.5 \times \frac{2}{2} \right) \left(1 + 2 - \frac{2}{3} \right) \\ - \left(1.5 \times \frac{1}{2} \right) \left(1 + 2 - \frac{1}{3} \right) = \left\{ \frac{\pi \times (1.5)^2}{2} + 1 \times 2 + 1 \times \frac{1.5}{2} \right\} \bar{x} \end{aligned} \right\} (27)$$

in which $\bar{x}$ is the distance between the axis YY and a line parallel thereto passing through the center of gravity of the section.

From equation (27) $\bar{x} = 1.56$ ins.

The coordinates of the center of gravity of the figure with respect to the axes XX and YY are, therefore,

$$\bar{y} = 3.242 \text{ ins.}; \quad \bar{x} = 1.56 \text{ ins.}$$

Determination of the Moments of Inertia of Irregular Sections.

— The moment of inertia of an irregular section about any axis in its plane is the sum of the moments of inertia of the various partial areas into which it may be divided about the same axis. Each partial area should be of such regular form that its moment of inertia about an axis passing through its center of gravity is known or can readily be obtained by the methods of the calculus. Having the moment of inertia about an axis passing through the center of gravity of a figure, its moment of inertia about any other axis parallel to the first is obtained by adding the area of the figure multiplied by the square of the distance between the two axes to the moment of inertia of the figure about the axis passing through its center of gravity.

The moments of inertia of a number of plane figures of regular form about axes passing through their centers of gravity are given in Table 5.

Example. — To illustrate the method of determining the moment of inertia of an irregular section let it be required to determine the moment of inertia of the section shown in Fig. 44, (a) about the axis through its center of gravity parallel to XX , (b) about one through its center of gravity parallel to YY , and (c) about the axis passing through its center of gravity perpendicular to its plane. The moment of inertia about the last axis is called the polar moment of inertia and would be used in determining the torsional stress in the section.

Consulting Table 5 we find that the moment of inertia of the semicircular part of Fig. 44 about an axis parallel to XX passing through the center of gravity of the section is

$$.110 \times (1.5)^4 + 1.5708 \times (1.5)^2 \times \{1.5 + 2 + .4244 \times 1.5 - 3.242\}^2 = 3.385 \text{ ins.}^4$$

Similarly, the moment of inertia of the rectangular part about the same axis is

$$\frac{1 \times (2)^3}{12} + \{1 \times 2\} \{3.242 - (1.5 + 1)\}^2 = 1.768 \text{ ins.}^4$$

and the moment of inertia of the triangular part about the same axis is

$$\frac{1 \times (1.5)^3}{36} + \left\{1 \times \frac{1.5}{2}\right\} \left\{3.242 - \left(1.5 - \frac{1.5}{3}\right)\right\}^2 = 3.864 \text{ ins.}^4$$

The moment of inertia of the section about this axis is, therefore,

$$3.385 + 1.768 + 3.864 = 9.017 \text{ ins.}^4$$

The moments of inertia of the partial areas of the section about an axis parallel to YY passing through the center of gravity of the section are:

Of the semicircle,

$$.3937 \times (1.5)^4 + 1.5708 \times (1.5)^2 \times (1.56 - 1.50)^2 = 2.006 \text{ ins.}^4$$

Of the rectangle,

$$\frac{2 \times (1)^3}{12} + \{1 \times 2\} \{1.56 - (1 + .5)\}^2 = .174 \text{ ins.}^4$$

Of the triangle, (see Fig. 44.)

$$\begin{aligned} \frac{1.5 \times (2)^3}{36} + \left\{2 \times \frac{1.5}{2}\right\} \left\{1 + \left(2 - \frac{2}{3}\right) - 1.56\right\}^2 - \left[\frac{1.5 \times (1)^3}{36} \right. \\ \left. + \left\{1 \times \frac{1.5}{2}\right\} \left\{1 + \left(2 - \frac{1}{3}\right) - 1.56\right\}^2\right] = .270 \text{ ins.}^4 \end{aligned}$$

The moment of inertia of the section about this axis is, therefore,

$$2.006 + .174 + .270 = 2.45 \text{ ins.}^4$$

Having obtained the moments of inertia of the section about two axes in its plane perpendicular to each other and passing through its center of gravity, the polar moment of inertia I_p may be obtained from the principle that the sum of the moments of inertia of a plane figure about two axes in its plane perpendicular to each other is the moment of inertia of the figure about an axis perpendicular to its plane passing through the point of intersection of the first two axes. Whence

$$I_p = 9.017 + 2.45 = 11.467 \text{ ins.}^4$$

Cubical Contents, Centers of Gravity, and Moments of Inertia, of Irregular Volumes.—The cubical contents and the positions of the centers of gravity of a number of regular volumes, and the moments of inertia of the volumes about axes passing through their centers of gravity, may be found tabulated in most books on mechanics. To determine these data for irregular volumes the procedure is the same as for irregular plane sections. The irregular volume is divided into a number of partial volumes

of regular form or approaching some regular form sufficiently closely for the purpose. The cubical contents of the whole is then equal to the sum of the cubical contents of the various partial volumes; and the position of the center of gravity of the whole may be obtained by the principle of moments. The moment of inertia of the irregular volume about any axis is equal to the sum of the moments of inertia about that axis of the various partial volumes into which it is divided.

When the moment of inertia of a volume about an axis passing through its center of gravity is known, its moment of inertia about any other axis parallel to the first is obtained by adding the cubical contents of the volume multiplied by the square of the distance between the two axes to the moment of inertia of the volume about the axis passing through its center of gravity.

Weight and Mass of any Volume Composed of a Given Material. Moment of Inertia of the Mass in any Volume. — The weight of a volume of any material is equal to the cubical contents of the volume multiplied by the weight of a unit volume of the material of which it is composed.

The mass in a volume of any material is equal to the cubical contents of the volume multiplied by the number of units of mass in a unit volume of the material of which it is composed; or the mass in a volume of any material is equal to the weight of the volume divided by the acceleration due to the force of gravity.

The moment of inertia about any axis of the mass in a volume of any material is equal to the moment of inertia of the volume about that axis multiplied by the number of units of mass in a unit volume of the material of which it is composed.

75. Permissible Stresses in Gun Carriage Parts. — Printed specifications are published by the Ordnance Department, U. S. Army, in which are given the elastic limit in tension, the tensile strength, the elongation per unit of length at rupture, and the contraction of area at rupture demanded by the department for the most important materials used for parts of gun carriages. In some materials such as cast iron, copper, and bronze, the elastic limit is not clearly defined in the testing machine and on this account it is omitted in the specifications. Sometimes also the contraction of area at rupture is omitted.

The elastic limit in compression is assumed to be equal to the elastic limit in tension, and the elastic limit in shear is taken as four-fifths of the elastic limit in tension.

When the elastic limit is specified for a given material the dimensions of the part to be made thereof should be so regulated that the stresses developed in it shall not exceed one-half of the elastic limit. When the tensile strength only is specified the dimensions of the part should be such that the stresses therein shall not exceed one-fifth of the tensile strength. The object of prescribing a certain percentage of elongation per unit of length and of contraction of area at rupture for materials used in parts of gun carriages is to prevent their being unduly brittle and, therefore, likely to break under shock.

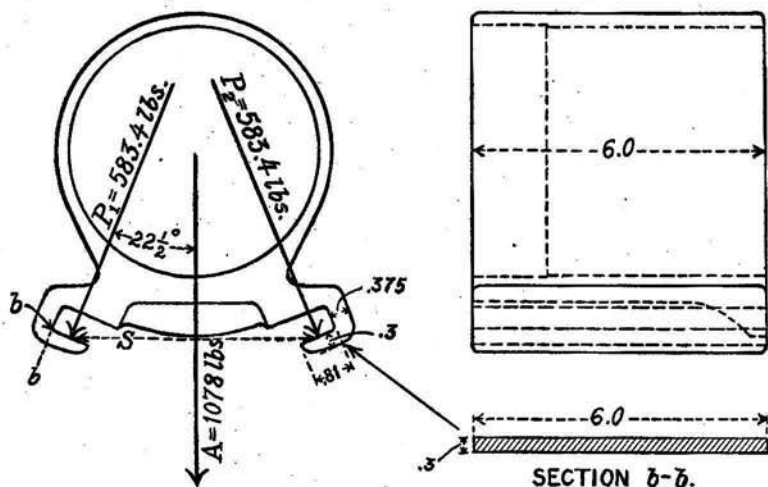


Fig. 46.

CALCULATION OF STRESSES IN PARTS OF GUN CARRIAGES. 3-INCH FIELD GUN.

76. Bending and Shearing Stresses in the Front Clip of the Gun. — The front clip of the gun is shown in Fig. 46. The force $A = 1078 \text{ lbs.}$, see Figs. 15 and 16, acts vertically downward on the clip. It is the resultant of two equal forces P_1 and P_2 whose action lines are normal to the surfaces S of the clip as shown in the figure. The parts of the clip on which the forces

P_1 and P_2 act make an angle of $22\frac{1}{2}^\circ$ with the horizontal and the intensities of P_1 and P_2 are, therefore,

$$P_1 = P_2 = \frac{1078}{2 \cos 22\frac{1}{2}^\circ} = 583.4 \text{ lbs.}$$

These parts of the clip are .81 in. long and each may be considered as a cantilever over which the force is uniformly distributed, and which tends to break under the action of the force through the section bb shown in the figure. The bending moment at this section is, from equation (19),

$$M = \frac{583.4 \times (.81)^2}{.81 \times 2} = 236.3 \text{ in. lbs.}$$

Section bb is 6 ins. wide and .3 ins. deep and its neutral axis is midway between the top and the bottom. I , the moment of inertia of the section with respect to its neutral axis, is

$$bd^3/12 = 6 \times (.3)^3/12 = .0135 \text{ in.}^4$$

$$\text{and} \quad y = .15 \text{ in.}$$

The stress in the fibre most distant from the neutral axis is, therefore, from equation (6),

$$S'''y = 236.3 \times .15 / .0135 = 2626 \text{ lbs. per sq. in.}$$

tension at the top of the section and compression at the bottom.

Considering the shearing stress as uniformly distributed over section bb its intensity per unit of area is

$$\frac{583.4}{6 \times .3} = 324 \text{ lbs. per sq. in.}$$

The force P_1 or P_2 also causes a combined bending and tensile stress in the part of the clip above section bb where the dimension .375 is placed. The bending moment with respect to the section there may be obtained by considering the force as concentrated at the middle of the length of .81 in., whence

$$M = 583.4 \left\{ \frac{.81}{2} + \frac{.375}{2} \right\} = 345.7 \text{ in. lbs.}$$

This section is 6 ins. wide and .375 in. deep and its moment of inertia with respect to its neutral axis, half way between the inner and outer edges of the section, is

$$I = 6 \times (.375)^3 / 12 = .0264 \text{ in.}^4$$

The maximum bending stress is, therefore,

$$S'''y = \frac{345.7 \times .375 / 2}{.0264} = 2455 \text{ lbs. per sq. in.}$$

tension on the inside and compression on the outside of the section.

The tensile stress in the section is

$$\frac{583.4}{6 \times .375} = 259 \text{ lbs. per sq. in.}$$

The maximum stress in the section is, therefore, one of tension which occurs on the inner edge and is equal to

$$2455 + 259 = 2714 \text{ lbs. per sq. in.}$$

So far as the stresses in the clip are concerned its length might safely be reduced to .75 in. since the elastic limit of the material of which it is made is required to be not less than 53000 lbs. per sq. in. But when one part slides upon another as in this case it is desirable to keep the pressure between the sliding surfaces as low as possible to prevent wear and consequent loose fitting of the parts after they have been in service for a considerable time.

STRESSES IN THE RECOIL LUG OF THE GUN.

77. Maximum Intensity of the Force P . Force Required to Accelerate the Recoil Cylinder. — When the gun is fired the recoil lug draws the cylinder filled with oil to the rear compressing the counter-recoil spring and reducing its length from 70 to 25 inches. The resistance to drawing the cylinder to the rear including the compression of the spring is the force P , see Figs. 15 and 16, which when the gun is at its extreme position to the rear during recoil was found, equation (32), page 50, to be 3849 lbs., corresponding to a total resistance to recoil of 4077 lbs.

Evidently, however, the value of P will be considerably greater than this when the gun is just commencing to recoil for then the

total resistance to recoil, equation (12), page 44, is 4923 lbs. The larger value of P will, therefore, be calculated and used in determining the stresses in the recoil lug. Fig. 47 shows the forces on the gun when it is at 15° elevation and just commencing to recoil.*

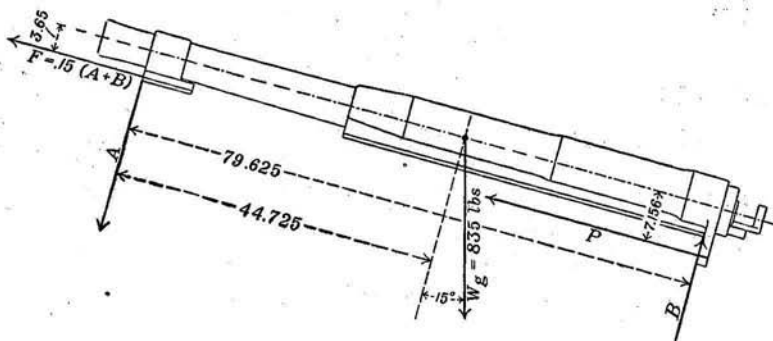


Fig. 47.

Taking moments about the center of mass of the gun, the equations expressing the relations between the forces (see article 37) are as follows:

$$P - 835 \sin 15^\circ + .15(A+B) = R_1 = 4923 \quad (28)$$

$$A - B + 835 \cos 15^\circ = 0 \quad (29)$$

$$P \times 7.156 + .15(A+B) \times 3.65 - B \times 34.9 - A \times 44.725 = 0 \quad (30)$$

From which

$$A = 101 \text{ lbs.} \quad (31) \quad F = 151 \text{ lbs.} \quad (33)$$

$$B = 908 \text{ lbs.} \quad (32) \quad P = 4988 \text{ lbs.} \quad (34)$$

It will be seen that at the beginning of recoil the force P is considerably greater than at the end, but that the contrary is the case with the forces A , B , and F .

The force P , however, is not the only one acting on the recoil lug of the gun, for at the instant of maximum powder pressure the recoil cylinder being required to move with the gun to the rear

* In determining the forces brought into action between the gun and the carriage by a given total resistance to recoil, R , the force of the powder gases does not have to be considered.

has the same acceleration as the gun, and the force required to produce this acceleration of the cylinder must be transmitted to it through the recoil lug. The total weight of the recoiling parts is 960 lbs., and since the maximum pressure in the gun is 33,000 lbs. per sq. in. the total pressure tending to move the gun to the rear is

$$33000 \times \pi (3)^2 / 4 = 233264 \text{ lbs.}^*$$

and the corresponding acceleration is

$$233264 \times 32.16 / 960 = 7814 \text{ ft. per sec. per sec.}$$

The weight of the cylinder filled with oil is 61.5 lbs. and the force required to give it this acceleration is

$$\frac{61.5}{32.16} \times 7814 = 14944 \text{ lbs.}$$

The counter-recoil spring is not capable of transmitting a force sufficient to give a material acceleration to any considerable part of its mass, and, therefore, at the instant of maximum acceleration of the gun the first and possibly the second and third coils at the front end of the spring will be very much deflected while the balance of the spring will have undergone practically no displacement. On this account the acceleration of the spring will not be considered.

The total force transmitted through the recoil lug because of the force P and the acceleration of the recoil cylinder is, then,

$$4988 + 14944 = 19932 \text{ lbs.}$$

The recoil lug of the gun and the point of application of this force are shown in Fig. 48. The weakest section of the lug to resist the shearing action of the force is AB and the weakest section to resist the bending moment is CD . The lever arm of the force with respect to section CD is 3 ins.

* Theoretically the resultant force that causes the recoil of the gun is the total powder pressure diminished by the total resistance to recoil. The latter, however, is so small compared with the maximum powder pressure that it is not worth while to consider it in computing the maximum acceleration of the recoiling parts of this carriage.

Shearing Stress in Section AB.— This section is shown in Fig. 48. The portion of the recoil lug below it weighs about 1.18 lbs. and to give this weight the acceleration of the gun requires a force of 286 lbs., which theoretically should be added to the

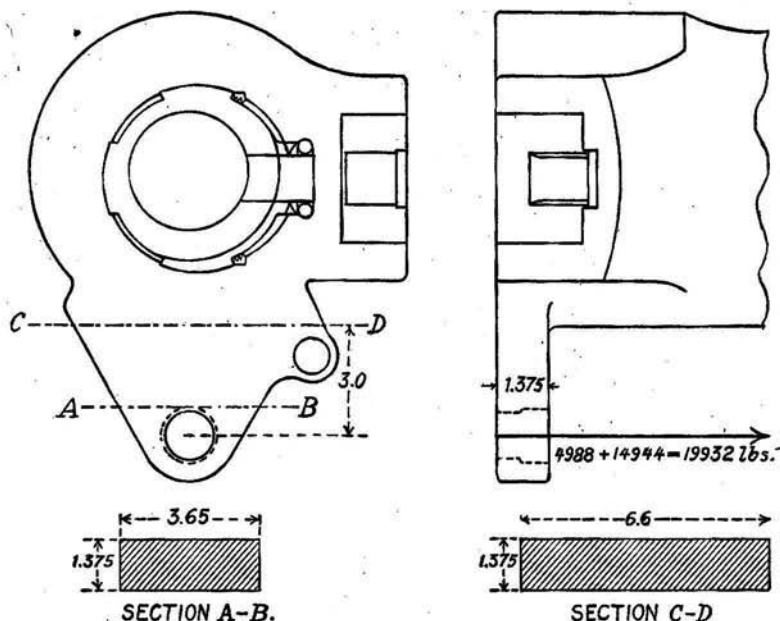


Fig. 48.

force of 19932 lbs. to determine the shearing stress in the section, but on account of its insignificance when compared with the latter force it will be neglected in these computations.

The shearing stress in section AB is, therefore,

$$\frac{19932}{3.65 \times 1.375} = 3972 \text{ lbs. per sq. in.}$$

Bending Stress in Section CD.— This section is shown in Fig. 48. The portion of the recoil lug below it weighs about 5.9 lbs. and to give this weight the acceleration of the gun requires a force of 1434 lbs. The point of application of this force is at the center of gravity of the portion of the lug below the section and its lever arm with respect to the section is about 1.25 ins.

While its bending moment with respect to section *CD* is relatively unimportant compared with that of the force of 19932 lbs. it will nevertheless be considered in determining the bending stress in the section. The total bending moment at the section is, therefore,

$$19932 \times 3 + 1434 \times 1.25 = 61589 \text{ in. lbs.}$$

and the maximum bending stress is

$$S'''_y = \frac{61589 \times 1.375/2}{6.6 \times (1.375)^3/12} = 29615 \text{ lbs. per sq. in.}$$

tension at the rear edge of the section and compression at the front edge. The elastic limit of the material in the recoil lug is not permitted to be less than 65000 lbs. per sq. in.

STRESSES IN THE RECOIL CYLINDER.

78. Tensile Stress in the Rear End of the Recoil Cylinder. — The method of connecting the recoil cylinder to the recoil lug of the gun is shown in Fig. 49. The rear end of the cylinder is closed

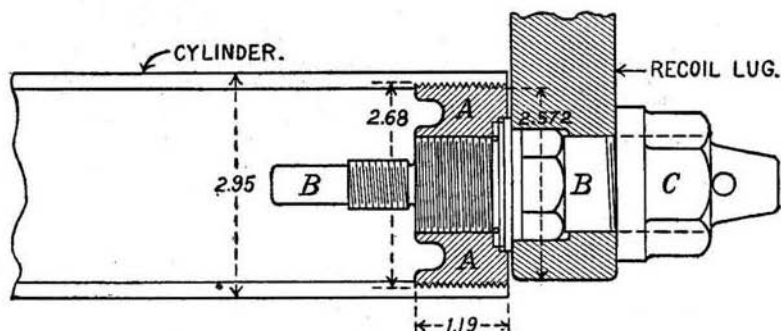


Fig. 49.

by the cylinder end *AA* screwed into it as shown. A cylinder end stud *BB* in turn screws into the cylinder end and passing through a hole in the recoil lug is held thereto by the nut *C*. The thinnest section of the cylinder occurs through the bottom of the threads into which the cylinder end is screwed. At this section the outer diameter is 2.95 ins. and the inner diameter 2.68 ins. The force of 19932 lbs., consisting of the resistance *P* and

the force required to accelerate the cylinder, must be transmitted through this section, and the tensile stress therein is, therefore,

$$\frac{19932}{\pi[(2.95)^2 - (2.68)^2]/4} = 16694 \text{ lbs. per sq. in.}$$

Shearing Stress in the Threads of the Cylinder End.—The force of 19932 lbs. also tends to shear the threads connecting the cylinder end to the cylinder. The U. S. standard thread used by the Ordnance Department is of the shape shown in Fig. 50

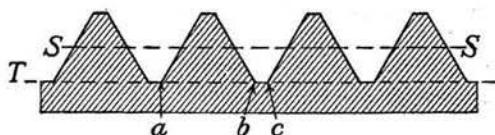


Fig. 50.

and the distance ab is seven times the distance bc . If shearing occurs along any line SS above the line TT at the bottom of the threads, the threads on both parts of the pieces screwed together will have to be shorn through but, if it occurs along the line TT , only seven-eighths as much metal will be shorn through as in the former case. It will, therefore, be assumed that shearing if it occurs would take place along the bottom of the threads and, since the cylinder end has the smaller diameter, that its threads would shear before those of the cylinder. The diameter at the bottom of the threads of the cylinder end is 2.572 ins. and the length of the threaded surface is 1.19 ins. Of this length only seven-eighths is effective on account of the flats bc at the bottom of the threads. The total area resisting the shearing action of the force of 19932 lbs. is, therefore, equal to seven-eighths of the outer surface of a cylinder whose length is 1.19 ins. and outer diameter 2.572 ins., and the shearing stress in the threads is

$$\frac{19932}{.875 \times \pi \times 2.572 \times 1.19} = 2369 \text{ lbs. per sq. in.}$$

Stress in the Walls of the Recoil Cylinder Due to the Interior Hydraulic Pressure.—The pressure in lbs. per sq. in. of the oil in the recoil cylinder must be such that it will exert a total pressure on the front end of the cylinder, tending to prevent its

movement to the rear, equal to P minus the force exerted by the counter-recoil spring.

The intensity of P when recoil is just beginning, equation (34), is 4988 lbs. and at this time the force exerted by the spring is 516 lbs., whence the total pressure on the end of the cylinder is 4472 lbs. The effective area at the front end of the cylinder is equal to the effective area of the piston, which is the area of a circle whose diameter is that of the piston, minus the sectional area of the piston-rod, minus the area of the slots in the piston

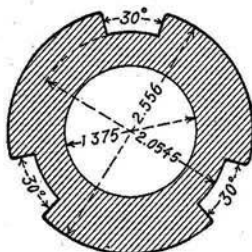


Fig. 51.

for the throttling bars. Fig. 51 shows a cross-section of the piston and piston-rod.

The shaded part is the effective area of the piston, the unshaded central part representing the piston-rod. The effective area is equal to

$$\frac{\pi}{4} \left\{ (2.556)^2 - (1.375)^2 \right\} - \frac{\pi}{4} \left\{ (2.556)^2 - (2.0545)^2 \right\} \frac{3 \times 30}{360} = 3.192 \text{ sq. ins.}$$

and the pressure per sq. in. is

$$4472 / 3.192 = 1401 \text{ lbs. per sq. in.}$$

This pressure occurs in front of the piston and between it and the front end of the cylinder. The thinnest section of this part of the cylinder occurs in front of the place where the throttling bars begin and has an outer diameter of 2.95 ins. and an inner diameter of 2.5717 ins.

Applying the formula for the maximum stress produced by the application of an interior pressure to a simple cylinder we obtain

$$\theta = \frac{2 (1.28585)^2 + 4 (1.475)^2}{3 [(1.475)^2 - (1.28585)^2]} \times 1401 = 10740 \text{ lbs. per sq.in.}$$

tension at the inner surface.

This stress is not the resultant tension in the section for the force P produces a tensile stress therein, in the direction of the axis of the cylinder and at right angles to the stress produced by the interior pressure, equal to

$$\frac{4988}{\pi[(2.95)^2 - (2.5717)^2]/4} = 3040 \text{ lbs. per sq. in.}$$

and, as has been shown in the deduction of the formulas used in calculating the stresses in a gun, a force of tension or compression produces a stress in a direction perpendicular to its action line equal to one-third of the stress produced in that direction but of opposite sign. The force P , therefore, produces a compression of $3040/3 = 1013$ lbs. per sq. in.

in the direction of the tension due to the interior pressure, reducing that tension from 10740 to 9727 lbs. per sq. in.

This discussion relates to periods after the acceleration of the gun and recoiling parts has ceased for, as has been shown, at the instant of maximum acceleration the tensile stress in the cylinder due to the force required to accelerate it is largely in excess of that due to the force P or to the interior pressure.

STRESSES IN THE CRADLE.

79. Maximum Value of the Forces Acting on the Cradle. —

It was shown in determining the stresses in the recoil lug that when the gun is just commencing to recoil the force P is considerably greater than it is when recoil is about to end, while the contrary is the case with forces A and B . Before computing the stresses in some of the parts of the cradle, therefore, the forces acting on it when the gun is just commencing to recoil will be determined so that the larger forces occurring under either one of the two conditions may be used in the computations.

Fig. 52 shows the position of the forces acting on the cradle when the gun is just commencing to recoil, and the values of the forces A , B , F , and P already determined in article 77, page 133.

Taking moments about the center of mass of the cradle, the equations representing the relations between the forces are

$$D - E + 409 \cos 15^\circ - 101 + 908 = 0 \quad (35)$$

$$C - 409 \sin 15^\circ - 4988 - 151 = 0 \quad (36)$$

$$D \times 2.277 + E \times 22.848 - C \times 3.939 - 101 \times 49.277 - 151 \times 3.506 - 908 \times 30.348 = 0 \quad (37)$$

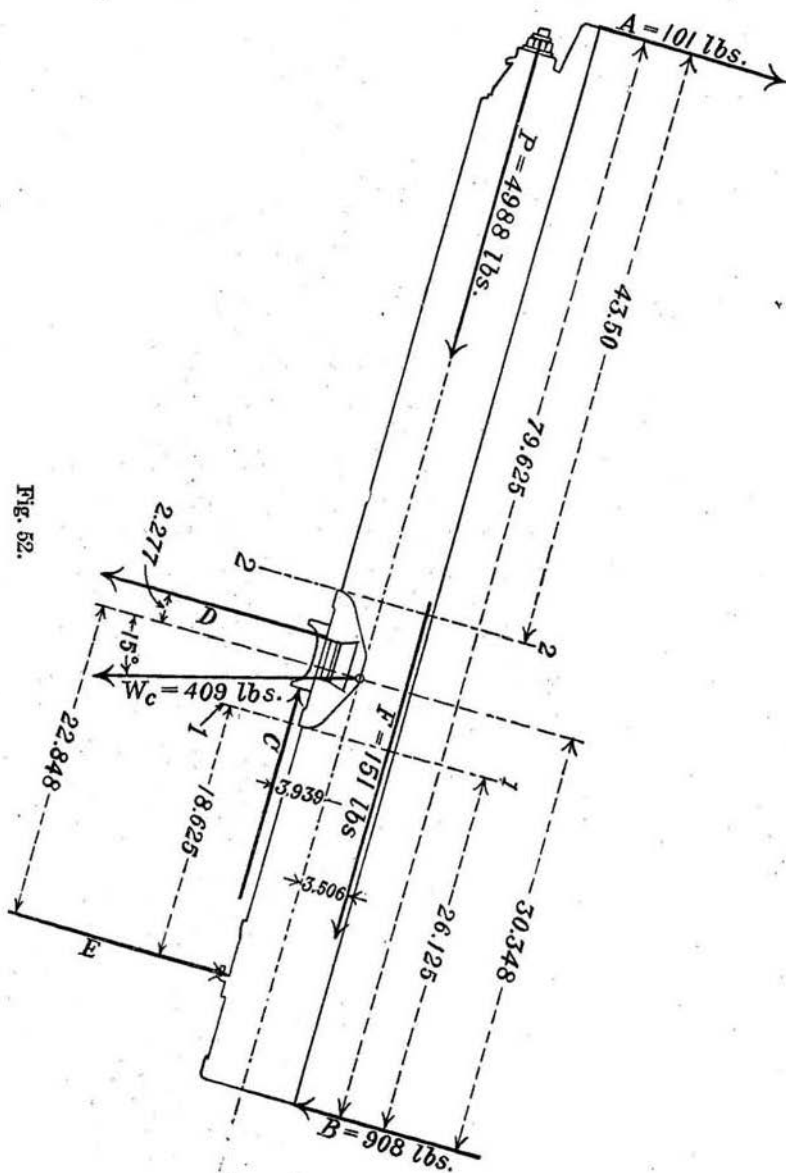


Fig. 52.

Whence

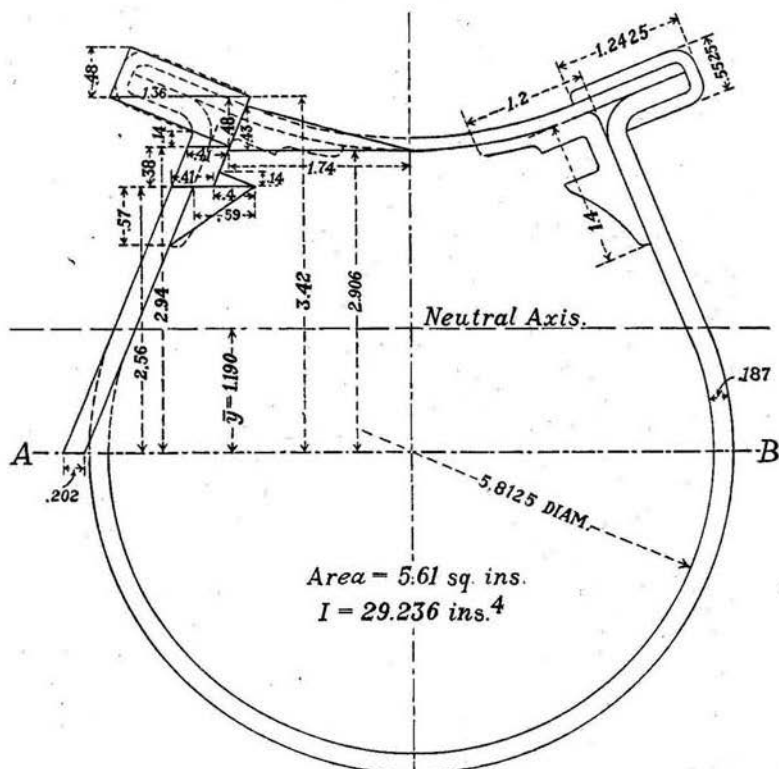
$$C = 5245 \text{ lbs.} \quad (38)$$

$$D = 1046 \text{ lbs.} \quad (39)$$

$$E = 2248 \text{ lbs.} \quad (40)$$

Stresses in Section 1-1 of the Cradle.—This section is indicated in Figs. 16 and 52. It is shown in Fig. 53.

Moment of Inertia of Section 1-1.—As this section is subjected to a bending stress due to the forces B , E , part of F equal



Section 1-1.

Fig. 53.

to .15 B , and a component of the weight of that part of the cradle in rear of the section, it will be necessary to determine its moment of inertia about a horizontal axis passing through its center of gravity. The true shape of the section is given in the

right half of Fig. 53 and, while quite irregular, it may be divided approximately into a number of regular parts whose bases are horizontal as shown in the left half of the figure. Such approximations as these are generally necessary to determine the moments of inertia of irregular sections. It will be observed that the approximate figures include in some cases small areas not found in the real section and also that some small areas included in the real section are omitted in the approximate figures. When this is necessary the areas lost and gained should balance each other both in extent and position so far as possible, in order that the moment of inertia computed from the approximate regular figures shall vary as little as possible from that of the real section.

To determine the position of a horizontal line passing approximately through the center of gravity of the real section consider only the left half of the figure, since the real section is symmetrical with respect to its vertical axis, and take moments with respect to the horizontal line AB .

Whence

$$\begin{aligned}
 & 1.36 \times \frac{.48}{2} \left\{ 3.42 + \frac{.48}{3} \right\} + 1.36 \times \frac{.48}{2} \left\{ 3.42 - \frac{.48}{3} \right\} \\
 & + 1.74 \times \frac{.43}{2} \left\{ 2.906 + \frac{.43}{3} \right\} + .41 \times \frac{.14}{2} \left\{ 2.94 + \frac{.14}{3} \right\} \\
 & + .41 \times .38 \left\{ 2.56 + \frac{.38}{2} \right\} + .4 \times \frac{.14}{2} \left\{ 2.56 + \frac{.14}{3} \right\} \\
 & + .59 \times \frac{.57}{2} \left\{ 2.56 - \frac{.57}{3} \right\} + .202 \times 2.56 \times \frac{2.56}{2} - \\
 & \frac{\pi [5.8125 + 2 \times .187]^2}{16} \times .4244 \times \frac{5.8125 + 2 \times .187}{2} \\
 & + \frac{\pi (5.8125)^2}{16} \times .4244 \times \frac{5.8125}{2} = (\text{Area of half section}) \bar{y}
 \end{aligned}$$

The total area of the half section is

$$\begin{aligned}
 & 1.36 \times .48 + 1.74 \times \frac{.43}{2} + .41 \times \frac{.14}{2} + .41 \times .38 + .4 \times \frac{.14}{2} \\
 & + .59 \times \frac{.57}{2} + .202 \times 2.56 + \frac{\pi [5.8125 + 2 \times .187]^2}{16} \\
 & - \frac{\pi (5.8125)^2}{16} = 2.806 \text{ sq. ins.}
 \end{aligned}$$

and

$$\bar{y} = 1.190 \text{ ins.}$$

The area of the whole section is 5.61 sq. ins.

The moment of inertia of the half section with respect to the horizontal axis passing through its center of gravity is

$$\begin{aligned}
 & \frac{1.36 \times (.48)^3}{36} + 1.36 \times \frac{.48}{2} \left\{ 3.42 + \frac{.48}{3} - 1.190 \right\}^2 \\
 & + \frac{1.36 \times (.48)^3}{36} + 1.36 \times \frac{.48}{2} \left\{ 3.42 - \frac{.48}{3} - 1.190 \right\}^2 \\
 & + \frac{1.74 \times (.43)^3}{36} + 1.74 \times \frac{.43}{2} \left\{ 2.906 + \frac{.43}{3} - 1.190 \right\}^2 \\
 & + \frac{.41 \times (.14)^3}{36} + .41 \times \frac{.14}{2} \left\{ 2.94 + \frac{.14}{3} - 1.190 \right\}^2 \\
 & + \frac{.41 \times (.38)^3}{12} + .41 \times .38 \left\{ 2.56 + \frac{.38}{2} - 1.190 \right\}^2 \\
 & + \frac{.4 \times (.14)^3}{36} + .4 \times \frac{.14}{2} \left\{ 2.56 + \frac{.14}{3} - 1.190 \right\}^2 \\
 & + \frac{.59 \times (.57)^3}{36} + .59 \times \frac{.57}{2} \left\{ 2.56 - \frac{.57}{3} - 1.190 \right\}^2 \\
 & + \frac{.202 \times (2.56)^3}{12} + .202 \times 2.56 \left\{ \frac{2.56}{2} - 1.190 \right\}^2 \\
 & + \frac{.110}{2} \left\{ \frac{5.8125 + 2 \times .187}{2} \right\}^4 + \frac{\pi}{16} [5.8125 + 2 \times .187]^2 \\
 & \times \left\{ .4244 \times \frac{5.8125 + 2 \times .187}{2} + 1.190 \right\}^2 - \frac{.110}{2} \times \left\{ \frac{5.8125}{2} \right\}^4 \\
 & - \frac{\pi}{16} (5.8125)^2 \times \left\{ .4244 \times \frac{5.8125}{2} + 1.190 \right\}^2 = 14.618 \text{ ins.}^4
 \end{aligned}$$

The moment of inertia of the whole section with respect to the horizontal axis through its center of gravity is, therefore,

$$I = 29.236 \text{ ins.}^4$$

Bending Moment and Stress in Section 1-1. — The bending moment at this section is obtained by taking the algebraic sum of the moments of the forces acting on the right (to the rear) of it. In doing this it will be found that while the forces are greater when recoil of the gun is about to end, the resultant bending moment is slightly less than when the gun is just commencing to recoil. The stresses under the latter assumption will, therefore, be computed. In the computations which follow the weight

of the cradle will be neglected as only that part of it in rear of the section produces a bending moment. This bending moment is slight and neglecting it increases the resultant bending moment at the section, which is an error on the safe side.

In determining the bending moment it should be noted that only that part of F which is due to the friction produced by B must be considered since the part due to the friction produced by A acts on the left (in front) of the section. Furthermore the lever arm of $.15 B$ must be taken with reference to the horizontal axis through the center of gravity of the section which, from Fig. 53, is 1.190 ins. above the action line of the force P , or the axis of the lower, cylindrical, part of the cradle. The bending moment at the section is, then, see Figs. 52 and 53,

$$M = E \times 18.625 - B \times 26.125 - .15 B (3.506 - 1.190) = 17832 \text{ in. lbs.}$$

since

$$E = 2248 \text{ lbs. and } B = 908 \text{ lbs.}$$

The lowest point of the section is at the greatest distance from the neutral axis, this distance being

$$1.190 + \frac{5.8125}{2} + .187 = 4.283 \text{ ins.}$$

and the bending stress at that point is

$$S'''y = 17832 \times 4.283 / 29.236 = 2612 \text{ lbs. per sq. in. tension.}$$

The force $.15 B$ produces a tensile stress distributed uniformly over the section equal to $136 / 5.61 = 24$ lbs. per sq. in. which increases very slightly the tension below the neutral axis and decreases by the same amount the compression above it.

The forces B and E produce a shearing stress in the section which is greatest when recoil is about to end. At this time its intensity is, see equations (40) and (31), chapter II,

$$[3511 - 1885] / 5.61 = 290 \text{ lbs. per sq. in.}$$

Stresses in Section 2-2 of the Cradle.—This section is indicated in Figs. 16 and 52. It is practically identical in shape and area with section 1-1, Fig. 53, and, therefore, the area, position of neutral axis, and moment of inertia determined for that section will also serve for it. The stresses in the section are greatest when the gun is just commencing to recoil.

The forces P and $.15 A$, see Fig. 52, produce a stress of compression uniformly distributed over the section which is equal to

$$[4988 + 15] / 5.61 = 892 \text{ lbs. per sq. in.}$$

The section is also subjected to a bending moment which may be found by taking the algebraic sum of the moments of the forces acting on the left (in front) of it. Neglecting the weight of the cradle as in the case of section 1-1 and noting by reference to Fig. 53 that the force P has a lever arm of 1.190 ins. with respect to the neutral axis of the section, we obtain, see Fig. 52,

$$M = A \times 43.50 + .15 A (3.506 - 1.190) - P \times 1.190 = -1507 \text{ in. lbs.}$$

since

$$A = 101 \text{ lbs. and } P = 4988 \text{ lbs.}$$

The maximum bending stress in the section occurs at its lowest point and is

$$S'''y = 1507 \times 4.283 / 29.236 = 221 \text{ lbs. per sq. in. compression.}$$

The latter stress increases the compressive stress uniformly distributed over the section so that the maximum resultant stress is

$$892 + 221 = 1113 \text{ lbs. per sq. in. compression}$$

and occurs at the lowest point of the section.

The force A also produces a shearing stress in the section which is greatest when the gun is about .5 in. from the end of its recoil. At this time the intensity of A is practically 1078 lbs. and the corresponding shearing stress is

$$1078 / 5.61 = 192 \text{ lbs. per sq. in.}$$

It will be observed that the cradle, so far as the stresses in it due to the firing of the gun are concerned, could be made much lighter, but its walls are now only .187 in. thick and if made much thinner they would be likely to be deformed and injured in rapid transportation or maneuvering of the battery.

Stresses in the Pintle and in the Rivets fastening it to the Cradle. — Fig. 54 shows the pintle of the 3-inch field carriage with the forces C and D acting thereon.

The pintle is fastened to the cradle by twenty-three .375-in. rivets through the rivet holes shown in the figure. The

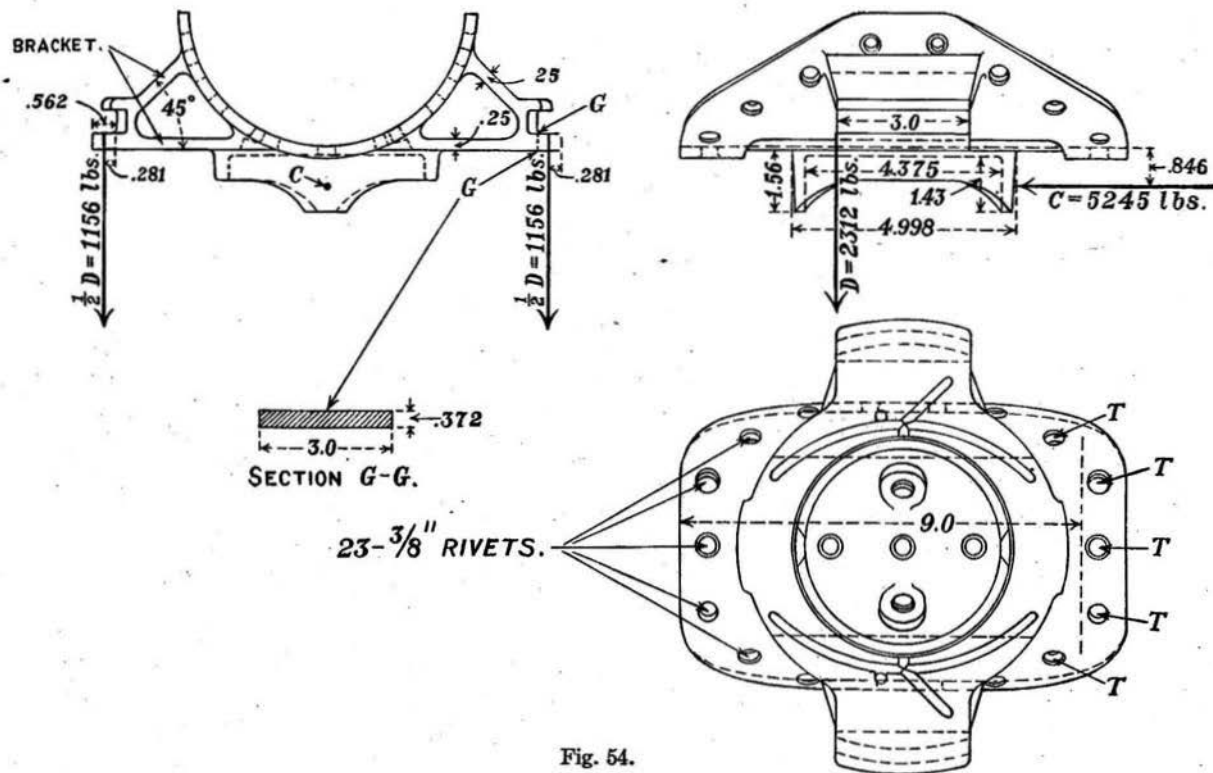


Fig. 54.

greatest value of the force C (5245 lbs.) occurs when the gun is just commencing to recoil while that of the force D (2312 lbs.) occurs when recoil is about to end. These values will be used in the computations that follow.

Stresses Caused by the Force C . — The lower projecting part of the pintle against which the force C acts is practically cylindrical in shape (it has a very slight taper) and forms a cantilever 1.56 ins. long over which the force is uniformly distributed. The dangerous section, see Fig. 54, occurs at a distance of 1.43 ins. above the lower edge of the pintle and the bending moment there is, from equation (18),

$$M = \frac{5245}{1.56} \times \frac{(1.43)^2}{2} = 3438 \text{ in. lbs.}$$

The dangerous section is a circular ring, its outer diameter being approximately 4.998 ins. and its inner diameter 4.375 ins. Its moment of inertia about its neutral axis is, therefore,

$$.0491 \{ (4.998)^4 - (4.375)^4 \} = 12.651 \text{ ins.}^4$$

and the bending stress at the extreme fibre is

$$S'''y = \frac{3438 \times 4.998 / 2}{12.651} = 679 \text{ lbs. per sq. in.}$$

compression at the front of the section and tension at the rear.

The shearing stress in the section is

$$\frac{5245 \times 1.43 / 1.56}{.7854 \{ (4.998)^2 - (4.375)^2 \}} = 1049 \text{ lbs. per sq. in.}$$

The force C tends to shear the twenty-three rivets fastening the pintle to the main body of the cradle. These rivets are .375 in. in diameter and consequently .11045 sq. in. in area. The shearing stress in them is

$$\frac{5245}{23 \times .11045} = 2065 \text{ lbs. per sq. in.}$$

C also tends to crush the rivets and the walls of the rivet holes in the pintle and the cradle. As the thickness of the cradle is less than that of the pintle the greater crushing stress will occur between the rivets and the walls of the rivet holes in the cradle. The crushing stress between a rivet and the walls of the rivet hole

is considered to be uniformly distributed over an area equal to the thickness of the plate multiplied by the diameter of the rivet. Under this assumption the crushing stress due to the force C is

$$\frac{5245}{23 \times .187 \times .375} = 3252 \text{ lbs. per sq. in. compression,}$$

since there are twenty-three rivets having a diameter of .375 in., and the thickness of the cradle wall is .187 in.

The force C also tends to rotate the pintle about the line cut from the surface of contact with the cradle by a plane perpendicular to the axis of the cradle passing through the center of the pintle. This tendency to rotation is mainly resisted by the compression between the contact surfaces of the cradle and pintle in front of the central plane, and by the stress in the rivets behind that plane. The manner of distribution of the stress in these rivets is not unlike that of the tensile stress on one side of the neutral axis of a section of a beam subjected to a bending force. The greatest stress will occur in the rivets at the rear of the pintle and will be mainly a tensile stress. It can be obtained approximately with the error on the safe side in the following manner: Assume that the tendency of the pintle to rotate under the action of the force C is resisted entirely by the pressure along the line of contact at the front edge of the pintle and by the tension in the five rear rivets marked T in Fig. 54. Assume also that the mean distance of the five rivets from the front edge of the pintle is 9 ins. From Fig. 54 the lever arm of C with respect to the line of contact at this edge is .846 in. Now since the pintle is in equilibrium, the moments of the forces about the line of contact at the front edge must be zero and, calling T the force which the five rivets oppose to the rotation of the pintle, we have

$$T \times 9 = 5245 \times .846$$

or

$$T = 493 \text{ lbs.}$$

and the tensile stress in each of the five rivets is

$$\frac{493}{5 \times .11045} = 893 \text{ lbs. per sq. in.}$$

Stresses Caused by the Force $D/2$. — The force $D/2 = 1156$ lbs. acts downward on the lower lip of each wing of the pintle

as shown in Fig. 54. These lips may be considered as cantilevers .562 in. long, approximately 3 ins. wide, and .372 in. deep with the forces uniformly distributed over them. The area of the dangerous section *GG* shown in Fig. 54 is 1.116 sq. ins. and its moment of inertia with respect to its neutral axis is .01287 in.⁴ The bending moment at this section is, from equation (19),

$$\frac{1156}{.562} \times \frac{(.562)^2}{2} = 324.8 \text{ in. lbs.}$$

and the maximum bending stress is

$$S'''y = 324.8 \times .186 / .01287 = 4695 \text{ lbs. per sq. in.}$$

The shearing stress in the section is

$$1156 / 1.116 = 1036 \text{ lbs. per sq. in.}$$

The bracket connecting the wing to the rest of the pintle, see Fig. 54, acts as a triangular frame supporting the force $D/2$. The upper arm of the bracket makes an angle of 45° with the lower arm, and the force is perpendicular to the lower arm. The total stress in the upper arm is, therefore, one of tension equal to

$$\frac{1}{2} D / \cos 45^\circ = 1635 \text{ lbs.}$$

The greatest stress per unit of area in the upper arm occurs at the smallest section thereof which is .25 in. thick and 3 in. wide. This stress is

$$\frac{1635}{.25 \times 3} = 2180 \text{ lbs. per sq. in.}$$

The total stress in the lower arm is one of compression equal to $D/2 = 1156$ lbs. The area of cross-section of the lower arm being

$$.25 \times 3 = .75 \text{ sq. in.}$$

the stress per unit of area is

$$1156 / .75 = 1541 \text{ lbs. per sq. in.}$$

The total tensile stress of 1635 lbs. in the upper arm of the bracket puts a tensile stress in a number of the rivets fastening the pintle to the main body of the cradle. This stress is mainly confined to the four rivets near the junction of the upper arm with

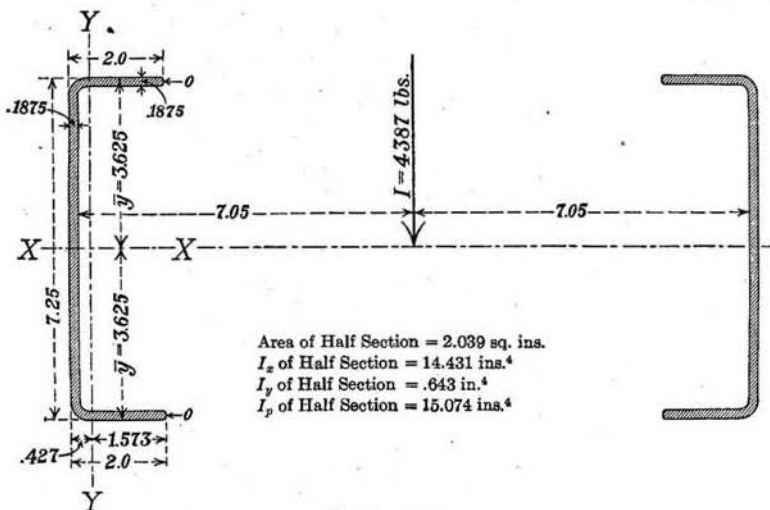
the body of the pintle and, assuming it to be entirely so confined and uniformly distributed over the four rivets, its intensity is

$$\frac{1635}{4 \times .11045} = 3701 \text{ lbs. per sq. in.}$$

Maximum Resultant Stress in the Rivets. — To obtain the maximum resultant stress in the rivets the simple stresses occurring in them should be combined in accordance with the methods already explained, but on account of the low intensities of the simple stresses this is unnecessary.

STRESSES IN SECTION 3-3 OF THE TRAIL.

80. Forces Causing Stress in the Section. — **Area and Moment of Inertia of the Section.** — This section is indicated in Fig. 18 and shown in Fig. 55. It is taken through the elevating screw



Section 3-3.

Fig. 55.

bearings and, as the force $I = 4387$ lbs. is contained in its plane, it is the dangerous section of the trail. The stresses in it are greatest when recoil is about to end and they will be determined under that condition.

The stresses in the section, see Figs. 18 and 55, are compression and bending due to the force $S = 3938$ lbs., bending and shear

due to the force $T = 2139$ lbs., and torsion due to the force $I = 4387$ lbs. The force I lies in the plane of the section and has no bending moment with respect to it. Because of the torsion produced by I , which is somewhat special and will be explained later, it will simplify the problem to consider each half of the symmetrical section 3-3 by itself, in which case the forces acting at the spade should be taken as

$$S/2 = 1969 \text{ lbs. and } T/2 = 1069.5 \text{ lbs.}$$

The area of the half section is 2.039 sq. ins. Its center of gravity is given by the intersection of the horizontal and vertical axes XX and YY , the former being at a distance of 3.625 ins. from the lower edge of the section and the latter at a distance of .427 in. from the outer edge. The moment of inertia I_x about the axis XX is 14.431 ins.⁴, $I_y = .643$ in.⁴, and $I_p = 15.074$ ins.⁴.

Stresses in Section 3-3 due to the Forces S and T .—The compression in the half section due to the force $S/2$ is

$$1969/2.039 = 966 \text{ lbs. per sq. in.}$$

The lever arm of $T/2$ with respect to the half section, see Fig. 18, is

$$112.19 - 21.55 = 90.64 \text{ ins.}$$

and that of $S/2$ with respect to the neutral axis XX , see Figs. 18 and 55, is

$$4 + 16 + 3.625 = 23.625 \text{ ins.}$$

The resulting bending moment at the half section is, then,

$$1069.5 \times 90.64 - 1969 \times 23.625 = 50422 \text{ in. lbs.}$$

and the maximum bending stress is

$$S''y = 50422 \times 3.625/14.431 = 12666 \text{ lbs. per sq. in.}$$

compression at the upper and tension at the lower edge of the section. The shearing stress in the half section due to the force $T/2$ is

$$1069.5/2.039 = 525 \text{ lbs. per sq. in.}$$

Torsional Stress in Section 3-3 caused by the Force I .—Referring to Fig. 55, it will be seen that the force I , which is the thrust on the elevating screw, acts midway between the

two flasks of the trail. It is transmitted to each flask by two cross transoms riveted thereto which together form a fixed beam with the force applied at its middle. The length of the beam is the distance between the webs of the flasks equal to 14.1 ins. The bending moment at the ends of the transoms under these conditions is given by equation (11) and is

$$M = 4387 \times 14.1 / 8 = 7732 \text{ in. lbs.}$$

This bending moment will produce a twisting couple of equal value on each flask which will cause a torsional stress in the half section 3-3 and in the sections on each side of it, between it and the axle and between it and the tool box. The axle and the plates forming the top, bottom, and front end of the tool box prevent twisting in the parts of the flasks connected by them so that the right sections of the flasks tangent to the axle at its rear and those at the front end of the tool box may be considered as fixed ends so far as the torsional stress in the flasks is concerned.

The distance between the half section 3-3 and the axle is 20.902 ins. and between it and the front end of the tool box the distance is 15.224 ins., so that the torsional moment in the half section is, from equation (2),

$$M_t = 7732 \times 20.902 / 36.126 = 4474 \text{ in. lbs.}$$

and the maximum torsional stress, which occurs at the points *O*, Fig. 55, is

$$rS'''_t = M_t / I_p = 4474 \sqrt{(3.625)^2 + (1.573)^2} / 15.074 = 1173 \text{ lbs. per sq. in. shear.}$$

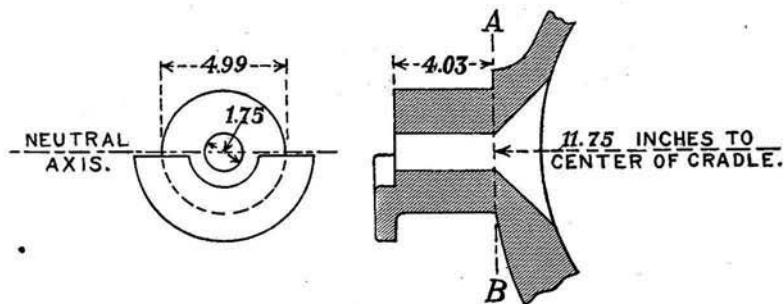
Combination of Compressive, Bending, and Torsional Stresses in Section 3-3. — The compressive stress of 966 lbs. per sq. in. is uniformly distributed over the section; it therefore increases the compression of 12666 lbs. per sq. in. at the upper edge of the section, due to the bending stress, to 13632 lbs. per sq. in., and reduces the tension at the lower edge to 11700 lbs. per sq. in. The compression of 13632 lbs. per sq. in. being the greater stress, it will be combined with the torsional stress of 1173 lbs. per sq. in. at *O* to obtain the maximum combined stress in the section. From equation (24) this maximum stress is found to be

$$S_{tc} = \frac{13632}{2} + \sqrt{(1173)^2 + \frac{(13632)^2}{4}} = 13732 \text{ lbs. per sq. in.}$$

compression at *O* on the upper edge of the section. The torsional stress at *O* is so slight that it will not be necessary to determine how much it is increased by the compression at that point.

**STRESSES IN PARTS OF THE 5-INCH BARBETTE
CARRIAGE, MODEL OF 1903.**

81. **Stresses in the Trunnions of the Cradle.**—The cradle may be considered as a beam carried in the trunnion beds of the pivot yoke. The diameter of the trunnions of the cradle, see Fig. 56, is .01 in. smaller than the diameter of the trunnion beds and this fact, in connection with the immense depth of the sections of the cradle in a direction parallel to the action lines of the principal forces acting on it, makes of the cradle a beam merely supported at its ends instead of a fixed beam. A fixed beam is one whose ends are held so firmly by the supports that they cannot accommodate themselves to any deflection of the part of the beam between the supports caused by the bending



Area of Section *AB* = 17.15 sq. ins.

I of Section *AB* = 29.98 ins.⁴

Fig. 56.

forces acting upon it. A free beam is one whose ends are merely supported in such manner that they can freely accommodate themselves to the deflection of the part of the beam between the supports. By the methods discussed in works on strength of materials it can be shown that the maximum deflection of the cradle, which occurs midway between the trunnion beds, is less than .00002 in. so that it is entirely negligible, and the clearance between the trunnions and their beds is many times greater than it need be to permit the trunnions to conform to the deflection of the rest of the cradle.

One of the trunnions of the cradle is shown in Fig. 56.

The forces acting on each trunnion, see Fig. 21, are the horizontal force $C/2 = 46647$ lbs. and the vertical force $D/2 = 5359$ lbs. The resultant of these forces is

$$\sqrt{(46647)^2 + (5359)^2} = 46954 \text{ lbs.}$$

It is uniformly distributed over the surface of contact between the trunnion and its bed in the pivot yoke. This surface is 4 ins. long, being .03 in. shorter than the trunnion to allow a clearance of .015 in. between the rimbase and the side of the trunnion bed on the inside, and between the half collar on the end of the trunnion and the side of the trunnion bed on the outside.

The bending moment at section AB is greater than at any other section of the trunnion. It may be obtained by considering the resultant force concentrated at the middle of the trunnion length, whence

$$M = 46954 \times 2.015 = 94612 \text{ in. lbs.}$$

and the corresponding maximum bending stress is

$$S''y = \frac{94612 \times 2.495}{29.98} = 7874 \text{ lbs. per sq. in.}$$

tension at the rear and compression at the front of the section.

The shearing stress in section AB is

$$46954/17.15 = 2738 \text{ lbs. per sq. in.}$$

Stress in the Recoil Cylinder of the Cradle due to the Interior Hydraulic Pressure. — The cylinder liner in which the recoil grooves are cut is made in halves and adds nothing to the strength of the cylinder, which outside of the liner has an inner diameter of 7.5 ins. and an outer diameter of 11 ins. The force $P = 84665$ lbs., see Fig. 21, is the sum of the forces exerted by the piston-rod of the recoil cylinder and the spring rods of the spring cylinders. The force exerted by the spring rods, which is due to the springs, is least when the gun is just commencing to recoil, at which time it has a value of 6540 lbs., so that the corresponding force exerted by the piston-rod of the recoil cylinder is 78145 lbs. The piston of the recoil cylinder has a diameter of 6.49 ins. and the rod a diameter of 3 ins. and consequently the area against which the oil in the cylinder must act to produce a total force of 78145 lbs. is

$$.7854 \{ (6.49)^2 - (3)^2 \} = 26.01 \text{ sq. ins.}$$

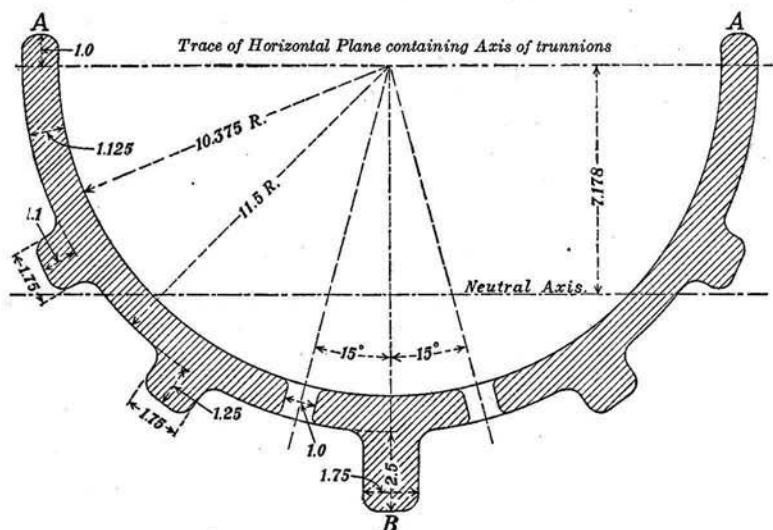
and the corresponding interior pressure per sq. in. in the cylinder is

$$78145/26.01 = 3004 \text{ lbs. per sq. in.}$$

The maximum stress in the cylinder due to this interior pressure is one of tension and occurs at its inner surface. It is as follows:

$$\theta = \frac{2(3.75)^2 + 4(5.5)^2}{3\{(5.5)^2 - (3.75)^2\}} \times 3004 = 9226 \text{ lbs. per sq. in.}$$

Stresses in Section 4-4 of the Cradle. — This section, taken just in front of the spring cylinders, is indicated in Fig. 21 and shown in Fig. 57.



Area = 51.24 sq. ins. $I = 729.85 \text{ ins.}^4$

Section 4-4.

Fig. 57.

There is a bending moment at the section, see Fig. 21, which may be obtained by considering all the forces acting to the right (in rear) of it. The weight of the cradle will be neglected as only a part of it acts to the right of the section and neglecting the weight increases the bending moment slightly. The bending moments of the forces perpendicular to the section must be taken with respect to its neutral axis which is 7.178 ins. below the horizontal plane containing the axis of the trunnions.

Whence

$$M = 84665 (15.75 - 7.178) + 4662 (10 - 7.178) \\ - 31084 (44.875 - 20) + 4311 \cos 14^\circ (35 \cos 14^\circ - 20) \\ + 4311 \sin 14^\circ (35 \sin 14^\circ - 7.178) = 25433 \text{ in. lbs.}$$

and the corresponding maximum bending stresses are

$$S'''y = 25433 \times 8.178 / 729.85 = 285 \text{ lbs. per sq. in.}$$

compression at *A* and *A*, Fig. 57, and

$$S'''y = 25433 \times 6.822 / 729.85 = 238 \text{ lbs. per sq. in.}$$

tension at *B*, Fig. 57.

It will be noted that while the bending moments of the individual forces at this section are large, they nearly neutralize each other so that the resultant bending moment is small and, in connection with the large moment of inertia of the section, produces very little bending stress.

The forces *P*, *F'*, and *E* sin 14° produce a tensile stress in the section equal to

$$[84665 + 4662 + 4311 \sin 14^\circ] / 51.24 = 1764 \text{ lbs. per sq. in.}$$

The forces *B* and *E* cos 14° produce a shearing stress in the section equal to

$$[31084 - 4311 \cos 14^\circ] / 51.24 = 525 \text{ lbs. per sq. in.}$$

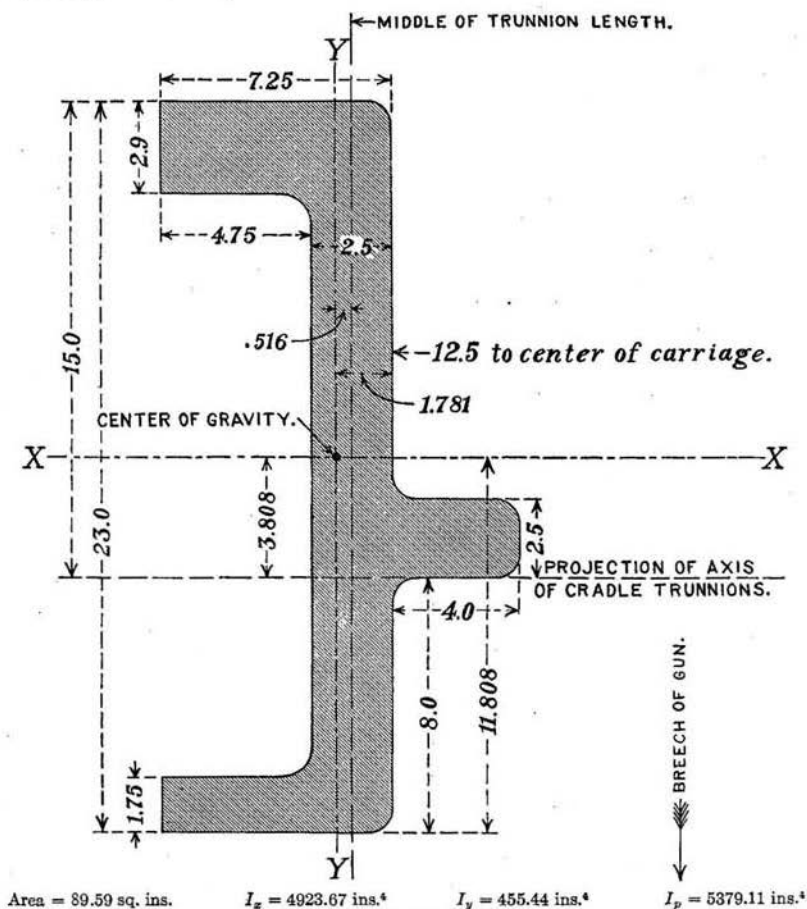
As the force *E* is applied at a distance of 16.125 ins. to the left of the center of the section, its horizontal component causes a bending stress in the section about a vertical neutral axis, and its vertical component causes a torsional stress. Owing to the large moments of inertia of the section these stresses will be small and will, therefore, not be computed.

The maximum combined stress in the section occurs at *B* and is 2002 lbs. per sq. in. tension obtained by adding the tension due to the bending stress and that due to the direct action of the forces *P*, *F'*, and *E* sin 14°. If the torsion be considered this maximum stress will be increased slightly.

STRESSES IN THE PIVOT YOKE.

82. Stresses in Section 5-5 of the Pivot Yoke. — This section is indicated in Figs. 22 and 23 and shown in Fig. 58.

It is selected rather than one at which the bending moment is greater because the increase in the strength of the sections as we pass further down is greater than the increase of the bending moment.



Section 5-5.

Fig. 58.

Forces Producing Stress in Section 5-5.—Neglecting the weight of the pivot yoke, most of which is applied below the section, the forces acting are $C/2$, $D/2$, and E . The whole of the force E is to be considered instead of one-half of it because it

is applied, through the elevating and platform brackets, to the left side of the pivot yoke only.

The force $C/2 = 46647$ lbs. produces a bending stress about the axis XX , Fig. 58, a shearing stress, and a torsional stress. Its lever arm in bending, see Fig. 22, is 11.7 ins. Its lever arm in torsion, see Fig. 58, is .516 in. obtained by considering the force concentrated at the middle of the trunnion length and measuring the distance from its action line to a line perpendicular to the plane of the section passing through its center of gravity.

The force $D/2 = 5359$ lbs. produces a bending stress about the axis XX , a stress of compression, and a bending stress about the axis YY , Fig. 58. Its lever arm with respect to the axis XX is 3.808 ins., and with respect to the axis YY it is .516 in., the lever arms being determined by considering the force concentrated at the axis of the trunnions and at the middle of the trunnion length.

The force E acts on the left side of the carriage only, being applied to the teeth of the elevating pinion mounted on a short horizontal shaft carried in bearings in the elevating bracket. The elevating bracket is bolted to the platform bracket and the latter is bolted by two upper and two lower bolts to the rear face of the upright arm of the pivot yoke. E tends to rotate the platform bracket around the lower edge of the surface of contact between it and the pivot yoke. This tendency is resisted by the tension in the two top bolts fastening the bracket to the pivot yoke. The lever arm of E with respect to the edge about which the bracket tends to rotate being 21.61 ins. and that of the tension in the two top bolts being 20 ins., the intensity of the tension, which will be called T_h , is

$$T_h = 4311 \times 21.61 / 20 = 4658 \text{ lbs.}$$

This tension is transmitted to the pivot yoke as a horizontal force whose point of application may be taken as midway between the centers of the two top bolts fastening the platform bracket to the pivot yoke. This point is 19 ins. to the left of the center of the carriage and 3.25 ins. below the axis of the trunnions.

The bracket is prevented from moving downward under the action of the vertical component of E by the four bolts fastening the bracket to the pivot yoke. As the shearing stress may be

considered as uniformly distributed over the four bolts, the total shearing stress in the two top bolts is

$$4311 \cos 14^\circ / 2 = 2092 \text{ lbs.}$$

This shearing stress, which will be called T_v , is transmitted to the pivot yoke as a vertical force, its point of application being the same as that of T_h .

The two bottom bolts and the lower edge of the surface of contact between the bracket and the pivot yoke being below section 5-5, the shearing stress in the bottom bolts and the pressure along the contact edge must not be considered in determining the stresses in the section.

The force $T_h = 4658$ lbs. causes a bending stress about the axis XX , a shearing stress, and a torsional stress. Its point of application being 3.25 ins. below the axis of the trunnions, its lever arm in bending is $11.7 - 3.25 = 8.45$ ins. As its point of application is also 19 ins. to the left of the center of the carriage, its lever arm in torsion, see Fig. 58, is $19 - 12.5 - 1.781 = 4.719$ ins. The torsional stress produced by T_h is opposed to that produced by $C/2$.

The force $T_v = 2092$ lbs. causes a bending stress about the axis XX , a stress of compression, and a bending stress about the axis YY . Its point of application being in the vertical plane containing the rear face of the upright arm of the pivot yoke, its lever arm with respect to the axis XX , see Fig. 58, is 11.808 ins. As its point of application is also 19 ins. to the left of the center of the carriage, its lever arm with respect to the axis YY is $19 - 12.5 - 1.781 = 4.719$ ins.

Bending Stress in Section 5-5 about XX as a Neutral Axis. — The bending moment is

$$M = 46647 \times 11.7 + 5359 \times 3.808 + 4658 \times 8.45 + 2092 \times 11.808 = 630239 \text{ in. lbs.}$$

and the corresponding bending stresses are

$$S'''y = 630239 \times 11.808 / 4923.67 = 1511 \text{ lbs. per sq. in.}$$

compression at the rear edge of the section, and

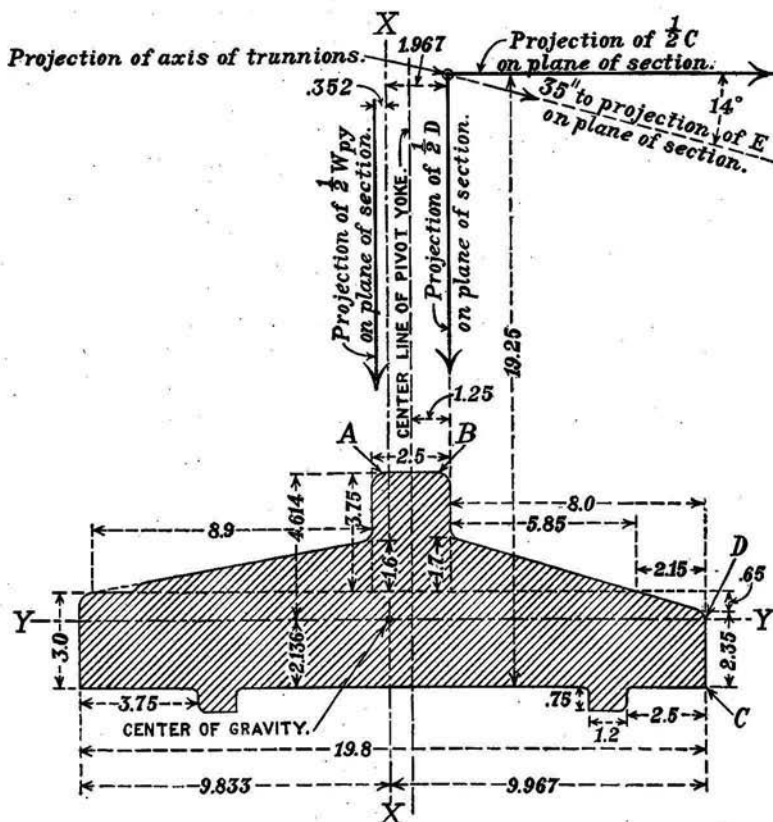
$$S'''y = 630239 \times 11.192 / 4923.67 = 1433 \text{ lbs. per sq. in.}$$

tension at the front edge of the section.

Other Stresses. — The shearing stress is

$$[46647 + 4658] / 89.59 = 573 \text{ lbs. per sq. in.}$$

It is apparent by inspection that the compressive stress, the



Area = 81.96 sq. ins.

$I_x = 2164.66 \text{ ins.}^4$

$I_y = 186.91 \text{ ins.}^4$

$I_p = 2351.57 \text{ ins.}^4$

Section 6-6.

Fig. 59.

bending stress about the axis YY , and the torsional stress are so small that it is unnecessary to compute them.

Stresses in Section 6-6 of the Pivot Yoke. — This section is indicated in Fig. 23 and shown in Fig. 59.

Forces Producing Stress in Section 6-6. — The force $C/2 = 46647$ lbs., see Figs. 22 and 23, produces in the section a torsional stress, a bending stress about the axis XX , and a shearing stress. Its lever arm in torsion is the perpendicular distance between its action line and the line normal to the plane of the section passing through its center of gravity, which distance, see Fig. 59, is $19.25 - 2.136 = 17.114$ ins. Its lever arm in bending is the perpendicular distance between its action line and the plane of the section equal, see Fig. 23, to 5.75 ins.

The force $D/2 = 5359$ lbs., see Figs. 22 and 23, produces in the section a torsional stress, a bending stress about the axis YY , and a shearing stress. Its lever arm in torsion, see Fig. 59, is $9.967 - 8.0 = 1.967$ ins. Its lever arm in bending, see Fig. 23, is 5.75 ins.

The greater part of the weight of the pivot yoke and shields is applied above this section so that $W_{pv}/2 = 7967$ lbs. will be considered as acting upon it. It produces in the section a torsional stress, a bending stress about the axis YY , and a shearing stress. Its lever arm in torsion, see Figs. 23 and 59, is .352 in. Its lever arm in bending, see Fig. 23, may be taken as approximately 11.75 ins.

The force $E \sin 14^\circ = 1043$ lbs. produces in the section a torsional stress, a bending stress about the axis XX , and a shearing stress. Its lever arm in torsion, see Figs. 22 and 59, is $19.25 - 35 \sin 14^\circ - 2.136 = 8.647$ ins. Its lever arm in bending, see Fig. 23, is 8.125 ins.

The force $E \cos 14^\circ = 4183$ lbs. produces in the section a torsional stress, a bending stress about the axis YY , and a shearing stress. Its lever arm in torsion, see Fig. 59, is $35 \cos 14^\circ + 1.967 = 35.927$ ins. Its lever arm in bending, see Fig. 23, is 8.125 ins.

Bending Stress in Section 6-6 about XX as a Neutral Axis. — The bending moment is

$$46647 \times 5.75 - 1043 \times 8.125 = 259746 \text{ in. lbs.}$$

and the corresponding maximum bending stress, see Fig. 59, is

$$259746 \times 9.967 / 2164.66 = 1196 \text{ lbs. per sq. in.}$$

compression occurring at the surface CD .

Bending Stress in Section 6-6 about YY as a Neutral Axis. — The bending moment is

$$5359 \times 5.75 + 7967 \times 11.75 + 4183 \times 8.125 = 158413 \text{ in. lbs.}$$

and the corresponding maximum bending stress, see Fig. 59, is

$$158413 \times 4.614 / 186.91 = 3911 \text{ lbs. per sq. in.}$$

tension occurring at the surface *AB*.

The bending stress at the point *C*, Fig. 59, due to this bending moment is

$$158413 \times 2.136 / 186.91 = 1810 \text{ lbs. per sq. in. compression.}$$

Torsional Stress in Section 6-6. — The torsional moment is

$$46647 \times 17.114 + 5359 \times 1.967 - 7967 \times .352 - 1043 \times 8.647 \\ + 4183 \times 35.927 = 947318 \text{ in. lbs.}$$

and the corresponding maximum torsional stress, which occurs at *C*, is

$$947318 \sqrt{(9.967)^2 + (2.136)^2} / 2351.57 = 4106 \text{ lbs. per sq. in.}$$

Shearing Stress in Section 6-6. — The shearing stress due to the forces *C*/2 and *E* sin 14° is

$$[46647 - 1043] / 81.96 = 556 \text{ lbs. per sq. in.,}$$

and that due to the forces *D*/2, *W_{vu}*/2, and *E* cos 14° is

$$[5359 + 7967 + 4183] / 81.96 = 214 \text{ lbs. per sq. in.}$$

As these two shearing stresses are due to forces whose action lines are perpendicular to each other, the resultant shearing stress is

$$\sqrt{(556)^2 + (214)^2} = 596 \text{ lbs. per sq. in.}$$

Maximum Resultant Stress in Section 6-6. — The maximum resultant stress in the section occurs at *C*. It is a compressive stress due to combining the torsional stress at this point with the resultant compressive stress there caused by the bending moments about the axes *XX* and *YY*, respectively. The stress due to the bending moment about the axis *XX* being 1196 lbs. per sq. in. compression and that due to the bending moment about *YY* being 1810 lbs. per sq. in. compression, their resultant is $1196 + 1810 = 3006$ lbs. per sq. in. compression. Combining this with the

torsional stress of 4106 lbs. per sq. in. at C by equation (24) we have

$$S_{lc} = (3006/2) + \sqrt{(4106)^2 + (3006)^2/4} = 5876 \text{ lbs. per sq. in.}$$

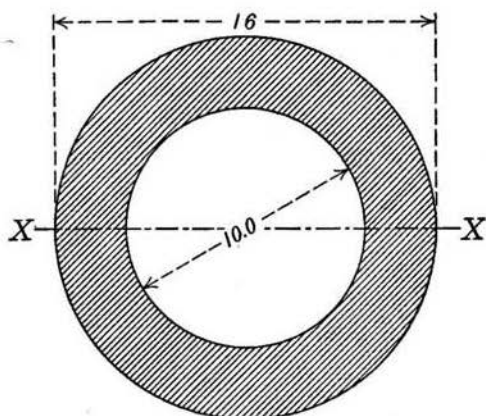
compression as the maximum resultant stress in the section.

The resultant shearing stress at C due to the torsion and the compression is, from equation (25),

$$S_{sc} = \sqrt{(4106)^2 + (3006)^2/4} = 4373 \text{ lbs. per sq. in.}$$

The shearing stress caused by the bending forces (but not the torsional stress) in reality varies from a maximum at the neutral axis to zero at the extreme fiber and is, therefore, negligible at the point C .

Stresses in Section 7-7 of the Stem of the Pivot Yoke. — This section is indicated in Fig. 22 and shown in Fig. 60. Con-



Area = 122.52 sq. ins.

$I = 2726.79 \text{ ins.}^4$

Section 7-7.

Fig. 60.

sidering only the forces applied below the section, the force $H = 62807$ lbs., see Fig. 22, produces bending and shear in the section, and the force $I = 30833$ lbs. produces bending and compression. The weight of the pivot yoke is not taken into account as it is considered as applied above this section.

The total bending moment is

$$62807 \times 28 + 30833 \times 7 = 1974427 \text{ in. lbs.}$$

and the corresponding maximum bending stress is, see Fig. 60,

$$1974427 \times 8 / 2726.79 = 5793 \text{ lbs. per sq. in.}$$

compression at the rear and tension at the front of the section.

The shearing stress due to the force H is

$$62807 / 122.52 = 513 \text{ lbs. per sq. in.}$$

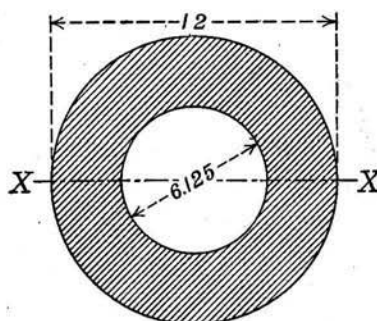
The compression due to the force I is

$$30833 / 122.52 = 252 \text{ lbs. per sq. in.}$$

The maximum resultant stress, therefore, is one of compression. It occurs at the rear of the section and is equal to

$$5793 + 252 = 6045 \text{ lbs. per sq. in.}$$

Stresses in Section 8-8 of the Stem of the Pivot Yoke. — This section is indicated in Fig. 22 and shown in Fig. 61.



Area = 83.63 sq. ins. $I = 949.01 \text{ ins.}^4$

Section 8-8.

Fig. 61.

The force H , see Fig. 22, produces bending and shear in the section, and the force I produces bending and compression.

The total bending moment is

$$62807 \times 12 + 30833 \times 7 = 969515 \text{ in. lbs.}$$

and the corresponding maximum bending stress is, see Fig. 61,

$$969515 \times 6 / 949.01 = 6130 \text{ lbs. per sq. in.}$$

compression at the rear and tension at the front of the section.

The shearing stress due to the force H is

$$62807 / 83.63 = 751 \text{ lbs. per sq. in.}$$

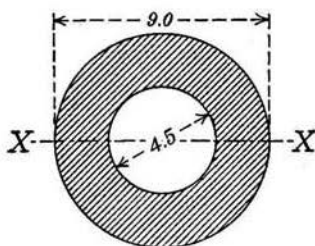
The compression due to the force I is

$$30833 / 83.63 = 369 \text{ lbs. per sq. in.}$$

The maximum resultant stress, therefore, is one of compression. It occurs at the rear of the section and is equal to

$$6130 + 369 = 6499 \text{ lbs. per sq. in.}$$

Stresses in Section 9-9 of the Stem of the Pivot Yoke. — This section is indicated in Fig. 22 and shown in Fig. 62.



$$\text{Area} = 47.71 \text{ sq. ins.} \quad I = 302.01 \text{ ins.}^4$$

Section 9-9.

Fig. 62.

The force H , see Fig. 22, produces bending and shear in the section. The force I which acts on sections 7-7 and 8-8 cannot be considered as acting in this case for its point of application is above the section, and only the forces whose points of application are below it are being considered, in accordance with the general principles already explained.

The bending moment at the section is

$$62807 \times 7.875 = 494605 \text{ in. lbs.}$$

and the corresponding maximum bending stress is, see Fig. 62,

$$494605 \times 4.5 / 302.01 = 7370 \text{ lbs. per sq. in.}$$

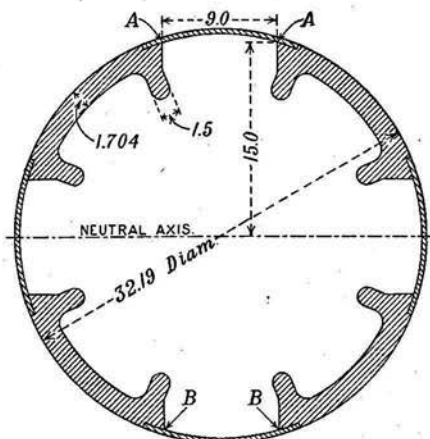
compression at the rear and tension at the front of the section.

The shearing stress is

$$62807 / 47.71 = 1316 \text{ lbs. per sq. in.}$$

STRESSES IN THE PEDESTAL

83. **Stresses in Section 10-10 of the Pedestal.** — This section is indicated in Fig. 24 and shown in Fig. 63. It is a horizontal section taken through the centers of the four hand-holes in the pedestal. It is assumed that the axis of the gun when fired is immediately above the line joining the centers of two of the hand-holes; and the cover plates over the holes are neglected. Neg-



Area = 134.28 sq. ins.

$I = 14196 \text{ ins.}^4$

Section 10-10.

Fig. 63.

lecting the weight of the pedestal as the bulk of it is below this section, and considering the forces whose points of application are above the section, the only force acting, see Fig. 24, is $G = 155057$ lbs. This produces bending and shear in the section.

Its bending moment is

$$155057 \times 16.37 = 2538283 \text{ in. lbs.}$$

and the corresponding maximum bending stress is

$$2538283 \times 15 / 14196 = 2682 \text{ lbs. per sq. in.}$$

tension at A and compression at B.

The shearing stress in the section is

$$155057 / 134.28 = 1155 \text{ lbs. per sq. in.}$$

Stresses in the Foundation Bolts. — There are sixteen foundation bolts fastening the pedestal to the gun platform, each of which is 1.75 ins. in diameter at the top of the threads and 1.564 ins. in diameter at the bottom of the threads. The positions of these bolts are shown in Fig. 64.

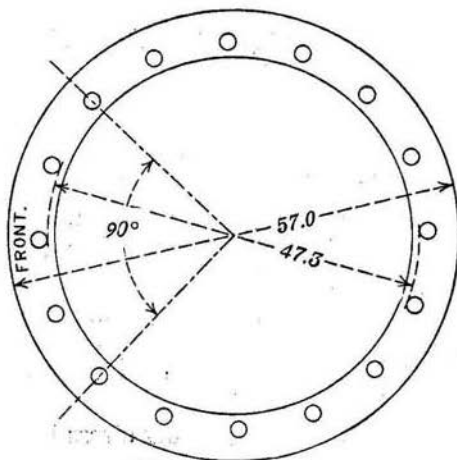


Fig. 64.

Assume that the holding down force $L = 69441$ lbs., see Fig. 24, is exerted through the five bolts included in the quadrant at the front of the pedestal as indicated in Fig. 64. Then, the area of each bolt at the bottom of the threads being 1.921 sq. ins., the stress in each is

$$69441 / [5 \times 1.921] = 7230 \text{ lbs. per sq. in. tension.}$$

Stresses in the Flange at the Rear of the Pedestal. — The stresses in this part of the flange are larger than those which occur at the front owing to the greater bending moment at the rear.

Although the force T is shown in Fig. 24 as acting at the extreme rear point of the flange of the pedestal it is in reality distributed over the rear half of the flange,* the intensity of the pressure per unit of area varying from a maximum at the extreme

* Part of the force T is also distributed over the bottom of the pedestal inside the flange but in its vicinity.

rear point to zero at the diameter perpendicular to the axis of the gun. Under the circumstances, however, it will be safe to assume that the force is uniformly distributed over the portion of the flange within the quadrant at the rear and outside the dangerous section thereof, which occurs along the line where the fillet, see Fig. 24, joins the upper surface of the flange, this line being the circumference of a circle 47.3 ins. in diameter tangent to the foundation bolt holes on the inside. The portion of the flange under consideration may be taken as a cantilever whose dangerous section is $\pi \times 47.3 / 4 = 37.15$ ins. wide, and, since the flange is 2 ins. thick, the dangerous section has a moment of inertia about its neutral axis of 24.77 ins.⁴ and an area of 74.3 sq. ins. The outer radius of the flange being 28.5 ins. and the radius of the dangerous section 23.65 ins., the cantilever can be taken as $28.5 - 23.65 = 4.85$ ins. long, and the bending moment of the force T uniformly distributed over it is, from equation (19),

$$\frac{103897 \times (4.85)^2}{4.85 \times 2} = 251950 \text{ in. lbs.}$$

The corresponding maximum bending stress is

$$251950 \times 1 / 24.77 = 10172 \text{ lbs. per sq. in.}$$

compression at the upper and tension at the lower surface of the flange.

The shearing stress is

$$103897 / 74.3 = 1398 \text{ lbs. per sq. in.}$$

The compression due to the force S , see Fig. 24, is

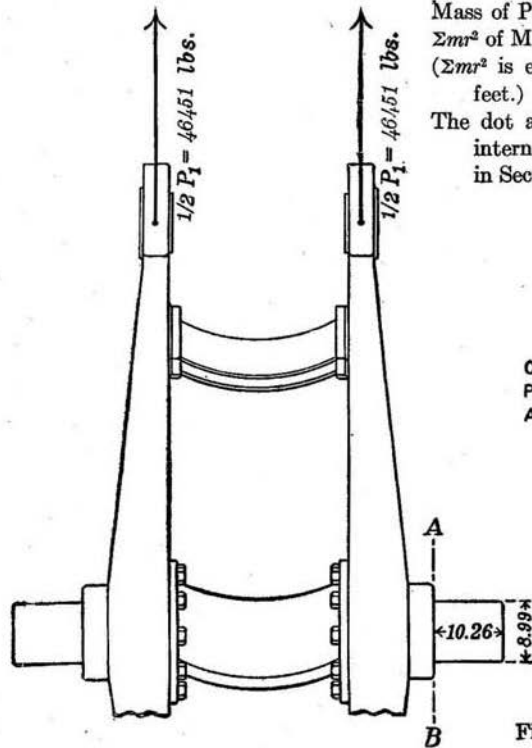
$$92250 / 74.3 = 1242 \text{ lbs. per sq. in.}$$

The maximum resultant stress, which occurs at the upper edge of the section is, therefore,

$$10172 + 1242 = 11414 \text{ lbs. per sq. in. compression.}$$

STRESSES IN PARTS OF THE 6-INCH DISAPPEARING CARRIAGE, MODEL OF 1905 ML.

84. Stresses in Section 11-11 of the Gun Levers. — The gun-lever axle and the part of the gun levers above it are shown in Fig. 65, in which are also indicated the forces (in full lines)



Mass of Part above Sec. 11 - 11 = 40.
 Σmr^2 of Mass above Sec. 11 - 11 = 230.
 (Σmr^2 is expressed in mass units and feet.)

The dot and dash force lines indicate internal stresses (total stresses) in Section 11 - 11.

CENTER OF GRAVITY OF
 PART OF GUN LEVER
 ABOVE SECTION 11-11.

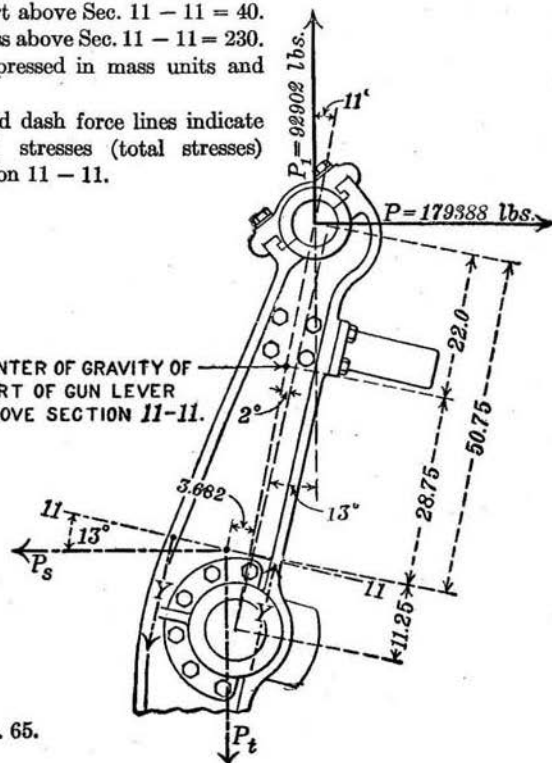
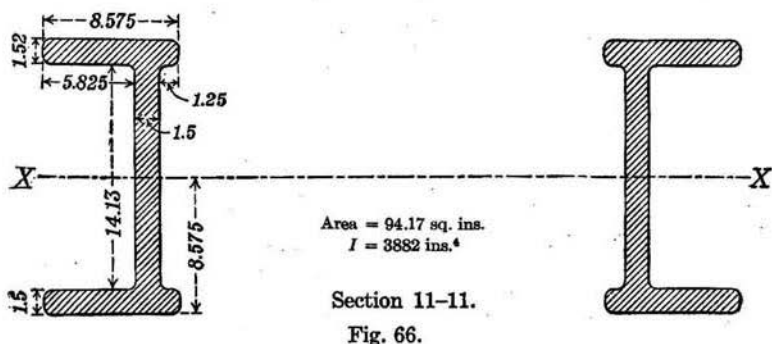


Fig. 65.

exerted on the levers by the gun and the position of section 11-11. The values of the forces P and P_1 were determined in Chapter III, see equations (1''') and (2'''), page 89, to be 179338 lbs. and 92902 lbs., respectively.

Section 11-11 is shown in Fig. 66.



Method of Computing Stresses in Parts not in Equilibrium. —

The stresses in this section cannot be obtained in the same way as were the stresses in the parts of the 3-inch field carriage and the 5-inch barbette carriage because those parts were in equilibrium under the forces acting upon them; while the gun levers have linear accelerations in the vertical and horizontal directions and an angular acceleration about their center of mass, and a part of each of the forces acting on them is taken up in producing these accelerations without causing internal stresses in the parts. If, however, a section such as 11-11, Fig. 65, be taken through any part of a body not in equilibrium, the internal stresses acting in that section must be such that, in connection with the external forces acting on the part of the body on one side (either side) of the section, they will produce in that part the accelerations of translation and rotation possessed by it. Therefore, considering the part of the gun levers above section 11-11, we may form three equations between the known external forces and the unknown stresses in section 11-11 stating: (a) that the sum of the components of both forces and stresses in a vertical direction is equal to the mass of the part of the gun levers above section 11-11 multiplied by its acceleration in that direction; (b) that the sum of the components in the horizontal direction is equal to the mass multi-

plied by its acceleration in the horizontal direction; and (c) that the sum of the moments of both forces and stresses with respect to the center of mass of the part of the gun levers above section 11-11 is equal to the moment of inertia of that part, with respect to an axis passing through its center of mass perpendicular to the plane of the forces, multiplied by the angular acceleration of the part about that axis. If these three equations do not contain more than three unknown quantities, the internal stresses in section 11-11 can be determined.

Resolution of the Total Stresses in Section 11-11 into Unknown Horizontal and Vertical Components P_s and P_t and an Unknown Couple YY .—Referring to Fig. 65, the total stresses in section 11-11 can be resolved into unknown horizontal and vertical components P_s and P_t , respectively, and an unknown couple YY as shown by the dot and dash force lines, these being the most general assumptions that can be made with regard to any set of unknown co-planar forces or stresses acting there. The point of application of P_s and P_t can be taken at the center of the section. They will be assumed to act in opposite directions to the external forces P and P_1 , and if this assumption is not correct it will be shown by a negative sign opposite the numerical value of one or both of the stresses obtained from the equations. The assumption of wrong directions for the component stresses will not affect the correctness of the numerical results. The sum of the components of P_s and P_t parallel to section 11-11 is the total shearing stress in the section, and the sum of their components normal to the section is the total stress of tension or compression therein. Since P_s and P_t are applied at the center of the section they can have no tendency to rotate it and will, therefore, have no effect on the bending moment at the section. The couple YY can produce neither shear nor simple tension or compression in the section since the individual forces of a couple are equal in intensity but opposite in direction. It does, however, tend to rotate the section about the neutral axis XX , Fig. 66, and it is, therefore, the bending moment at the section.

Determination of the Values of P_s , P_t , and YY .—Writing the three equations expressing the relations between the known forces, $P = 179338$ lbs. and $P_1 = 92902$ lbs., and the unknown total stresses in section 11-11, and noting that as YY is a couple

it need only appear in the equation of moments, we have, since $M = 40$ and $\Sigma mr^2 = 230$, see Fig. 65,

$$179338 - P_x = 40 d^2x/dt^2 \quad (41)$$

$$92902 - P_t = 40 d^2y/dt^2 \quad (42)$$

$$\begin{aligned} 179338 \times \frac{22 \cos 11^\circ}{12} + P_x \times \left\{ \frac{28.75 \cos 11^\circ - 3.682 \sin 13^\circ}{12} \right\} \\ - 92902 \times \frac{22 \sin 11^\circ}{12} - P_t \times \left\{ \frac{28.75 \sin 11^\circ + 3.682 \cos 13^\circ}{12} \right\} \\ - YY = 230 d^2\phi/dt^2 \quad (43) \end{aligned}$$

d^2x/dt^2 and d^2y/dt^2 being the accelerations of the center of mass of the part of the gun levers above section 11-11 in the horizontal and vertical directions, respectively, and $d^2\phi/dt^2$ being the angular acceleration of that part about the axis through its center of mass perpendicular to the plane of the forces. Since the unit of force is the pound, the linear accelerations must be expressed in feet per second per second and the moment of inertia in mass units and feet.

The horizontal acceleration in equation (41) may be obtained from equation (43), page 82, by neglecting the term containing $(d\phi/dt)^2$ as being too small to be considered; by substituting for $d^2\phi/dt^2$ its value of 224.98 radians per sec. per sec. from equation (7^{IV}), page 89; by substituting for α , ϕ , and $(\phi - \beta)$ their values of 4.3333 ft., 13° , and 11° ,* respectively, from table 3, page 85; and by replacing c , the distance of the axis of the gun trunnions from the axis of the trunnions of the gun-lever axle, by 40/12 ft., the distance of the center of mass of the part of the gun-levers above section 11-11 from the axis of the trunnions of the gun-lever axle; whence

$$\begin{aligned} d^2x/dt^2 &= (4.3333 \cos 13^\circ + \frac{40}{12} \cos 11^\circ) 224.1 \\ &= 1679 \text{ ft. per sec. per sec.} \end{aligned}$$

and

$$M d^2x/dt^2 = 40 \times 1679.5 = 67180.$$

* See article 56, page 86.

By making the same omission and substitutions in equation (48), page 83, and substituting for α its value of $1^\circ 20'$ from table 3, we obtain as the vertical acceleration in equation (42)

$$\begin{aligned} d^2y/dt^2 &= (4.3333 \tan 1^\circ 20' \cos 13^\circ - \frac{40}{12} \sin 11^\circ) 224.08 \\ &= -120.5 \text{ ft. per sec. per sec.} \end{aligned}$$

and $M d^2y/dt^2 = 40 (-120.5) = -4820$

The angular acceleration of the part of the gun levers above section 11-11 about an axis passing through its center of mass perpendicular to the plane of the forces is the same as that of the gun levers, complete, about the axis of the gun-lever pins, and is 224.08 radians per sec. per sec., whence

$$\Sigma mr^2 \times d^2\phi/dt^2 = 230 \times 224.08 = 51543$$

Substituting these values of $M d^2x/dt^2$, $M d^2y/dt^2$, and $\Sigma mr^2 d^2\phi/dt^2$ in equations (41), (42), and (43), respectively, the only unknown quantities remaining therein are P_s , P_t , and YY which may be determined to have the following values:

$$P_s = 112158 \text{ lbs.}$$

$$P_t = 97722 \text{ lbs.}$$

$$YY = 420839 \text{ ft. lbs.} = 5050068 \text{ in. lbs.}$$

Shearing Stress in Section 11-11. — The total shearing stress in section 11-11 is the algebraic sum of the components of P_s and P_t parallel to the section. It is equal to

$$112158 \cos 13^\circ - 97922 \sin 13^\circ = 87300 \text{ lbs.}$$

and the shearing stress per unit of area is, see Fig. 66,

$$87300/94.17 = 927 \text{ lbs. per sq. in.}$$

Tensile Stress in Section 11-11. — The total longitudinal stress in this section is the sum of the components of P_s and P_t perpendicular to the section; and, since these components act away from the section, the stress is one of tension. It is equal to

$$112158 \sin 13^\circ + 97922 \cos 13^\circ = 120446 \text{ lbs.}$$

and the tensile stress per unit of area is

$$120446/94.17 = 1279 \text{ lbs. per sq. in.}$$

Bending Stress in Section 11-11. — The bending moment YY is 5050068 in. lbs. and the corresponding maximum bending stress is, see Fig. 66,

$$5050068 \times 8.575 / 3882 = 11155 \text{ lbs. per sq. in.}$$

tension at the front and compression at the rear of the section.

Maximum Resultant Stress in Section 11-11. — The maximum resultant stress in the section is one of tension; it occurs at the front of the section and is equal to

$$1279 + 11155 = 12434 \text{ lbs. per sq. in.}$$

85. Stresses in the Trunnions of the Gun-Lever Axle. — The gun levers, see Fig. 65, are shrunk and bolted to the gun-lever axle, and the whole may be regarded as a beam carried in bearings in the top carriage and subjected to bending forces applied to each end of the gun levers, as shown in Fig. 31, Chapter III. Because of the great stiffness of the beam formed by the gun levers and the gun-lever axle in connection with the fact that the trunnions are .01 in. smaller in diameter than their bearings in the top carriage, the levers and axle may, like the cradle of the 5-inch barbette carriage, model of 1903, be considered as a beam that is merely supported at its ends.

The forces acting on each trunnion, see equations (4''') and (5'''), page 89, are the horizontal force $P_5/2 = 39111.5$ lbs. and the vertical force $P_4/2 = 67874$ lbs. The resultant of these forces is

$$\sqrt{(39111.5)^2 + (67874)^2} = 78328 \text{ lbs.}$$

and it is uniformly distributed over the surface of contact between the trunnion and its bed in the top carriage. This surface is 10 ins. long, being .26 in. shorter than the trunnion to provide a clearance of .01 in. on the inside and to allow the trunnion to project through for a distance of .25 in. on the outside.

The bending moment at section AB , Fig. 65, is greater than at any other section of the trunnion. It may be obtained by considering the resultant force concentrated at the middle of the length of the surface of contact between the trunnion and its bed, whence

$$M = 78328 \times 5.01 = 392423 \text{ in. lbs.}$$

The diameter of the trunnion being 8.99 ins., the area of any right section thereof is 63.48 sq. ins. and the moment of inertia of the section with respect to its neutral axis is 320.72 ins.⁴ The maximum bending stress in the section is, therefore,

$$392423 \times 4.495 / 320.72 = 5500 \text{ lbs. per sq. in.}$$

tension at the rear and compression at the front of the section.

The shearing stress in section *AB* is

$$78328 / 63.48 = 1234 \text{ lbs. per sq. in.}$$

86. Stresses in the Elevating Arm and Other Parts of the 6-inch Disappearing Carriage, Model of 1905 Ml. — The stresses in any section of the elevating arm, which rotates about an axis on the rear transom of the carriage, must be determined by the same method as the stresses in section 11-11 of the gun levers. The method of computing the stresses in the remaining parts of the carriage is the same in principle as that followed throughout this text in the determination of the stresses in parts of the 3-inch field carriage and of the 5-inch barbette carriage.

CHAPTER V.

TOOTHED GEARING.

87. Definition. — Ratio of Angular Velocities. — In order to transmit rotary motion from one shaft to another of a gun carriage, or of any other machine, toothed gearing is employed. This consists of a toothed wheel fastened to one shaft which when the shaft rotates moves with it, and by the engagement of its teeth with those of a second wheel fastened to a second shaft causes a rotation of the second wheel and shaft.

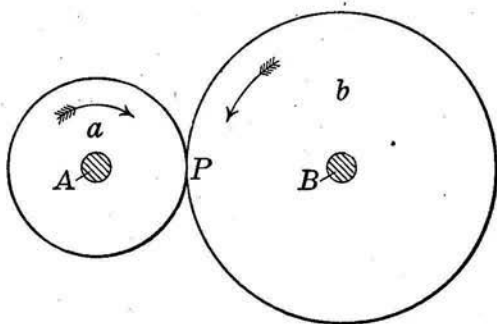


Fig. 67.

Let a and b , Fig. 67, be two wheels mounted on parallel shafts A and B and pressed together with considerable force so that the friction at their line of contact projected on P is sufficient to make either wheel turn if the other is turned. Suppose the shaft B be rotated counter-clockwise as shown by the arrow. Then b will rotate in the same direction and because of the friction at P the wheel a and the shaft A will be forced to rotate in the opposite direction as shown by the arrow on a .

Let V be the surface velocity of the wheel b ,
 r_b the radius of the wheel b ,
 ω_b the angular velocity of the wheel b ,

N_b the number of revolutions per minute of the wheel b ,
 r_a the radius of the wheel a ,
 ω_a the angular velocity of the wheel a , and
 N_a the number of revolutions per minute of the wheel a .

Assuming that there is no slipping at the line of contact, the surface velocities of a and b must be the same and, consequently,

$$V = r_b \omega_b = r_a \omega_a$$

or

$$\omega_a / \omega_b = r_b / r_a$$

That is, *the angular velocities of the two wheels are inversely as their radii.*

Since the number of revolutions per minute of a wheel varies directly as its angular velocity we may write

$$N_a / N_b = \omega_a / \omega_b = r_b / r_a$$

or *the numbers of revolutions per minute of the two wheels are inversely as their radii.*

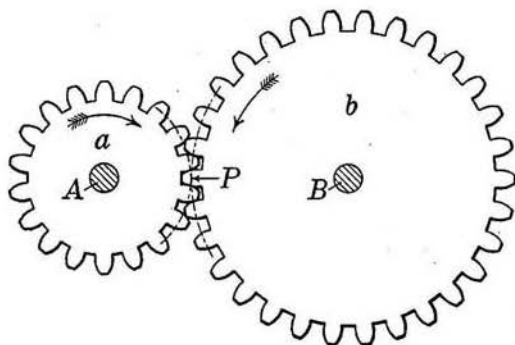


Fig. 68.

88. Necessity for Teeth. — **Tooth Surfaces Must Have Certain Definite Forms.** — The wheels a and b are called friction wheels and are used to a limited extent in machinery to transmit rotary motion from one shaft to another. If any great amount of power has to be transmitted, however, the wheels will slip as the greatest force that can be transmitted between their surfaces is equal to the normal pressure there multiplied by the coefficient of friction. To prevent slipping the surface of each wheel is provided with projections called teeth which engage with corresponding teeth on the other wheel as shown in Fig. 68.

It is evident whatever be the shape of the teeth, provided they are of the same uniform size and uniformly spaced on both wheels and the recesses between them are large enough for them to enter, that the numbers of revolutions per minute will be inversely proportional to the radii of the wheels or to the numbers of the teeth, since these must vary directly as the radii; and it may not, therefore, be apparent at first why the shape of the teeth should not be arbitrarily selected. It is necessary, however, that the machinery shall run smoothly, without jerks, and in most cases it is very essential that the velocity of one moving part of a machine shall at all times have definite relations to the velocities of the other moving parts. Therefore, not only must the numbers of revolutions per minute of the wheels have a definite relation to each other but the angular velocities at each instant of time must have this same definite ratio. This is possible only when the tooth surfaces have certain definite forms.

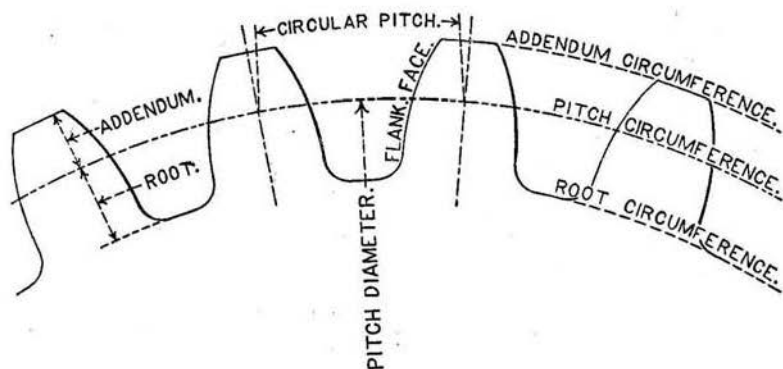


Fig. 69.

89. Outline of Gear-Teeth. — Pitch Circumference. — Circular Pitch. — Diametral Pitch. — Fig. 69 shows the outline of several gear-teeth.

The *pitch circumference** shown in the figure is the projection of the imaginary cylindrical surface which, rolling on a similar

* The circumferences of the pitch, addendum, and root circles are generally referred to as circles, not circumferences.

cylindrical surface of another gear-wheel, would cause the two wheels to rotate together with the same relative angular velocities as would be caused by the engagement of the teeth of the wheels; or the pitch circumference may be considered as the projection of the cylindrical surface of a friction wheel such as shown in Fig. 67 on which teeth have been fastened as shown in Fig. 68 to make the motion more reliable, it being understood that the ratio between the angular velocities of the wheels in both figures is the same.

The circle drawn through the outer ends of the teeth is called the *addendum circle* and that drawn through their inner ends is called the *root circle*. The distance between the pitch and addendum circumferences measured on a radial line is called the *addendum* and that between the pitch and root circumferences measured on a radial line is called the *root*. The term *addendum* is also frequently used to designate the part of the tooth outside the pitch surface, and the term *root* to designate the part of the tooth inside that surface. The working surface of the portion of the tooth outside the pitch surface is called the *face* of the tooth and the working surface inside the pitch surface is called the *flank* of the tooth. The distance between corresponding points of two adjacent teeth measured on the pitch circumference is called the *circular pitch*. The *diametral pitch* is the number of teeth per inch of diameter of the pitch circle; thus if p_1 represent the diametral pitch, the number of teeth of a gear-wheel whose pitch diameter is d is $p_1 d$. From the definition of circular pitch the number of teeth of the same wheel expressed in terms of the circular pitch, which will be represented by p , is $\pi d/p$. Since $p_1 d = \pi d/p$ we have $p_1 = \pi/p$, or the diametral pitch is equal to π divided by the circular pitch. The diametral pitch is the one more frequently used to express the pitch of the teeth of gear-wheels.

90. Angular Velocity of Rotating Wheel. — Let the wheel a shown in Fig. 70 be rotating about its center O and let the velocity of any point b on its circumference be represented in magnitude and direction by the line marked V . Then if r be the radius of the wheel its angular velocity ω will be V/r . Resolve the velocity V into two components normal to each other, V_n and V'_n as shown. Let α be the angle between the directions of V and V_n and let r_n

be the perpendicular distance from the center of the wheel to the direction line of V_n . Then

$$V_n = V \cos \alpha$$

and

$$r_n = r \cos \alpha$$

whence

$$\frac{V_n}{r_n} = \frac{V \cos \alpha}{r \cos \alpha} = \frac{V}{r} = \omega$$

and, similarly

$$V'_n / r'_n = \omega.$$

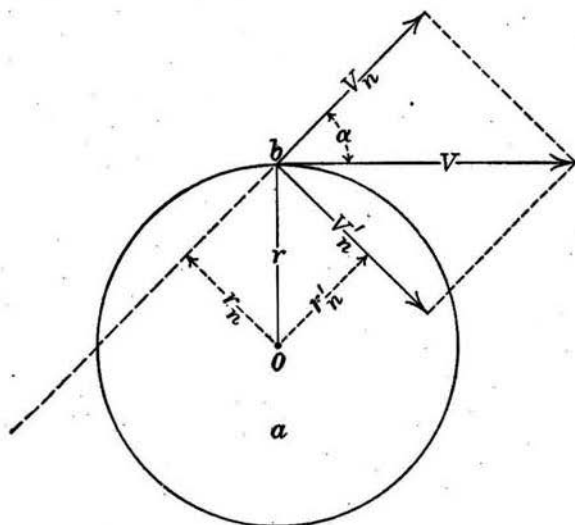


Fig. 70.

If the wheel be increased in size so that the point b no longer lies on its circumference, the relations just established will still hold for the velocity at the point b , r in this case, however, will be the perpendicular distance from the center of the wheel to the direction line of $\dot{V}$; whence we may write

If the velocity of any point of a wheel rotating about its center be resolved into two components normal to each other, the angular velocity of the wheel will be equal to either component divided by the perpendicular distance from the center of the wheel to the direction line of that component.

91. Condition to be Fulfilled by Tooth Curves. — Let T_a and T_b , Fig. 71, be two teeth in contact at the point m , and C and C_1 be the centers of the wheels to which T_a and T_b belong. Since the tooth curves are tangent at m they have at this point a common normal NN_1 which intersects the line of centers CC_1 at some point P . Suppose the wheel B to be rotating counter-clockwise

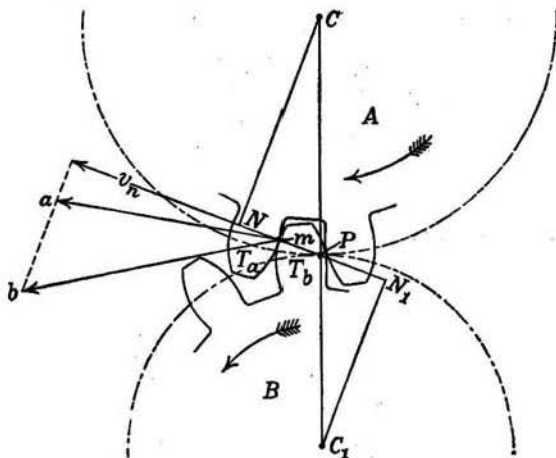


Fig. 71.

around its center C_1 driving the wheel A around its center C in a clockwise direction; then the velocity of the point m on the tooth T_b may be represented by the line mb drawn tangent to the circle described through m about the center C_1 , and, similarly, the velocity of the point m on the tooth T_a may be represented by the line ma drawn tangent to the circle described through m about the center C . Let the point m on the tooth T_b be called m_b and the point m on the tooth T_a be called m_a , and resolve the velocities of m_b and m_a into components parallel and perpendicular to the common normal NN_1 .

While the velocities of m_b and m_a vary in direction and in magnitude, the components of these velocities in the direction of their common normal must be equal during the time m_b and m_a are in contact, for otherwise the teeth would separate or one surface would penetrate the other. Representing by v_n the equal component velocities of m_b and m_a in the direction of the common

normal, and dropping the perpendiculars C_1N_1 and CN on this normal, the angular velocities ω_b and ω_a of the wheels B and A , respectively, are (see article 90)

$$\omega_b = v_n / C_1N_1 \text{ and } \omega_a = v_n / CN$$

whence

$$\omega_b / \omega_a = CN / C_1N_1 = CP / C_1P$$

or

The angular velocities of the two wheels are inversely as the segments into which the line of centers is cut by the common normal to the tooth curves at their point of contact.

The condition to be fulfilled by tooth curves in order that the ratio of the angular velocities of the wheels shall be constant at every instant is, therefore, *that the common normal to the tooth curves at their point of contact shall always pass through a fixed point on the line of centers.*

In order that the wheels shall have the same angular velocities as their pitch circles the *common normal to the tooth curves at their point of contact must intersect the line of centers at the pitch point, which is the point of tangency of the two pitch circles.* Any number of curves will fulfill this condition but the two generally adopted for tooth outlines are the involute curve and the cycloidal curve.

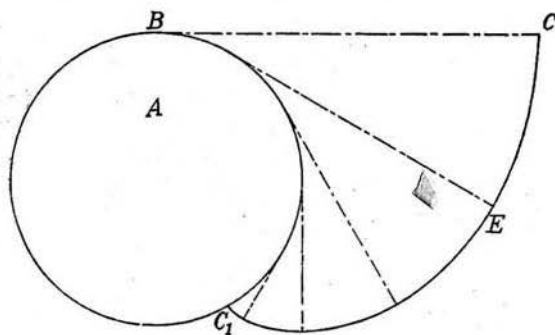


Fig. 72.

92. The Involute Curve. — If a cord be wound around a circle and then unwound, keeping the unwound portion straight, a point of the cord will describe an involute to the circle. Let BC , Fig. 72, be a portion of a cord unwound from the circle A . The point

C of the cord before it was unwound was at the point C_1 of the circle, and the curve C_1EC is the involute to the circle traced by the point C . At each instant C is rotating about the point of tangency of the cord to the circle and, consequently, its direction at that instant is at right angles to the tangent line drawn from it to the circle. Any normal to the involute is, therefore, tangent to the circle.

The Involute System of Gear-Teeth. — Let APB and CPD , Fig. 73, be the pitch circles of two wheels on which it is desired to construct teeth of such profile that the ratio of their angular velocities when running in gear shall at each instant be exactly the same as the ratio of the angular velocities of the pitch circles when running together without teeth. Through the pitch point P draw a line NN_1 at an angle α with the common tangent to the pitch circles, and draw circles EF and GH tangent to NN_1 about the centers O and O_1 , respectively, of the pitch circles. Let the profiles of the teeth on APB be involutes to the circle EF and the profiles of those on CPD be involutes to the circle GH . Now let APB be rotated in the direction of the arrow until one of its teeth is brought in contact with a tooth on CPD . At the point of contact the curves on the two teeth will have a common normal which must be tangent to the circle EF because the profile of the tooth on APB is an involute to that circle, and which must also be tangent to the circle GH because the profile of the tooth on CPD is an involute to the circle GH . From the figure it is evident that NN_1 is the only line tangent to the circles EF and GH that can pass through the point of contact of the teeth, and this line must, therefore, be the common normal to the tooth curves at their point of contact. If, however, the wheel APB be rotated in the opposite direction the point of contact will change to the other sides of the teeth and the common normal to the tooth curves at their point of contact will be the line $N'N'_1$, which is also a common tangent to the circles EF and GH passing through the pitch point P . The point of contact of the teeth must, therefore, always lie on one or the other of the lines NN_1 and $N'N'_1$, both of which are common normals to the tooth curves passing through the pitch point. It is evident, therefore, that the involute to a circle fulfills strictly the condition required for tooth profiles of gear-wheels.

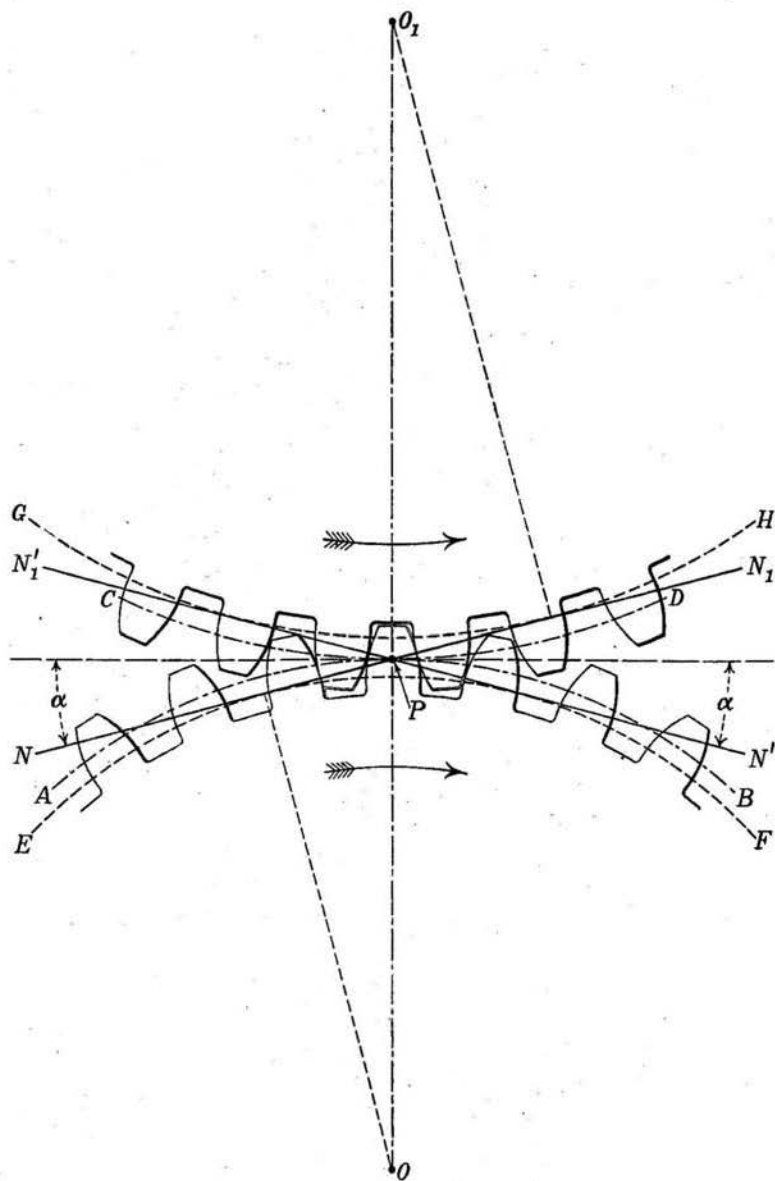


Fig. 73

The circles EF and GH are called base circles. Since the point of contact of the teeth is always on one or the other of the lines NN_1 and $N'N'_1$, these lines are called the lines of action. Neglecting friction the line of action is the direction of the pressure between the teeth. The angle α between the line of action and the common tangent to the pitch circles is called the angle of obliquity. In order that gears with involute teeth shall be interchangeable they must have the same circular pitch and the same angle of obliquity. The standard angle of obliquity is 15° .

The arc of the pitch circle that subtends the angle through which a wheel rotates from the time when one of its teeth comes in contact with a tooth of the other wheel until the point of con-

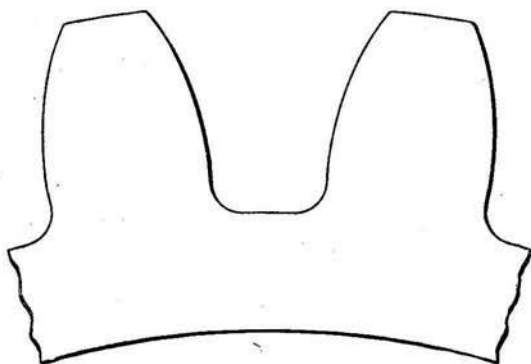


Fig. 74.

tact reaches the pitch point P is called the arc of approach; and the arc of the pitch circle that subtends the angle through which the wheel rotates from the time the point of contact leaves the pitch point until the teeth separate is called the arc of recess. The sum of these arcs is called the arc of action. In order that there shall always be contact between the teeth, the arc of action should be at least equal to the circular pitch. If it is desired that there shall always be contact between two pairs of teeth, the arc of action should be at least equal to twice the circular pitch. The arc of action may be increased by lengthening the teeth up to a certain limit. Fig. 74 shows the profile of the involute tooth.

93. The Cycloid. — Epicycloid. — Hypocycloid. — If a circle be rolled upon a straight line a point in its circumference will trace a curve called a cycloid. If the circle be rolled upon the

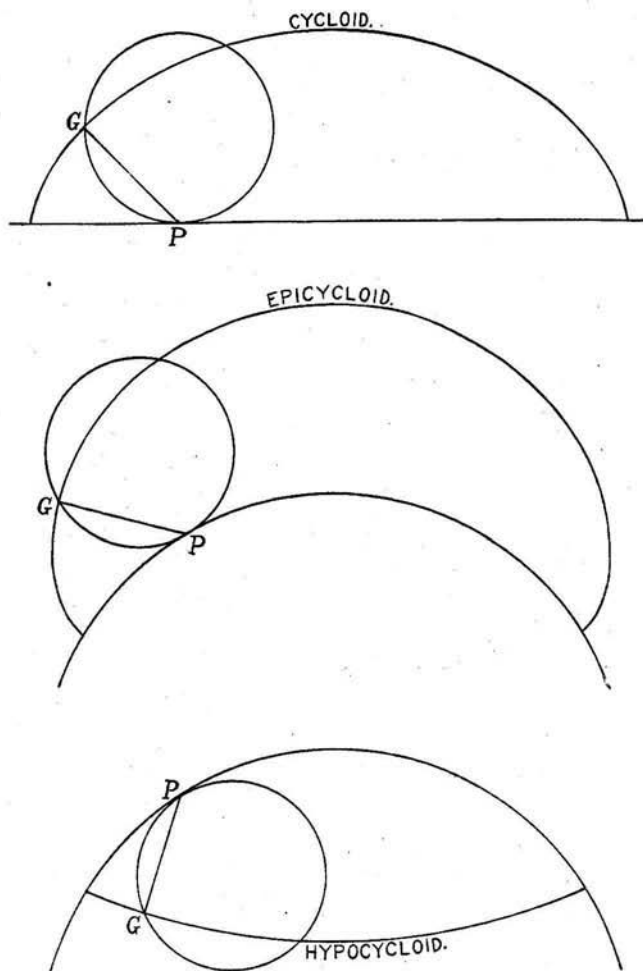


Fig. 75.

convex side of another circle a point in the circumference of the former circle will trace a curve called an epicycloid; and if rolled upon the concave side of the circle the curve traced is called a hypocycloid. These curves are shown in Fig. 75.

Let G be the point on the generating circle that traces each of the three curves shown in Fig. 75. Then, since the motion of the generating circle and the point G is at each instant a rotation about the point of tangency P of the generating circle and the line upon which it rolls, the direction of motion of G is at right angles to the line joining it to P , and this line GP is a normal to the cycloid, epicycloid, or hypocycloid as the case may be.

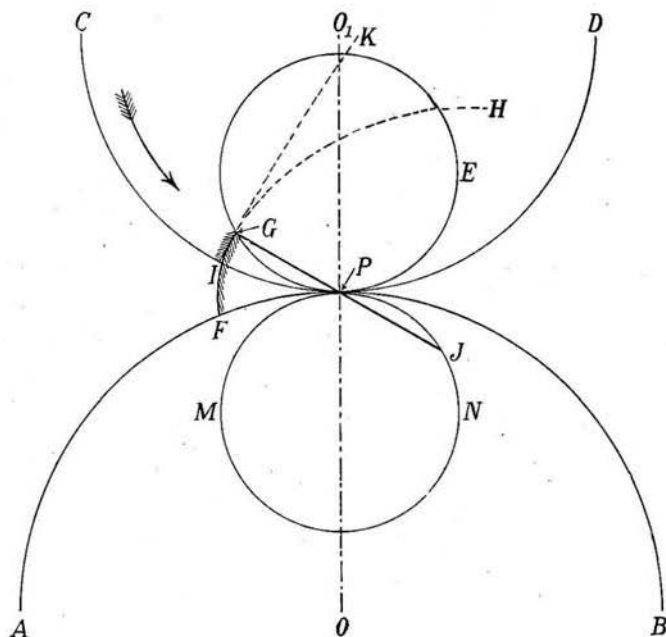


Fig. 76.

The Cycloidal System of Gear-Teeth. — Let APB and CPD , Fig. 76, be the pitch circles of two wheels on which it is desired to construct teeth of such profile that the ratio of their angular velocities when running in gear shall at each instant be exactly the same as that of the pitch circles when running together without teeth. Suppose the generating circle GPE be rolled on the outside of APB and that the point G traces the epicycloid FGH ; suppose also that the generating circle be rolled on the inside of CPD , the point G tracing the hypocycloid IGK . Let the portion

FG of the epicycloid be taken as the profile of the part of a tooth on APB outside the pitch circle, that is, as the profile of the face of a tooth on APB (see Fig. 69), and the portion IG of the hypocycloid be taken as the profile of the flank of a tooth on CPD ; and let FG and IG be brought into contact as shown. Then the common normal to the tooth curves at their point of contact must pass through the point at which the generating circle was tangent to the pitch circle APB at the instant the point G of the epicycloid was being traced, and it must also pass through the point at which the generating circle was tangent to the pitch circle CPD at the instant the point G of the hypocycloid was being traced; and further, since the point of contact G is a point of both the epicycloid and the hypocycloid, the generating circle must have been tangent to both of the pitch circles at the instant the point G was being traced. The common normal to the tooth curves at their point of contact must, therefore, pass through the pitch point P . As this is true for any point of contact of the tooth curves FG and IG , it follows that when the profile of the face of a tooth on one gear is part of the epicycloid traced by a point of the generating circle rolling on the outside of the pitch circle of that gear, and the profile of the flank of the tooth on the other gear is part of the hypocycloid traced by a point of the *same generating circle* rolling on the inside of the pitch circle of the second gear, the tooth curves fulfill strictly the condition required for tooth profiles of gear-wheels.

Referring again to Fig. 76, the generating circle GPE rolling on the outside of the pitch circle APB can only generate the profile for the faces of the teeth on the lower wheel, and rolling on the inside of CPD it can only generate the profile for the flanks of the teeth on the upper wheel. To generate the profile for the flanks of the teeth on the lower wheel a second generating circle MN must be rolled on the inside of the pitch circle APB , and to generate the profile for the faces of the teeth on the upper wheel the same circle MN must be rolled on the outside of CPD . For two wheels to gear together the circular pitch must be the same in each and the generating circle for the profile of the faces of the teeth on the one must be the same size as the generating circle for the profile of the flanks of the teeth on the other, but it is not necessary that the same generating circle be used for the profiles

of the faces and the flanks of the teeth on the same wheel. It is very important, however, that all gear-wheels of the same circular pitch shall be interchangeable, and to accomplish this the same generating circle must be used for the profiles of the faces and flanks of the teeth on all the wheels.

If the diameter of the generating circle is larger than the radius of the pitch circle the profile of the flank of the tooth will curve rapidly towards the center of the tooth at the root circumference, thereby diminishing its thickness at the root and weakening the tooth. For a set of wheels of the same circular pitch required to be interchangeable the size of the generating circle, which must be the same for all, is limited by the fact that, in order not to unduly weaken the teeth of the smallest wheel of the set, the diameter of the generating circle must not be greater than the pitch radius of that wheel. The number of teeth in the smallest wheel of the cycloidal system is taken as twelve, and for all wheels of the same circular pitch the diameter of the generating circle is made equal to the pitch radius of the twelve-tooth wheel.

Referring to Fig. 76, suppose the upper wheel be rotating counter-clockwise in the direction of the arrow. It will then rotate the lower wheel clockwise and, if G be the first point of contact between the flank of the tooth on the upper wheel and the face of the tooth on the lower, the line of pressure between the teeth at that instant, neglecting friction, will be GP . As the wheels advance the point of contact of the teeth will slide along the arc GP of the generating circle GE , and the direction of the line of pressure will continue to change until when the point of contact reaches P this line will be tangent to both pitch circles. After the point P has been reached the face of the tooth on the upper wheel will come in contact with the flank of the tooth on the lower, and the point of contact will slide along the arc PJ of the generating circle MN , the direction of the line of pressure between the teeth changing continually until the teeth are about to separate at the point J , when the line of pressure is PJ . The greatest angle between the common tangent to the pitch circles and the line of pressure between the teeth occurs when contact is just commencing or ending. This angle should ordinarily not be permitted to be greater than 30° .

As in the case of the involute system, the arc of the pitch circle that subtends the angle through which a wheel rotates from the time when one of its teeth comes in contact with a tooth of the other wheel until the point of contact reaches the pitch point P is called the arc of approach; and the arc of the pitch circle that subtends the angle through which the wheel rotates from the time the point of contact leaves the pitch point until the teeth separate is called the arc of recess. The sum of these arcs is called the arc of action. In order that there shall always be contact between the teeth the arc of action should be at least equal to the circular pitch. If it is desired that there shall always be contact between two pairs of teeth, the arc of action should be at least equal to twice the circular pitch. The arc of action may be increased by increasing the length of the teeth up to a certain limit.

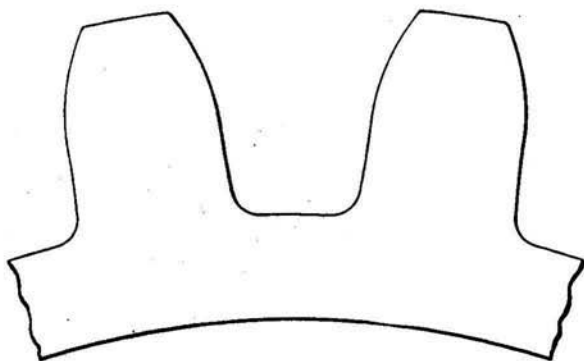


Fig. 77.

Fig. 77 shows the outline of the cycloidal tooth.

94. Spur Gears. — The gear-wheels shown in Fig. 68 are used to transmit rotary motion between parallel shafts. The pitch surfaces of the wheels are cylindrical and the tooth surfaces can be generated by moving the involute or cycloidal profile in a direction parallel to the axis of the shaft. Gear-wheels used to transmit rotary motion between parallel shafts are called *spur gears*. The wheel that imparts motion to the other is called the driver and the driven wheel is called the follower. Either one of a pair of spur gears in mesh may be the driver.

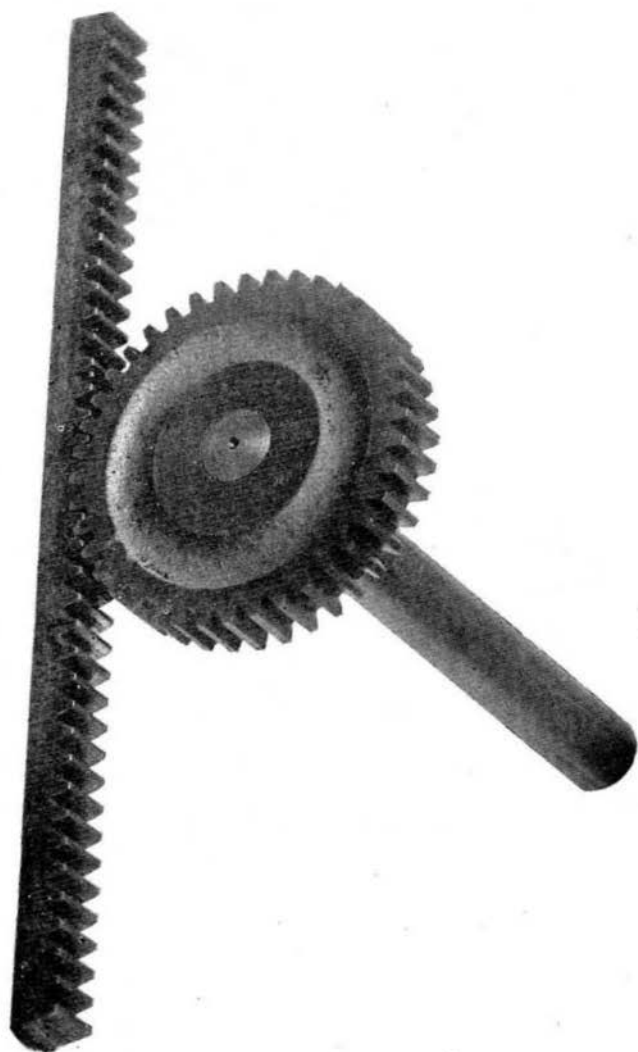


Fig. 78.—Rack and Spur Gear.

Rack and Spur Gear. — If the radius of one of the spur gears shown in Fig. 68 be increased until it becomes infinite, the spur gear becomes a rack and its pitch circumference a straight line whose linear velocity is the same as that of a point on the pitch circumference of the gear with which it meshes. A rack and spur gear are generally used to change a reciprocating rotary motion into a reciprocating motion of translation or vice versa.

Either the involute or the cycloidal system may be used for the teeth of a rack and spur gear. In the involute system the profile of the rack tooth is a straight line perpendicular to the line of action to which the base circle of the spur gear is tangent. In the cycloidal system the profile of the face of the rack tooth is a cycloid traced by a point on the generating circle while rolling on the outside of the pitch line of the rack. The profile of the flank of the rack tooth is also a cycloid traced by a point of the generating circle while rolling on the inside of the pitch line of the rack.

For interchangeability the diameter of the generating circle is made equal to the pitch radius of the twelve-tooth gear of the same circular pitch. Either the spur gear or the rack may be the driver.

Fig. 78 shows a rack and spur gear.

95. Bevel Gears. — If the shafts to be connected by gear-wheels would intersect if prolonged, the pitch surfaces cannot be cylindrical but must be the surfaces of frustums of cones whose apexes are at the point of intersection of the shafts, as shown in Fig. 79, in which *AB* and *CD* are the shafts which if prolonged would intersect at *E*, and *FGHI* and *HIJK* are the frustums of cones whose apexes are at *E* and whose surfaces are the pitch

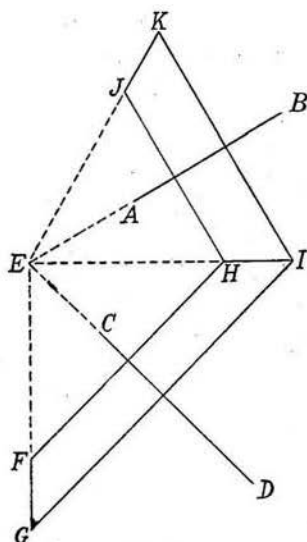


Fig. 79.

surfaces of gear-wheels transmitting rotary motion between the shafts. Gear-wheels with conical pitch surfaces transmitting rotary motion between shafts that would intersect if prolonged are called *bevel gears*. The angular velocities of a pair of bevel

gears in mesh are inversely proportional to the sines of the angles between the elements of the pitch surfaces in contact and the shafts to which the gears are attached or, more simply, inversely proportional to the numbers of the teeth on the gears. Either one of a pair of bevel gears in mesh may be the driver.

Both the involute and the cycloidal systems are used for the teeth of bevel gears but the tooth surfaces, while having rectilinear elements, cannot be formed by sliding a profile of the tooth along a straight line. For the involute system the tooth surfaces are obtained by rolling a plane on a base cone whose apex is at the point of intersection of the shafts, and for the cycloidal system the tooth surfaces are obtained by rolling generating cones on the inside and outside of the pitch cones. The tooth profile of a bevel gear at any section at right angles to the axis of the shaft can of course be obtained by rolling a line on the base circle cut by the section from the base cone, or by rolling a generating circle on the inside and outside of the pitch circle cut by the section from the pitch cone.

Fig. 80 shows a pair of bevel gears in mesh.

96. Screw Gearing. — Worm and Worm-Wheel. — Referring to Fig. 78, showing a rack and spur gear, suppose the teeth of the rack instead of being parallel to the axis of the gear be given an inclination to that axis as shown in Fig. 81. In order that the teeth of the gear shall mesh properly with the inclined teeth of the rack the former must be inclined also as shown in elevation in Fig. 81, what were formerly rectilinear elements of the teeth now becoming helices on the cylindrical surface of the gear. The inclination of the teeth will in no way affect the working of the rack and gear and a longitudinal movement of the rack at right angles to the axis of the gear will cause rotation of the latter as before. Owing to the inclination of the teeth, however, a movement of the rack at right angles to the axis of the gear is not the only movement of the rack that will rotate the gear, for an examination of Fig. 81 shows at once that a movement of the rack in a direction parallel to the axis of the gear, that is, perpendicular to the plane of the paper, will also cause rotation of the gear which will, however, be rather limited because of the comparatively limited width of the rack.

In the figure the distance ab is the pitch of the rack teeth,

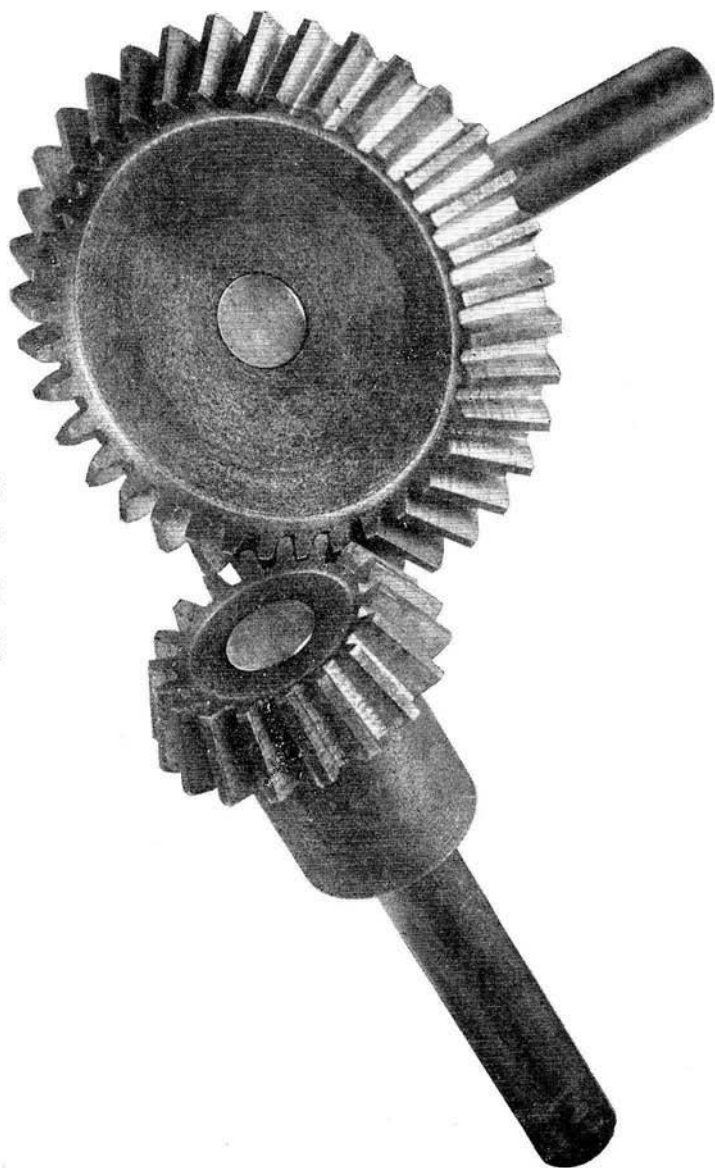


Fig. 80. — Bevel Gears.

equal to the circular pitch of the teeth of the gear, and ac is equal to the length of the arc measured on its pitch circumference through which the gear would be rotated by a movement of the rack over a distance equal to its width in a direction parallel to

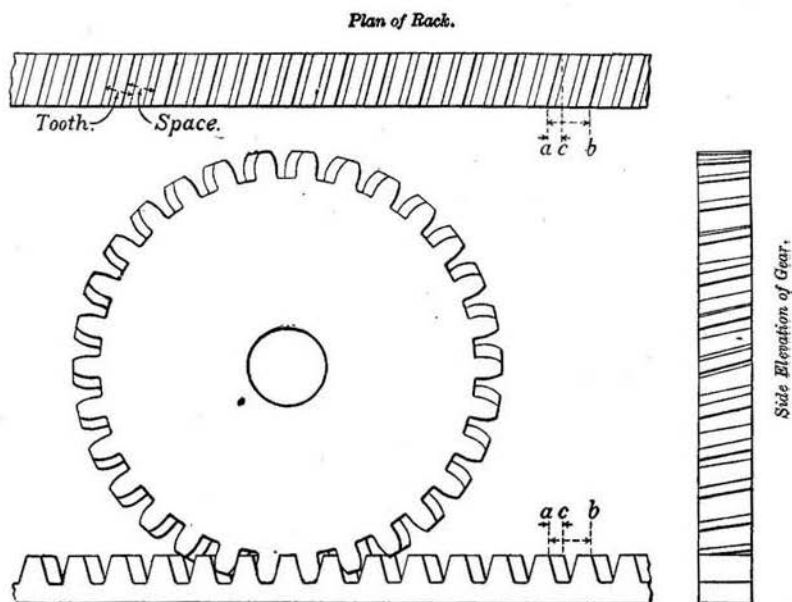


Fig. 81.

the axis of the gear. Now suppose the rack be rolled into a cylinder whose axis is parallel to the longest dimension of the original rack and whose circumference is equal to the width of the rack; then a tooth on the rack will become one complete turn of a helical projection or tooth on the cylinder, and the pitch of the helix so formed will be equal to the distance ac . If all teeth but one be removed from the cylinder and the latter be placed with its axis perpendicular to that of the gear and so that one end of the helix formed by the remaining tooth engages in a tooth space of the gear, the cylinder can be rotated about its axis and its helical tooth during one complete revolution will cause the gear to rotate through an arc measured on its pitch circumference equal to the distance ac , Fig. 81, in the same manner as did the inclined tooth

on the rack when the latter was moved over a distance equal to its width parallel to the axis of the gear. If the rack is of the width shown in the figure, it is evident that after one revolution of the cylinder formed therefrom further motion will be prevented by the end of its helical tooth coming against the end of one of the teeth on the gear, for the movement of the latter measured on its pitch circumference has been less than its circular pitch. If, however, the width of the rack be increased so that the distance ac , Fig. 81, which is the amount one end of an inclined tooth is in advance of the other end, is just equal to ab , the circular pitch of the gear, and the new rack be rolled into a cylinder, it will be seen that, when the helical tooth on the cylinder is engaged in a tooth space of the gear as before and the cylinder is rotated once about its axis, the gear will be rotated through an arc measured on its pitch circumference equal to its circular pitch. After one complete revolution of the cylinder the end of its helical tooth will occupy exactly the same position as when it was first engaged with a tooth space on the gear, but that space will have moved through an arc equal to the circular pitch of the gear so that the helical tooth can no longer engage with it; but as the distance between the centers of the tooth spaces of the gear is equal to its circular pitch, the space in rear will occupy exactly the same position as did the first before the cylinder was rotated and, consequently, the end of the helical tooth on the cylinder will now engage with it. Another complete revolution of the cylinder will again rotate the gear through an arc measured on its pitch circumference equal to its circular pitch, will restore the helical tooth on the cylinder to its original position, and will bring a third tooth space of the gear in position to be engaged by the tooth of the cylinder, and so on; so that continuous rotation of the cylinder will cause continuous rotation of the gear.

In this combination the gear with helical teeth is called a *worm-wheel* and the cylindrical rack with a helical tooth is called a *worm*. A worm and worm-wheel form a particular case of screw gearing, so called because of the screw-like action of the teeth of the gears.

In this discussion reference has been had to a length of the helical tooth on the worm sufficient to make one complete turn only about its axis. This length is all that is necessary to secure

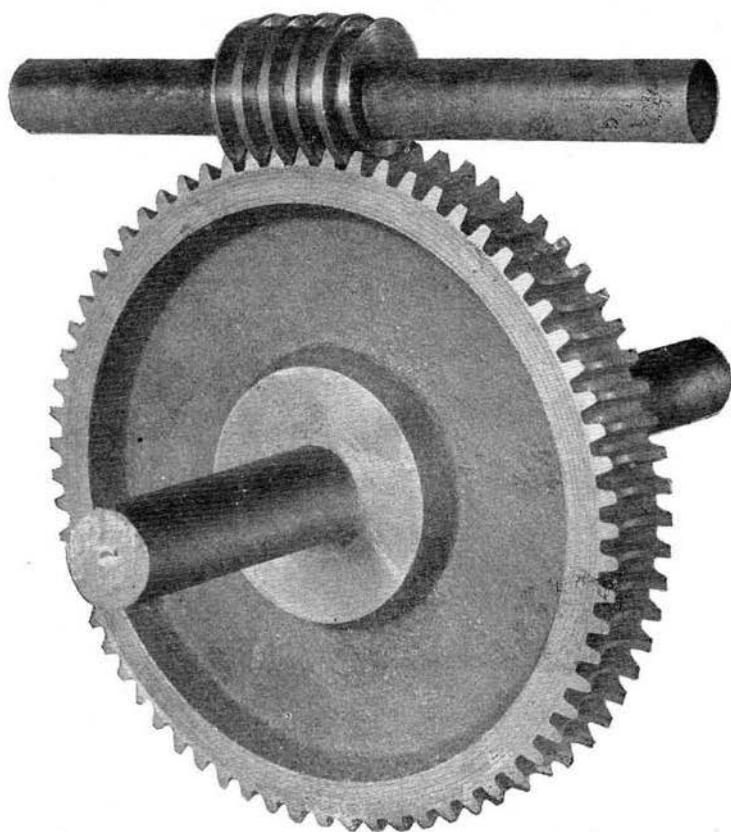
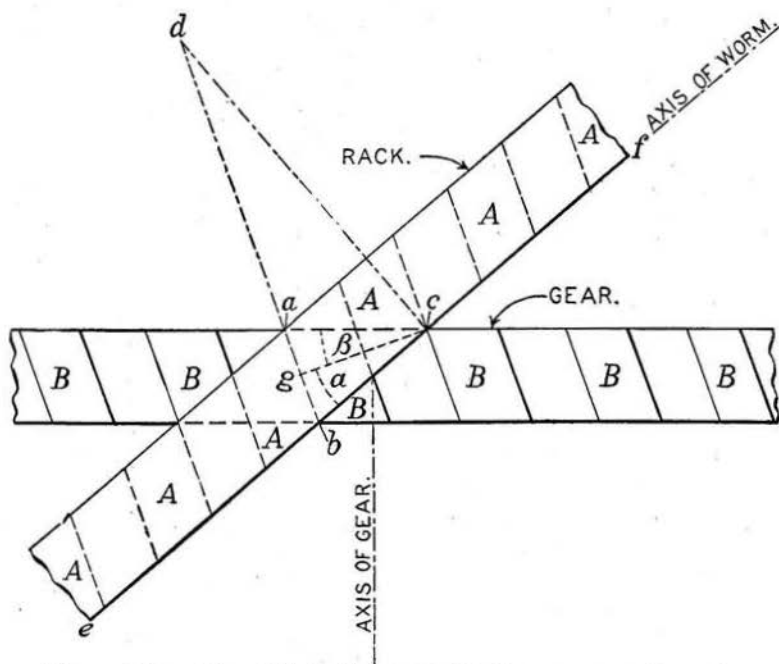


Fig. 82. — Worm and Worm-Wheel.

continuous rotation of the worm-wheel, but in practice several turns are used as this distributes the pressure over two or more teeth on the wheel.

Fig. 82 shows a worm and worm-wheel in mesh.

Referring again to Fig. 81, it is evident that the rack with inclined teeth need not be perpendicular to the axis of the spur gear provided the helical teeth on the gear make the same angle with its axis as do the inclined teeth on the rack. Such a rack and spur gear are shown in plan in Fig. 83, the teeth of the rack being marked *A* and those of the gear being marked *B*.



NOTE. — The outlines of the teeth shown in this figure correspond to sections of the teeth at the pitch surfaces.

Fig. 83.

When this combination is used as a rack and gear the rack must be held in guideways to compel it to move only in the direction of its longest dimension, and the gear must be prevented from moving in a direction parallel to the shaft by collars on the shaft or other suitable means. If the guideways be removed,

however, and the rack given a motion of translation at right angles to the direction of its longest dimension it will still cause a limited rotation of the gear; and, by giving the rack the proper width and wrapping it around a cylinder whose axis is parallel to the longest dimension of the rack, it will become a worm as before which, when rotated continuously about its axis, will cause continuous rotation of the gear, now called a worm-wheel. It is evident from this that it is not necessary that the axes of a worm and worm-wheel be at right angles to each other, although such is generally the case.

In Fig. 83 the circular pitch of the gear is equal to the distance ac , which is the distance between corresponding points of adjacent teeth of the rack measured in a direction at right angles to the axis of the gear. Now in order to rotate the gear through an arc measured on its pitch circumference equal in length to ac by a movement of the rack at right angles to its longest dimension over a distance equal to its width, it is evident that the width of the rack must be made equal to the distance cd , the point d being the intersection of the edge of the tooth passing through a by a line drawn through c at right angles to the longest dimension of the rack. It is also apparent that, when the width of the rack is made equal to cd and the rack is rolled into a cylinder whose axis is ef , the various teeth of the rack will form so many complete turns of a continuous helical tooth on the cylinder, and that one complete revolution of the cylinder or worm will rotate the gear, now called a worm-wheel, through an arc measured on its pitch circumference equal to its circular pitch, which is equal to ac . The pitch of the helix on the worm will then be equal to bc , and the circular pitch of the wheel, equal to ac , is equal to the oblique projection on a plane at right angles to the axis of the wheel of the pitch of the helix of the worm, the projecting lines being parallel to the teeth of the wheel.

Let cg be a line, in a plane parallel to the axes of the worm and wheel, drawn perpendicular to the teeth through their point of contact c , and let α be the angle which this line makes with the axis ef of the worm, and β be the angle which it makes with a plane at right angles to the axis of the wheel; then if α is greater than β a point on the pitch circumference of the wheel will, during one complete revolution of the worm, move through an arc less in

length than the pitch of the helix on the worm, and vice versa if α is less than β . If α is equal to β a point on the pitch circumference of the wheel will, during one complete revolution of the worm, move through an arc equal in length to the pitch of the helix on the worm, the same as it would if the axes of the worm and wheel were perpendicular to each other.

Since the axis of the worm is oblique to the axis of the wheel, the point of contact of the teeth during one revolution of the worm travels in a diagonal line across the rim of the wheel instead of remaining in its central plane as it does when the axes of the worm and wheel are perpendicular to each other. On this account the rim of the wheel must not be too narrow or the teeth will not remain in contact during one complete revolution of the worm.

Spiral Gears.— If instead of wrapping the rack shown in Fig. 83 around a cylinder whose axis is parallel to the longest dimension of the rack, it be wrapped around one whose axis is perpendicular to that dimension, the continuous rotation of this cylinder will also cause continuous rotation of the gear, and the combination so formed is called a pair of spiral gears. Fig. 84 shows a pair of spiral gears in mesh.

Spiral gears are also a particular form of screw gearing and do not differ in principle from a worm and worm-wheel as may be inferred from the manner in which they are developed from a rack and spur gear, or as may perhaps be more readily seen from the following discussion. Suppose the cylinder formed by wrapping the rack of Fig. 83 around an axis perpendicular to its longest dimension be increased in length until one of its helical teeth makes a complete turn about it, and that all teeth except this one be removed from the cylinder. Then, except for the fact that the pitch of the helix on the cylinder is relatively so great in comparison with the diameter and width of the gear that the tooth of the gear would probably rotate out of contact with that of the cylinder before the latter could make one complete revolution, one such revolution of the cylinder would cause a point on the pitch circumference of the gear to move through an arc equal in length to the oblique projection on a plane at right angles to the axis of the gear of the pitch of the helix on the cylinder, the projecting lines being parallel to the teeth of the gear; and, furthermore, by increasing the diameter of the gear, keeping the inclina-

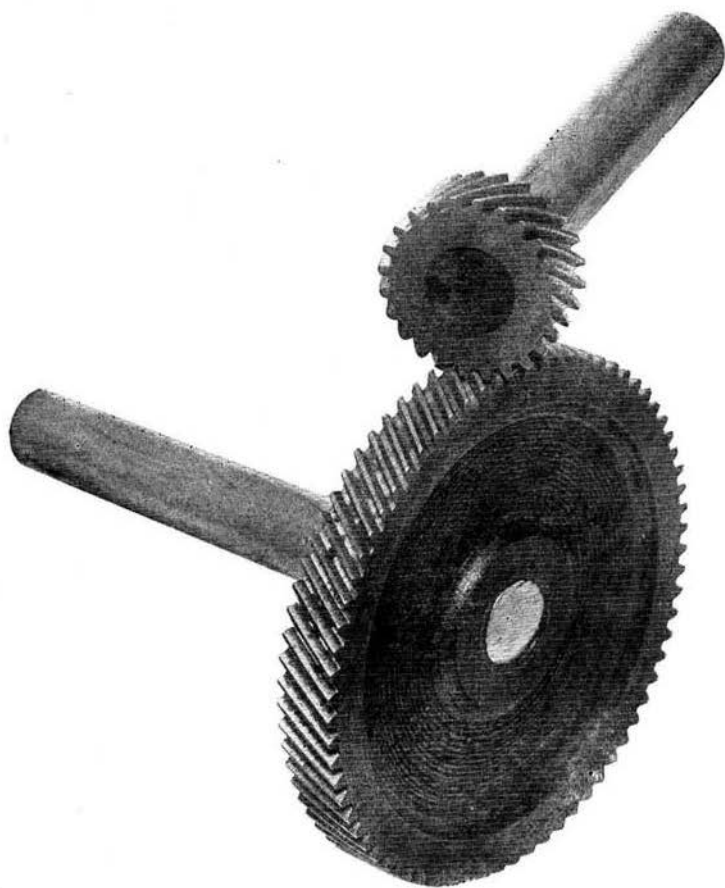


Fig. 84. — Spiral Gears.

tion of its helical teeth the same, the curvature of its pitch circle can be reduced to such an extent that, when the width of the gear is sufficiently increased, its tooth will remain in contact with that of the cylinder during one complete revolution of the latter.

It is thus seen that by increasing the length of one of the spiral gears developed from the rack and spur gear of Fig. 83 and increasing the diameter and the width of the other, keeping the inclination of the helical teeth the same in both cases, a combination has been produced in which, if the first gear be rotated through a complete revolution, the other will be rotated through an arc measured on its pitch circumference equal in length to the oblique projection on a plane at right angles to its axis of the pitch of the helix of the first; and, if the circular pitch of the teeth on the second gear be made equal to this arc, continuous rotation of the first gear will cause continuous rotation of the second. This combination has now become a worm and worm-wheel differing in no essential from the worm and wheel formed by wrapping the rack of Fig. 83 around a cylinder whose axis is parallel to the longest dimension of the rack; and this transformation has been effected without changing the angle between the axes of the gears or those between the teeth and the axes, but only by increasing the length of one gear and the diameter, width, and circular pitch of the other, which changes evidently do not affect the principle upon which the gears operate.

97. Distinction between Worm-Gearing and Spiral Gearing. —

If the helical teeth formed by wrapping the rack around a cylinder make so great an angle with its axis that they make one or more complete turns around it in the length of the cylinder, the resulting gear is called a worm and the gear-wheel with which it meshes is called a worm-wheel. By examination of Figs. 81 and 83 it is seen that the helical teeth formed by wrapping the rack around a cylinder whose axis is parallel to the longest dimension of the rack make several turns around that axis in the length of the cylinder, and this is why the resulting gear was called a worm and that with which it meshes, a worm wheel.

If the helical teeth formed by wrapping the rack around a cylinder make so small an angle with its axis that they do not make a complete turn about it in the length of the cylinder, the resulting gear and the one with which it meshes are called spiral

gears. By examination of Fig. 83 it is seen that when the rack is wrapped around a cylinder whose axis is perpendicular to the longest dimension of the rack, the helical teeth so formed will make but a small portion of a turn around the axis of the cylinder. For this reason the resulting gear and that with which it meshes were called spiral gears.

98. Velocity Ratio of Worm-Gears. — It has already been shown that by choosing a suitable width for the rack shown in Fig. 81 and then rolling it into a cylinder whose axis is parallel to the longest dimension of the rack, the pitch of the helix on the cylinder may be made equal to the circular pitch of the gear. Similarly, by choosing a suitable width for the rack shown in Fig. 83 and rolling it into a cylinder whose axis is parallel to the longest dimension of the rack, the pitch of the helix on that cylinder may be made of such a length that its oblique projection on a plane at right angles to the axis of the gear, by lines parallel to the teeth of the gear, will be equal to the circular pitch of the gear. Each separate tooth of either rack will then form one complete turn of a helix on the cylinder and, furthermore, the various turns formed by the teeth of the rack will become so many parts of one continuous helical tooth or thread on the cylinder, so that the resulting worm is called a *single-threaded worm*.

One revolution of a single-threaded worm will rotate the worm-wheel through an arc measured on its pitch circumference equal in length to its circular pitch, and to cause the wheel to make a complete revolution the worm must make as many revolutions as there are teeth on the wheel. Since the single thread on the worm is in reality a single tooth, the angular velocities of the worm and worm-wheel are to each other inversely as the numbers of their teeth, as is the case with all toothed gears.

If the width of the rack be increased to twice its former amount, the inclination of the teeth remaining the same as before, every alternate tooth of the rack, when it is rolled into a cylinder whose axis is parallel to that of the first cylinder, will form part of a continuous helical tooth or thread around the cylinder; and there will be two such threads, the worm in this case being called a *double-threaded worm*. Since the pitch of the helix on the worm is double what it was before, either one of the two threads will engage only with every alternate tooth of the gear, the remaining

teeth of the gear engaging with the other thread on the worm. One revolution of a double-threaded worm will rotate the worm-wheel through an arc measured on its pitch circumference equal in length to twice its circular pitch, and to cause the wheel to make a complete revolution the worm must make half as many revolutions as there are teeth on the wheel.

If there are three separate threads on the worm it is called a *triple-threaded worm*, and so on. When any multiple-threaded worm meshes with a worm-wheel the angular velocities of the worm and wheel are inversely as the numbers of the teeth or threads on the worm and the worm-wheel. The ratio of the angular velocities of a worm and worm-wheel in mesh is independent of their radii. The teeth on a worm are referred to as threads.

Velocity Ratio of Spiral Gears. — The angular velocities of a pair of spiral gears in mesh are inversely as the numbers of the teeth on the gears, and the ratio of the angular velocities is independent of the radii of the gears except in the one case where the axes of the gears are perpendicular to each other and the helix angle between the teeth of each gear and a line perpendicular to its axis is forty-five degrees. In this case the numbers of the teeth on the gears are directly proportional to, and the angular velocities inversely proportional to, the pitch radii of the gears.

Tooth Curves of Screw Gears. — In view of the development of screw gearing from a rack and spur gear it is evident that the tooth surfaces may belong to either the involute or the cycloidal system. The involute system is generally preferred for worms because the involute thread for racks and worms has straight sides, and is on this account more readily cut in the lathe than the cycloidal thread.

99. Shafts Connected by Screw Gearing. — Drivers. — Pitch of Screw Gearing. — Since the axes of spiral gears and worm and worm-wheels in mesh are in different planes and not parallel to each other, such gearing is used to transmit rotary motion between shafts that are neither parallel nor intersecting. The worm and worm-wheel are used particularly when it is desired to obtain a large velocity ratio between the shafts.

Either one of a pair of spiral gears in mesh may be the driver.

In practice, however, that spiral gear is taken as the driver which has the smaller helix angle between its teeth and a line perpendicular to its axis, since greater efficiency of the gears results from doing so.

In the case of a worm and worm-wheel the worm is the driver. When the angle between the threads of the worm and a line perpendicular to its axis is less than the angle of friction, which is generally the case, the worm-wheel cannot drive the worm. If, however, this angle is greater than the angle of friction the worm-wheel can drive the worm.

The *circular pitch* of a spiral gear is the distance between corresponding points of adjacent teeth measured on the cylindrical pitch surface at right angles to the axis of the gear. The distance measured on the pitch surface at right angles to the teeth is the *normal pitch*, and that measured in a direction parallel to the axis of the gear is the *axial pitch*. For two spiral gears to run together the normal pitches must be the same in both, and, if the axes of the gears are perpendicular to each other, the circular pitch of the one must also be equal to the axial pitch of the other.

The term circular pitch is not used in connection with a worm, but it is in connection with a worm-wheel and has the same significance with regard to the latter as for a spiral gear. The *axial pitch* of a worm is the distance measured on its pitch surface in a direction parallel to its axis between corresponding points of adjacent threads of a multiple-threaded worm or of adjacent parts of the same thread of a single-threaded worm. In the case of a single-threaded worm the axial pitch is the same as the pitch of the helix formed by its thread. The distance between corresponding points of adjacent threads of a multiple-threaded worm or of adjacent parts of the same thread of a single-threaded worm measured on its pitch surface at right angles to the threads is the *normal pitch*. The axial and normal pitches of a worm-wheel are the same as for a spiral gear. For a worm and worm-wheel to run together the normal pitches must be the same in both and, when their axes are perpendicular to each other, as is generally the case, the axial pitch of the worm must be equal to the circular pitch of the worm-wheel.

The *lead* of a worm is the distance any separate thread advances in the direction of the axis of the worm while making one

complete turn around it. It is the same as the pitch of the helix formed by the thread.

100. Wheel Train. — Velocity Ratio of First and Last Shafts Connected by a Wheel Train. — A series of gears interposed between two shafts is called a wheel or gear train. In Fig. 85

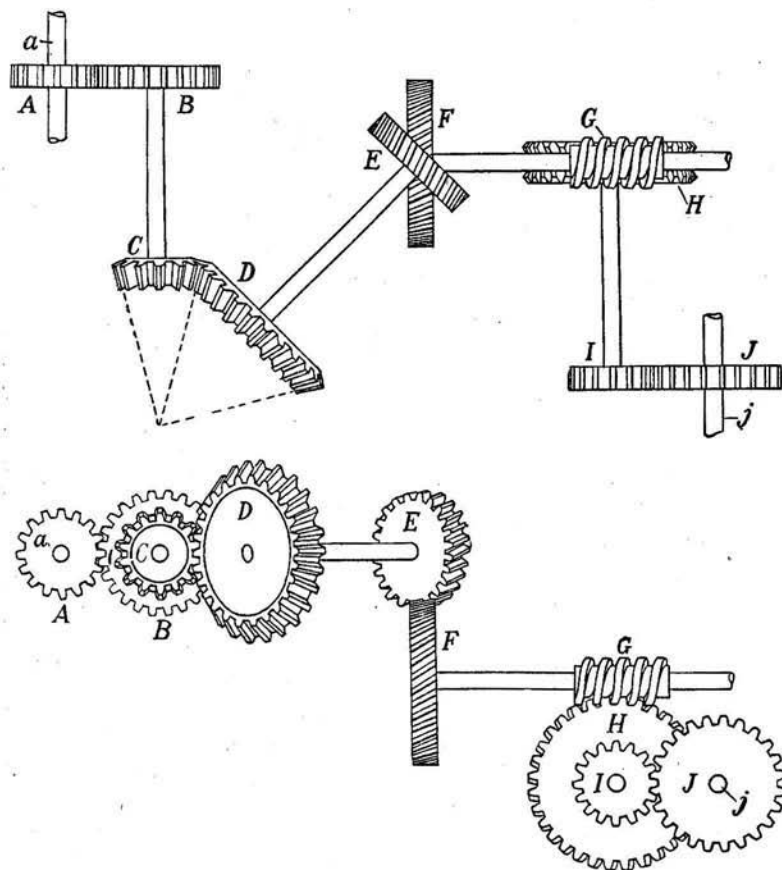


Fig. 85.

is shown a wheel train connecting the driving shaft *a* with the driven shaft *j*.

A spur gear *A* on the shaft *a* meshes with another *B*. On the same shaft with *B* is a bevel gear *C* meshing with another bevel gear *D* carried by a third shaft, on the other end of which is a

spiral gear *E*. The gear *E* meshes with a second spiral gear *F* and on the same shaft with *F* is a single-threaded worm *G* driving a worm-wheel *H*. On the other end of the shaft which carries *H* is a spur gear *I* that drives the spur gear *J* on the shaft *j*.

Let it be required to find the ratio of the angular velocities of the shafts *a* and *j* or, what is the same thing, the ratio of the numbers of revolutions per minute of the shafts. Let N_a represent the number of revolutions per minute of the shaft *a* and of the gear *A*, N_b the number of revolutions per minute of the gear *B*, N_c the number of revolutions per minute of the gear *C*, etc.; and let T_a represent the number of teeth of *A*, T_b the number of teeth of *B*, etc. Then

$$\frac{N_a}{N_b} = \frac{T_b}{T_a}; \quad \frac{N_c}{N_d} = \frac{T_d}{T_c}; \quad \frac{N_e}{N_f} = \frac{T_f}{T_e}; \quad \frac{N_g}{N_h} = \frac{T_h}{T_g}; \quad \frac{N_i}{N_j} = \frac{T_j}{T_i}$$

Multiplying together the first terms of the various equalities and placing the product equal to the product of all the last terms of the equalities, we have

$$\frac{N_a \times N_c \times N_e \times N_g \times N_i}{N_b \times N_d \times N_f \times N_h \times N_j} = \frac{T_b \times T_d \times T_f \times T_h \times T_j}{T_a \times T_c \times T_e \times T_g \times T_i} \quad (1)$$

But since *B* and *C* are on the same shaft $N_b = N_c$, and, similarly, $N_d = N_e$, $N_f = N_g$, and $N_h = N_i$. Making these substitutions in equation (1) we obtain

$$\frac{N_a}{N_j} = \frac{T_b \times T_d \times T_f \times T_h \times T_j}{T_a \times T_c \times T_e \times T_g \times T_i} \quad (2)$$

Referring to Fig. 85 it will be seen that the gears *B*, *D*, *F*, *H*, and *J* are all driven gears or followers and the gears *A*, *C*, *E*, *G*, and *I* are all drivers, and we may, therefore, write that *when two shafts are connected by a wheel train the ratio of the angular velocities of the driving and driven shafts or of the numbers of revolutions per minute of these shafts is equal to the product of the numbers of the teeth of all the followers divided by the product of the numbers of the teeth of all the drivers.*

101. Idlers. — If a wheel train connecting two shafts consists of three or more gears meshing directly the one into the other as

shown in Fig. 86, the velocity ratio of the driving and driven shafts is the same as if the gears on these shafts meshed directly into each other without the interposition of the intermediate gears, for the gear *B* is a follower with respect to *A* and a driver with respect to *C* and the number of its teeth will appear both in the numerator and the denominator of the fraction representing

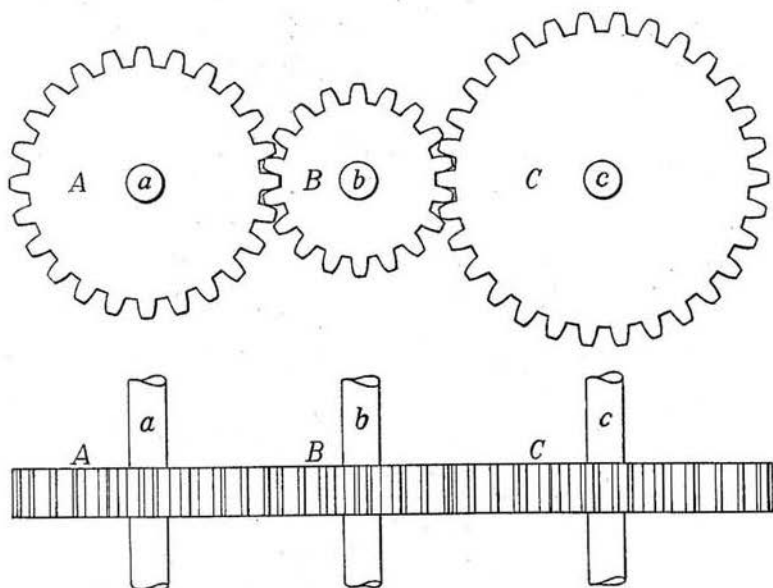


Fig. 86.

the ratio of the angular velocities of the shafts *a* and *c*, and will, consequently, cancel out of the fraction. If another gear be interposed between *B* and *C* it also will act both as a follower and a driver and can, consequently, have no effect on the velocity ratio of *a* and *c*.

By examination of Fig. 86 it will be seen that gears *A* and *B* turn in opposite directions as do gears *B* and *C* and, consequently, *A* and *C* and the shafts to which they are fastened turn in the same direction. If another gear be interposed between *A* and *C* the latter will turn in the opposite direction from *A*; and, in general, if the number of gears meshing directly the one into the other as shown in Fig. 86 is odd, the first and last gears will turn

in the same direction, but if the number is even the first and last gears will turn in opposite directions. All the gears in such a train except the first and last are called *idlers*, and their function is principally to change the direction of rotation of the driven shaft since they cannot change its angular velocity. In an exceptional case when shafts are so far apart that two gears connecting them would be inconveniently large, idlers might be used to bridge over the distance between them.

102. Relation between the Power and the Resistance in a Wheel Train. — Referring to Fig. 85, the shaft a is driven by some source of energy which applies a force P called the power at the end of a lever arm p with respect to the axis of the shaft. The source of energy may be human as when the shaft a is turned by the hand at the extremity of the lever p , or the shaft may be driven by still another gear-wheel not shown in the figure, or by a belt. If driven by another gear-wheel the power will be the force exerted between the teeth of the wheels and its lever arm may be taken as the pitch radius of the driven wheel on the shaft a . If the shaft is driven by a belt the power will be the tension in the belt, or, more strictly, the difference in tension between the upper and lower parts of the belt considered as horizontal, and its lever arm will be the radius of the pulley increased by one-half the thickness of the belt. The energy received from the source is transmitted through the wheel train and, neglecting friction, is all delivered to the shaft j where it is made to perform useful work in overcoming a resistance R having a lever arm r with respect to the axis of the shaft j .

Let it be required to find the power P that must be applied with a lever arm p to the shaft a to overcome through the wheel train shown in Fig. 85 a resistance R having a lever arm r with respect to the shaft j , (a) when friction is neglected, and (b) when the friction of the gearing is considered.

Friction Neglected. — Neglecting friction is equivalent to assuming that all the energy supplied to the shaft a is transmitted to the shaft j and, therefore, the work of the power P in one minute must be equal to the work of the resistance R in one minute. Since the number of revolutions per minute of a is N_a the path of the power in one minute is $N_a \times 2\pi p$ and the work performed by it in one minute is $P \times N_a \times 2\pi p$; and,

similarly, the work performed by the resistance in one minute is $R \times N_j \times 2 \pi r$, whence

$$P \times N_a \times 2 \pi p = R \times N_j \times 2 \pi r$$

or
$$P = R \times \frac{r}{p} \times \frac{N_j}{N_a} \quad (3)$$

and replacing the ratio $\frac{N_j}{N_a}$ in equation (3) by its value from equation (2)

$$P = R \times \frac{r}{p} \times \frac{T_a \times T_c \times T_e \times T_g \times T_i}{T_b \times T_d \times T_f \times T_h \times T_j} \quad (4)$$

and we may write that *the power required to overcome a resistance through any given wheel train, when friction is neglected, is equal to the resistance multiplied by the ratio of the lever arms of the resistance and the power with respect to the axes of the shafts of the last and first wheels, respectively, of the train, multiplied by the product of the numbers of the teeth of all the drivers in the train divided by the product of the numbers of the teeth of all the followers.*

Friction of Gearing Considered. — When energy is transmitted from one shaft to another through a wheel train, part of it is lost in performing wasteful work due to the friction between the teeth of the gears and that between the shafts and their bearings. To determine the latter it is necessary to know the normal pressures between all the shafts and their bearings; the radii and angular velocities of all the shafts; and the coefficients of friction, which vary with the normal pressures, the speeds of the rubbing surfaces, the materials in contact, the nature of the lubricant, if any, and the temperature. While the energy lost by friction between the shafts and their bearings can be taken into account with reasonable accuracy in any particular case, it will not be considered in this discussion as it is ordinarily considerably less than that lost through the friction of the gearing.

By analytical methods and by experiment the percentage of energy transmitted to energy received has been determined for a pair of each of the different classes of gears. These percentages are called efficiencies and they are always fractions less than unity. Represent the efficiencies of

spur gears by E_s ,
bevel gears by E_b ,

spiral gears by E_{sp} , and
worm-gears by E_w .

Now since the velocity ratio of the shafts a and j is constant and not affected by friction, the loss of energy must appear entirely in the reduction of the value of the resistance R and, therefore, the value of P that will produce a given value of R must be the theoretical value given by equation (4) divided by the product of the efficiencies of all the pairs of gears in the wheel train, whence

$$P = R \times \frac{r}{p} \times \frac{T_a \times T_c \times T_e \times T_g \times T_i}{T_b \times T_d \times T_f \times T_h \times T_j} \times \frac{1}{E_s \times E_b \times E_{sp} \times E_w \times E_s} \quad (5)$$

It should be noted that this equation is true only when R is the resistance and P the driving force. If P becomes the resistance and R the driving force, the term involving the efficiencies must be inverted since in this case it is P instead of R that must be multiplied by this term.

103. Efficiencies of Various Classes of Gears. — The efficiency of a pair of spur gears varies from about .90 at very slow speeds to about .985 when the linear speed of a point on the pitch circumference is equal to or greater than 200 feet per minute.

When the linear speed of a toothed gear is referred to it is to be understood as meaning the speed of a point on its pitch circumference.

The efficiency of a pair of spiral or worm-gears varies with the helix angle which the thread makes with a line perpendicular to the axis of the gear having the smaller helix angle, and also with the linear speed of a point on the pitch circumference. At very slow speeds the efficiency varies from about .30 for a helix angle of five degrees to about .75 for a helix angle of forty-five degrees. At speeds equal to or greater than 200 feet per minute the efficiency varies from about .75 for a helix angle of five degrees to about .90 for a helix angle of forty-five degrees. The low efficiency of spiral and worm-gearing is largely due to the thrusts of the helical threads in directions parallel to the axes of the shafts, which develop great pressures against the end bearings that are required to prevent longitudinal movement of the shafts. It is evident from this why the efficiency increases with the helix angle of the threads. By interposing ball or roller bearings between the shafts and their end bearings the efficiency of spiral and worm-gearing is much increased.

The efficiency of a pair of bevel gears varies from about .85 at very slow speeds to about .95 at speeds equal to or greater than 200 feet per minute.

By reference to equation (5) it is seen how great is the increase required in the value of the power over its theoretical value to produce a given value of the resistance when the number of gears in the train is large or when one or more of the pairs consist of screw gears. Screw gears are valuable, however, when it is desired to transmit motion between nonparallel shafts in different planes, and worm-gears are particularly useful when a large velocity ratio between shafts is required.

104. Example.—The elevating mechanism of the 5-inch barbette carriage, model of 1903, is shown diagrammatically in Fig. 87.

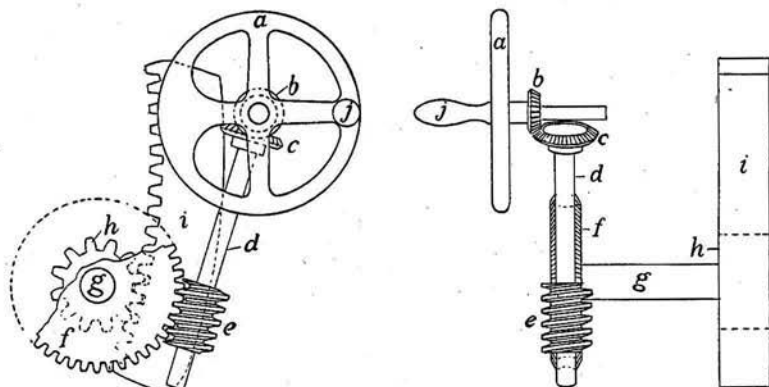


Fig. 87.

A hand-wheel *a* with crank-handle *j* is mounted on a short horizontal shaft carried in bearings in the elevating bracket. The elevating bracket is bolted to the platform bracket and the latter is bolted to the pivot yoke. On the shaft to which the hand-wheel *a* is attached is a bevel gear *b* engaging with another *c* on an inclined shaft *d* also carried in bearings in the elevating bracket. On the other end of this shaft is a worm *e* engaging with a worm-wheel *f*, on the same short horizontal shaft *g* with which is the spur gear or pinion *h* that meshes with the circular rack *i*. The shaft *g* is carried in bearings in the elevating bracket and the rack *i* is attached to the cradle carrying the gun. The

circular rack i forms a part of a spur wheel with internal teeth whose axis is the axis of the trunnions of the cradle. The trunnion may, therefore, be considered as the shaft to which i is attached. The following data are known from the construction of the carriage, viz.:

Lever arm of crank-handle with respect to axis of first shaft

Number of teeth in bevel gear $b = 18$. [= 5 ins.

Number of teeth in bevel gear $c = 30$.

Worm is single-threaded (number of teeth = 1).

Number of teeth in worm-wheel = 40.

Number of teeth in spur pinion = 12.

Number of teeth in spur gear of which rack is a part = 175.

Radius of trunnions = 2.505 ins.

Coefficient of friction at trunnions = .1.

Weight of parts resting on trunnions (gun and cradle) = 14900 lbs.

Efficiency of a pair of bevel gears = .85.

Efficiency of a pair of worm-gears = .35.

Efficiency of pinion and rack (spur gears) = .90.

Neglecting the friction between the shafts and their bearings, let it be required to determine the force P that must be exerted on the crank-handle j to start the gun in elevation, assuming that the center of mass of the parts resting on the trunnions is in the axis of the trunnions so that the weight has no moment with respect to that axis.

Solution. — The resistance to be overcome is the force of friction at the cylindrical surfaces of the trunnions, which is equal to $14900 \times .1 = 1490$ lbs. This resistance has a lever arm of 2.505 ins. with respect to the axis of the trunnions, and the lever arm of the power with respect to the axis of the first shaft is 5 ins. Therefore, noting from Fig. 87 that the gears b , e , and h are drivers and the gears c , f , and i are followers, we have, in accordance with the principles discussed in deducing equation (5),

$$P = 1490 \times \frac{2.505}{5} \times \frac{18 \times 1 \times 12}{30 \times 40 \times 175} \times \frac{1}{.85 \times .35 \times .90} = 2.87 \text{ lbs.} \quad (6)$$

2.87 lbs. is, therefore, the force on the crank-handle required to start the gun in elevation or, assuming that the coefficient of

friction at the trunnions is constant, it is the force required to *keep* the gun moving in elevation with a uniform angular velocity.

Force on the Crank-Handle Required to Produce in a given Time a given Angular Velocity of the Rotating Parts. — To attain a given angular velocity starting from rest it is necessary, however, to give the rotating parts an angular acceleration until the given angular velocity is reached. To determine the increase in the force required on the crank-handle to give the gun a desired angular velocity in elevation in a given time is not a difficult problem providing we know the moment of inertia of the rotating parts about the axis of the trunnions. Having decided on the angular velocity desired and the time in which it is to be attained, the angular acceleration is found by dividing the angular velocity by the time. To produce an angular acceleration $d^2\phi/dt^2$ about a fixed axis requires a moment in foot pounds about that axis equal to $\Sigma mr^2 \times d^2\phi/dt^2$, in which Σmr^2 is the moment of inertia in mass units and feet of the rotating parts about the axis. While the force required to produce this angular acceleration is actually applied at the pitch circumference of the rack fastened to the cradle, it is immaterial where it be taken as applied provided its moment with respect to the axis of the trunnions is $\Sigma mr^2 \times d^2\phi/dt^2$. For convenience assume the force to be applied at the surface of the trunnions as is the friction due to the weight of the parts. Then, the radius of the trunnions being 2.505 ins., the force is

$$\frac{\Sigma mr^2 \times d^2\phi/dt^2}{2.505/12} \text{ lbs.}$$

If now the value of the resistance, 1490 lbs., in equation (6) be increased by the value of the force just determined, the resulting value of P will be the force on the crank-handle required to overcome the resistance of the friction at the trunnions and to give the gun and other rotating parts the desired angular velocity in elevation in a given time. Computations such as these are important in designing the gearing of a gun carriage, for it is necessary that the gun can be elevated or traversed through a given number of degrees in a reasonably short time without too much effort being required at the crank-handles.

105. Pressure between the Teeth of any Pair of Gears in a Wheel Train. — Although it has been shown that the action line of the pressure between the teeth when the involute system is used makes an angle of 15° , and when the cycloidal system is used an angle varying from 30° to 0° , with the common tangent to the pitch circles, no serious error will be committed by assuming it to be coincident with this tangent. Referring to Fig. 85, the pressure between the teeth of any two gears as C and D can be determined if either the power P or the resistance R and the corresponding lever arm are known. If the resistance R and its lever arm are known the pressure between the teeth of C and D may be regarded as a power with respect to the rest of the train and determined as was the power P except that the part of the train between A and D must not be considered. The gears C and D being bevel gears, their pitch surfaces are conical and the lever arm of the pressure between the teeth with respect to the axis of either should be taken equal to the pitch radius at the middle of the length of the teeth. (In practice this length is called the face of the teeth.) If the power P and its lever arm are known the pressure between the teeth of C and D can be regarded as a resistance and its value determined by considering only that portion of the wheel train between A and D .

Frequently only the horse-power delivered to the first or last shaft of a wheel train and the speed of the shaft are known. Horse-power is a measure of the rate of doing work and one horse-power is equal to 33000 ft. lbs. per min. Let the number of horse-power delivered to the shaft a , Fig. 85, be represented by H and the number of revolutions per minute of the shaft by N_a ; then if the point of application of the force required to produce this number of horse-power be at a distance of p inches from the axis of the shaft, the path it will follow in one minute is $2\pi p N_a / 12$ ft., and the intensity of the force required to produce H horse-power when acting with a lever arm of p inches is

$$P = \frac{H \times 33000 \times 12}{2\pi p N_a}$$

The moment of this force (the power) with respect to the axis of the shaft is, in in. lbs.,

$$Pp = \frac{H \times 33000 \times 12}{2\pi N_a} \quad (7)$$

Similarly, if the horse-power delivered to the shaft j be represented by H and the number of revolutions per minute of the shaft by N_j , the moment of the resistance in in. lbs. is

$$Rr = \frac{H \times 33000 \times 12}{2 \pi N_j} \quad (8)$$

Having obtained either the moment of the power from equation (7) or the moment of the resistance from equation (8), the moment of the other can be obtained from equation (5), or more readily from equation (9) below, which is equation (5) put in a more convenient form for this purpose.

$$Pp = Rr \times \frac{T_a T_c T_e T_g T_i}{T_b T_d T_f T_h T_j} \times \frac{1}{E_s E_b E_{sp} E_w E_s} \quad (9)$$

If the moment of the power is known the moment of the pressure between the teeth of any pair of gears in the train can be obtained by regarding it as the moment of the resistance and considering only the part of the train between the point of application of the power and that of the pressure between the teeth. If the moment of the resistance is known the moment of the pressure between the teeth of any pair of gears in the train can be obtained by regarding it as the moment of the power and considering only the part of the train between the point of application of the resistance and that of the pressure between the teeth. The pressure between the teeth of the gears can then be obtained by dividing the moment of the pressure by the pitch radius of the driving gear in case the pressure is regarded as a resistance, and by the pitch radius of the driven gear in case the pressure is regarded as a power.

106. Stresses in the Shafts. — The pressure between the teeth of a pair of gear-wheels causes a torsional moment on each shaft equal to the pressure multiplied by the pitch radius of the gear on that shaft.* This pressure also causes a bending moment and a shearing stress in each shaft. The bending moment on each shaft will depend on the pressure and the distance between the supports (called the bearings) of the shaft. When the torsional and bending moments and the diameters of the shafts are known,

* Assuming that the action line of the pressure between the teeth is tangent to the pitch circles of both gears.

the stresses can be computed as explained in Chapter IV. The torsional and bending stresses should be combined by equations (24) and (25), Chapter IV.

107. Stresses in the Gear-Teeth.—The teeth of gears may be considered as cantilevers subjected to a bending force equal to the pressure between the teeth. In practice the height of a tooth above the pitch circumference is made equal to the circular pitch divided by π . The depth below the pitch circumference is equal to the height above it increased by a clearance equal to .05 times the circular pitch, the clearance being allowed to permit the rounding of the corners at the bottoms of the tooth spaces and to prevent the bottoming of the teeth in the tooth spaces. The thickness of the tooth at the pitch circumference is equal to one-half the circular pitch. The thickness at the root varies with the circular pitch and also with the size of the gear. Special formulas have been devised for determining the stresses in gear-teeth based upon their variation of form and, consequently, of the variation in position of the weakest section, which generally occurs a little above the root of the tooth. These formulas are given in mechanical engineering hand-books and in works on machine design. One of the best is Lewis's formula which is as follows:

$$S = \frac{W}{pf \left(.124 - \frac{.684}{n} \right)} \quad (10)$$

in which S is the maximum bending stress; W is the pressure between the teeth in pounds; p is the circular pitch in inches; f is the face of the teeth in inches, that is, the length of the teeth measured in a direction parallel to the axis of spur gears, and along the element of contact in bevel gears; and n is the number of teeth in the gear. This formula will answer for spur gears, spiral gears, and worm-wheels. The tooth of the worm is always stronger than that of the worm-wheel with which it meshes.

For bevel gears Lewis's formula is as follows:

$$S = \frac{W}{pf \left(.124 - \frac{.684}{n} \right)} \times \frac{D}{d} \quad (11)$$

in which S , W , f , and n have the same meanings as before, p is the circular pitch in inches at the large end of the bevel-gear tooth, D is the pitch diameter at the large end of the tooth, and d is the pitch diameter at the small end. To make allowance for shocks when the speed of the gearing is increased the factors of safety should increase with the speed.

In the absence of a special formula the maximum stress in gear-teeth may be approximately obtained with the error on the safe side by considering the whole pressure uniformly distributed along the top edge of the tooth and at right angles to the plane passing through the center of the tooth and the axis of the wheel, by taking the length of the cantilever as the total height of the tooth including the clearance, and by considering the dangerous section as located at the root and as having a thickness equal to that of the teeth at the pitch circumference, that is, equal to one-half the circular pitch. The height of bevel-gear teeth should be taken equal to half the sum of the heights at the large and small ends, and the thickness as half the sum of the thicknesses at the large and small ends.

108. Stresses in the Arms of Gear-Wheels. — The teeth of large gears, or gear-wheels, are formed on a rim connected to a nave or hub by several arms. The stresses in the arms may be determined by considering them as cantilevers subjected to a bending moment equal to the pressure between the teeth multiplied by the length of the arm divided by the number of arms. In order to err somewhat on the safe side the length of the arm is taken equal to the pitch radius of the gear.

Proportions for the Rims of Gear-Wheels. — Arbitrary proportions for the rims of gear-wheels based on experience have been adopted. These proportions are given in mechanical engineering hand-books and standard works on machine design. A standard thickness for rims of gear-wheels whose circular pitch is greater than one and one-half inches is one-half the circular pitch. For gear-wheels having pitches less than one and one-half inches the thickness of the rim is taken as

$$.4p + 1/8 \text{ in.} \quad (12)$$

in which p is the circular pitch in inches. A rib is added to the under side of the rim. The width of the rib is the same as that

of the arms of the gear-wheel and its height is equal to the thickness of the rim.

Proportions for the Hubs of Gear-Wheels. — Arbitrary proportions for the hubs of gear-wheels adopted as a result of experience are also given in mechanical engineering hand-books and standard works on machine design. If the gear-wheel has to transmit the full power of the shaft the thickness of the hub is made equal to the radius of the shaft. If the gear-wheel does not have to transmit the full power of the shaft the thickness of the hub is given by the following formula:

$$t = \sqrt[3]{fpR}/3 \quad (13)$$

in which t is the thickness of the hub in inches, f is the face of the teeth in inches, p is the circular pitch in inches, and R is the pitch radius in inches. The length of the hub varies from f to $1.4f$.

109. Gear Cutting. — The teeth of spur gears, spiral gears, and worm-wheels are cut in a universal milling machine such as that which the cadets have operated in their practical work. When a large amount of gear cutting has to be done special milling machines are used which are peculiarly adapted to the work and are not capable of performing the miscellaneous character of work done by the regular milling machine. The particular feature of the special machines which renders them most useful is that they are automatic in their action. After being started they continue to work without further attention until all the teeth are cut. After one tooth space has been cut the gear-wheel is moved past the cutter and rotated through the proper angle so as to be ready for the next cut, all by the automatic operation of the machinery. As the principles of the automatic machines are the same as those of the milling machine with which the cadets are familiar, the latter will be referred to in the descriptions that follow.

Rotary Cutters. — The profiles of the teeth of the milling cutters used are exact duplicates of those of the tooth spaces they are required to produce. The manufacture of milling cutters is a specialty not much practised by the ordinary machine shop for, by reason of the special appliances used and the quantity made at one time, the manufacturers of the cutters can furnish a better article for less money than it would cost to make it in an

ordinary shop. A brief description of the general method of making cutters for gear-teeth is as follows: A piece of tool steel is turned in a lathe to the proper diameter and width and then notches for the teeth are cut in the milling machine. The piece is then taken back to the lathe and the outline of the teeth cut by a lathe tool, shown in Fig. 88, the cutting edges of which have the outline of a tooth space and have been shaped by filing to a template prepared from a drawing of the tooth curves. As the top surfaces of the teeth of a milling cutter are not concentric with its axis but are of a spiral character to give proper clearance and keenness to the cutting edges, as shown by the lines *ab* in Fig. 88 in which the points *a* are at the cutting edges, the lathe

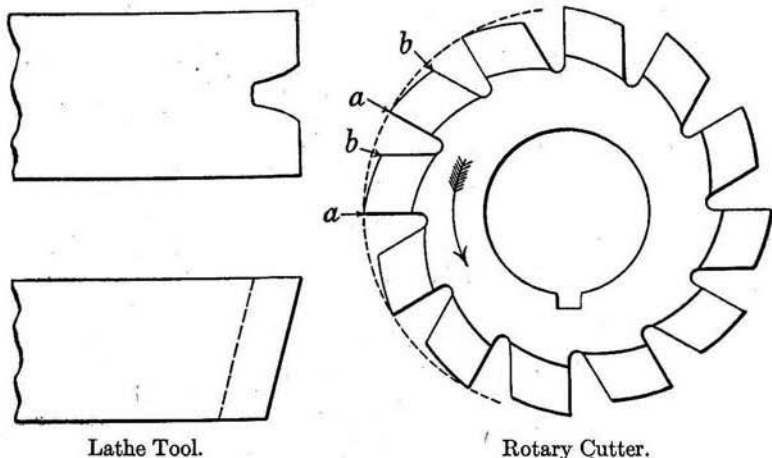


Fig. 88.

is provided with an attachment called a backing off or relieving attachment which forces the lathe tool gradually in as the work is revolving past it from *a* to *b* and then draws it out quickly to be in position to cut the edge *a* at the proper height on the next tooth. After the outlines of the teeth have been cut in the lathe the cutter is hardened and tempered.

110. Cutting the Teeth of Spur Gears. — The wheel or gear before the teeth are cut is called a gear-blank. This is turned to the required shape and size in the lathe. It is then mounted on centers between the index head and tail stock of the milling

machine. The rotary cutter is placed on the arbor with its center in the vertical plane containing the axis of the gear-blank, and the table raised to give the proper depth of cut. The machine is then started, the feed mechanism thrown in, and the table fed past the cutter. It is then drawn back by hand, the blank turned through the proper angle by the index mechanism, and another cut taken and so on.

Cutting the Teeth of Spiral Gears. — The method of cutting the teeth of spiral gears is essentially the same in principle as for spur gears, the only difference being that the teeth are helical. The table of the milling machine is, therefore, set at the proper angle with the axis of the spindle, and the index head is connected by the requisite gearing to the feed screw of the table in order to give the gear-blank such a combined motion of translation and rotation as will secure the proper helix angle, these operations being similar to those performed by the cadets in cutting the spiral teeth of a reamer.

111. Cutting the Teeth of a Worm and Worm-Wheel. — The worm is turned and threaded in the lathe, the thread being cut in practically the same way as the thread of an ordinary bolt except that a roughing tool is first used, being followed by the finishing tool. The roughing tool has a square nose slightly narrower than the tooth space at the bottom of the thread. When the diameter at the bottom of the thread has been reduced to the prescribed size with the roughing tool, the finishing tool is put in the tool post and the thread finished with it. The finishing tool is very carefully made and filed to a template whose profile is the same as that of the tooth space of the worm.

In order that the teeth of a worm and worm-wheel whose axes are perpendicular to each other shall be in contact over a considerable arc measured on the pitch surface of the worm in a direction parallel to its thread, the face of the wheel is made concave to fit the curvature of the worm and the teeth are cut in a milling machine using a rotary cutter called a hob. The worm-wheel blank is turned in a lathe. If the number of worm-wheels under manufacture justifies the expense, particularly if the wheels are large, an attachment is made or purchased for turning the concave face of the wheel. This attachment is placed on the

tool carriage of the lathe and permits the cutting tool to be fed around a fixed center on the carriage, thus turning the concave face to the required radius. The longitudinal feed of the lathe is not used with this attachment. The cross feed is used only to obtain the proper depth of cut. If only a few worm-wheels are to be made, particularly if the wheels are small, an attachment such as described would not be used. Instead the concave face of the wheel is first roughed out with a round-nose tool to approximately the correct outline as shown by a template, using the power longitudinal feed and operating the cross feed by hand. The cut is started at the right edge of the face of the wheel and the tool is fed to the left by power while being fed in by hand in such manner as to make it follow approximately the arc of a circle. When the tool reaches the center of the face it is withdrawn from the work and a new cut is started at the left edge of the face, the tool now being fed to the right by power while being fed in by hand as before. When the second cut reaches the first at the center of the face the tool is again withdrawn from the work, and the operations just described are repeated as often as may be necessary until the face of the wheel is rough turned to approximately the correct size and shape. The face is then finished to the template using a broad-nose finishing tool whose cutting edge is rounded to the same radius as the template. This tool is ordinarily narrower than the face of the wheel and is only used to dress off the high spots indicated by the template, being shifted from one spot to another of the face of the wheel for this purpose. When the finishing tool is used all feeding is done by hand.

The hob, see Fig. 89, is made of tool steel and is almost the exact counterpart of the worm, the only differences being that its diameter is slightly greater to provide for the clearances at the bottoms of the tooth spaces of the wheel, that it is fluted in a direction parallel to its axis so as to form cutting teeth, and that the top surfaces of the teeth like those of the milling cutter referred to in article 109 are of a spiral form to give proper clearance and keenness to the cutting edges. It is turned and threaded in the lathe and fluted in the milling machine. The top surfaces of the teeth are then filed to give them the proper shape and the hob is finally hardened and tempered.

To cut the teeth of the worm-wheel it is mounted on an arbor between the index centers of the milling machine and left free to rotate thereon. It is then brought directly under the hob which is fastened to an arbor in the spindle of the machine, the axes of the hob and worm-wheel being perpendicular to each other. The machine is started and the table raised until the hob teeth sink a certain distance into the face of the wheel. The rotation of the hob while cutting will, because of its cutting threads, cause the wheel to rotate in the same way as if it were in mesh with its worm. After the wheel has made one complete revolution the hob is sunk deeper into it and another cut taken on the teeth all around the circumference, these operations being repeated until the tooth spaces in the wheel are cut to the proper depth. Very frequently, and especially if the wheel is large, it is "gashed" by an ordinary milling cutter before being hobbled as when this is done there is no danger of the wheel not rotating properly with the hob, which sometimes occurs when the wheel is not gashed. A gash is cut over every tooth space, the table of the machine being turned so that the gashes are parallel to the helices of the finished teeth at the central plane of the wheel. The wheel does not rotate when the gashes are being made but it is turned through the proper angle between gashes by the index mechanism.

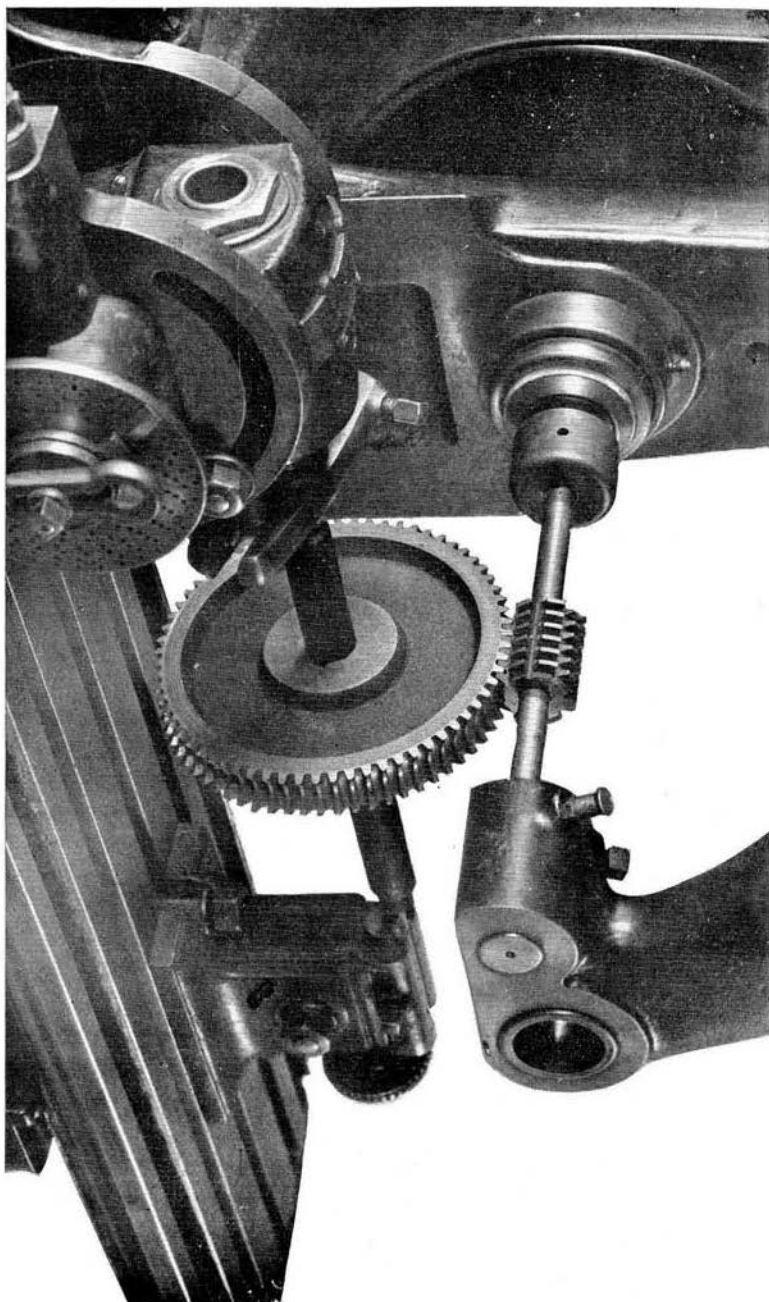
In some machines the worm-wheel while being hobbled is not left free to rotate on the centers but is required by gearing to rotate at exactly the correct speed with reference to that of the hob. When worm-wheels are hobbled in these machines the preliminary gashing is not needed.

Fig. 89 shows the operation of hobbing a worm-wheel.

When the axes of a worm and worm-wheel are not to be perpendicular to each other, the wheel is not hobbled but instead its teeth are cut with an ordinary rotary cutter in the same way as the teeth of a spiral gear.

112. Cutting the Teeth of Bevel Gears.—Because of the conical pitch surfaces of bevel gears a section of the tooth at one end is larger than at the other, and the tooth outline varies in size from the large end to the small end; consequently perfectly formed bevel-gear teeth cannot be cut by a rotary cutter. Nevertheless such teeth were in the past cut in the milling machine

Fig. 89. — Hobbing a Worm-Wheel.



with a rotary cutter and this practice is followed to a considerable extent at the present time.

After the gear-blank has been turned in the lathe, if the teeth are to be cut in a milling machine it is mounted on an arbor in the spindle of the index head, which is rotated around a horizontal axis until the middle element of the surface that will form the bottom of the tooth space is in a horizontal plane. A rotary cutter whose width is that of the tooth space at its small end and the profile of whose cutting edges is the same as that of the tooth space at its large end is so mounted on an arbor in the spindle of the machine that its center and the axis of the gear-blank are

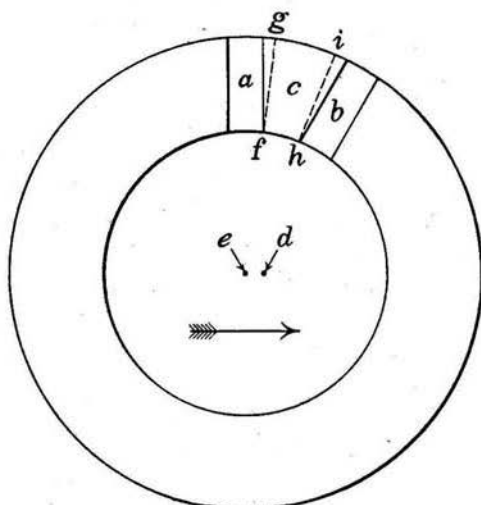


Fig. 90.

contained in the same vertical plane, and two spaces, *a* and *b*, are cut as shown in Fig. 90, the index mechanism being used to obtain the proper spacing.

This will leave the tooth *c* too wide at the large end. The table of the milling machine is then moved by the cross feed in the direction of the arrow a certain distance *ed*, which moves the center of the gear-blank to the right of the center of the cutter. The gear-blank is then revolved until the cutter will just enter the cut *a* at the small end of what will be the finished tooth space. The center of the blank being now to the right of the center of

the cutter, the path of the latter when the blank is fed up to it will not be parallel to the original cut a , but will make an angle with it; and the cutter will cut the left side of the tooth c as shown by the dotted line fg , the amount of metal removed varying from practically nothing at the small end of the tooth to an amount at the large end dependent on the distance ed which the center of the gear-blank has been moved to the right of the center of the cutter. The center of the blank is next moved as far to the left of the center of the cutter as it was to the right and, after the blank is revolved until the cutter will enter b at the small end of which will be the finished tooth space, a cut is taken on the right side of c as shown by the dotted line hi . The thickness of c at the pitch circumference at the large end is then measured; if too thick the operations just described are repeated, the center of the blank being set a little further to the right and to the left of the center of the cutter than it was before. When the correct thickness of the tooth at the pitch circumference at the large end is obtained, cuts are taken all around the blank with its center set at the proper distance to the right of the center of the cutter, and then with the center of the blank set at the same distance to the left of the center of the cutter. The index mechanism is used to rotate the blank through an angle corresponding to the circular pitch.

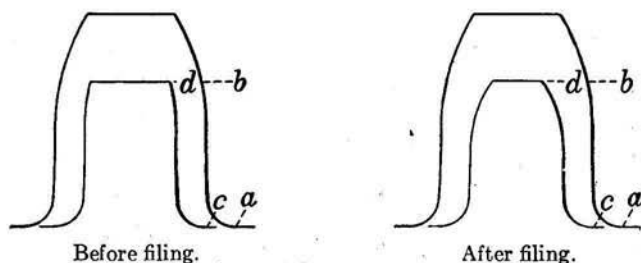


Fig. 91.

This method will produce a proper outline at the large end of the tooth but toward the small end it will not be curved enough, as will be seen from Fig. 91, which shows the large end of a bevel-gear tooth placed over the small end of the same tooth, it being understood that the part of the cutter that forms the portion ab of the large end must also form the whole of the side cd of the

small end of the tooth. The parts of the teeth near their small ends must, therefore, be rounded with a file.

113. Planing Bevel-Gear Teeth. — The proper way to cut bevel-gear teeth is to plane them, using for this purpose a specially designed shaper. The teeth of all bevel gears for gun carriages are required to be planed.

Since the tooth surfaces of bevel gears are generated by cones rolling on conical pitch surfaces whose apexes are at the point of intersection of the shafts, or by planes rolling on base cones whose apexes are at the same point, it is evident that the elements

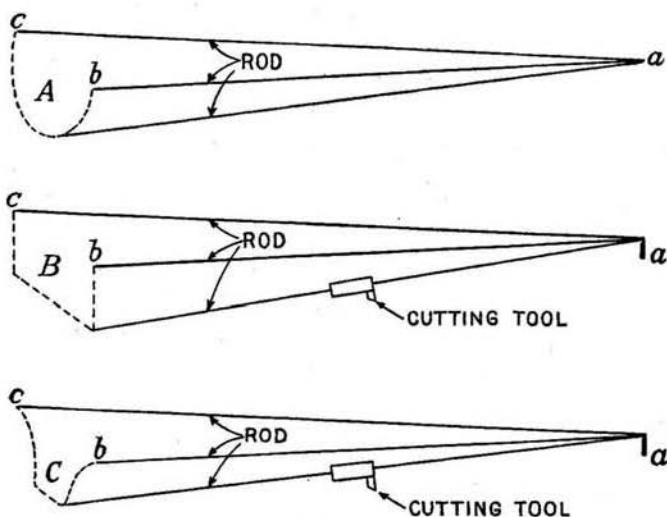


Fig. 92.

of the tooth surfaces are straight lines *converging to the point of intersection of the shafts*. If, therefore, the point of a planing tool can be made to follow these elements, the surfaces cut by it will be correct tooth surfaces.

If one end of a rod be pivoted at *a*, Fig. 92, by a universal joint and the other end be moved around the inside of a semi-circle *A*, it is apparent that it will by its motion generate the half surface of a cone, and that any section of this half cone parallel to the plane of the semi-circle *A* will also be a semi-circle whose size will vary with the distance of the section from *a*. If the end

of the rod instead of being moved continuously around the semi-circle be moved around it step by step, the successive positions of the rod being quite close to each other, a slide carrying a cutting tool can be placed on the rod and made to move forward toward *a* and back again at every successive position of the rod, which in this case would be bent downward for a short distance at its end before being pivoted at *a* in order to enable the point of the cutting tool to travel directly toward *a* and back. If a metal bar not too long is placed between *A* and *a* and the first position of the free end of the rod carrying the slide and cutting tool is at *b*, it is apparent that, during the successive movements of the free end of the rod around the semi-circle from *b* to *c*, the cutting tool on the slide, by moving forward and backward each time the rod takes a new position, will cut in the bar a hollow half cone whose apex is at *a* and whose sections parallel to the plane of *A* will all be semi-circles.* If instead of moving the free end of the rod around the inside of the semi-circle it be moved around the inside of the rectangle *B* from *b* to *c*, the tool can be made to cut a pyramidal trough whose apex is at *a* in a metal bar placed between *B* and *a*. Any section of this trough parallel to the plane of *B* will give three sides of a rectangle similar to *B*, the size of the rectangle varying with the distance of the section from *a*. And by substituting for the rectangle a template having the outline of a correctly shaped tooth space, the tool can be made to cut a correctly shaped tooth space in a bevel-gear blank so placed between *C* and *a* that the apex of its conical pitch surface is at *a*. All elements of the tooth surfaces so cut will converge toward *a* and the difference in the size of the tooth at its large and small ends will depend upon the length of its face and the angle at the apex of the pitch cone. The teeth of bevel gears of different size can be cut by varying the distance of the gear-blank from the point *a*.

The construction of a bevel-gear shaper is somewhat complicated but the principle upon which it operates is not difficult to understand after the discussion in the preceding paragraph. The ram of the shaper moves in guideways in a saddle pivoted

* To permit the end of the rod to be fed around the inside of the semi-circle, rectangle, or template while the tool is cutting the bar (or gear blank), the latter would ordinarily have to be first roughed out by central cuts.

at a point in front by a universal joint. The rear part of the saddle carries a roller which the mechanism of the machine keeps hard pressed against a template whose outline is that of a correctly shaped tooth space on a large scale. The ram carries the cutting tool and is moved backward and forward by the ordinary shaper mechanism. After each stroke of the shaper the feed mechanism moves the roller at the rear end of the saddle downward while at the same time it is kept hard pressed by springs or otherwise against the tooth outline of the template. In this way the rear part of the saddle, which corresponds to the rod in Fig. 92, follows the outline of a correctly shaped tooth space while its front end is pivoted at a fixed point. The ram of the shaper corresponds to the slide on the rod of Fig. 92. The bevel-gear blank on which the teeth are to be cut is placed in front of the ram with the apex of its pitch cone at the point to which the front end of the saddle is pivoted. The position of the gear-blank in front of the ram can be varied to suit the size of gear being cut. After one tooth space has been cut the blank is automatically rotated through the proper angle to place it in position for another cut by a mechanism similar to the index mechanism of a milling machine. In this machine the tool would be fed downward by the downward movement of the rear end of the saddle along the tooth outline of the template, and one side of a tooth would be cut at a time.

CHAPTER VI.

COUNTER-RECOIL SPRINGS.

114. Helical Springs. — The counter-recoil springs used in gun carriages for returning the gun into battery after recoil has ended are made of steel bars coiled into helices and hence are called helical springs. The cross-section of the bars from which the springs are made is generally either circular or rectangular. The steel from which the bars are forged must be of the best quality, and since an exceptionally high elastic limit is required the carbon content of the steel is high and the springs are hardened and tempered before use; and, as it is necessary to retain as much as possible of the increased elastic limit caused by the hardening process, the tempering temperature is low.

Fig. 93 shows two counter-recoil springs, one coiled from a bar of circular cross-section and the other coiled from a bar of rectangular cross-section.

In order that each end of a counter-recoil spring shall bear evenly against the piston of the spring rod or other part against which it acts, the ends of the end coils are closed down against the adjoining coils and ground flat so that the end surfaces of the springs are truly at right angles to its axis. When this is the case the force acting on the spring is uniformly distributed over its end surface and the effect is the same as if the force were concentrated at the axis of the spring and acted in the direction of that axis.

115. Torsional Stresses and Strains in a Straight Bar. — As the stresses and strains produced in the material of a helical spring by a force acting on it in the direction of its axis are almost entirely those of torsion, the effect of a twisting force on a straight bar will be considered before studying the stresses and strains produced in a helical spring by a force acting to compress or elongate it.



Fig. 93. — Counter-Recoil Springs.

Let BA , Fig. 94, be a bar fixed at the end B and subjected to a torsional moment caused by the force F . Then, as stated in article 71, page 111, the torsional stress per unit of area in the extreme fibre of any section of the bar will be

$$rS_t''' = S = M_t r / I_p \quad (1)$$

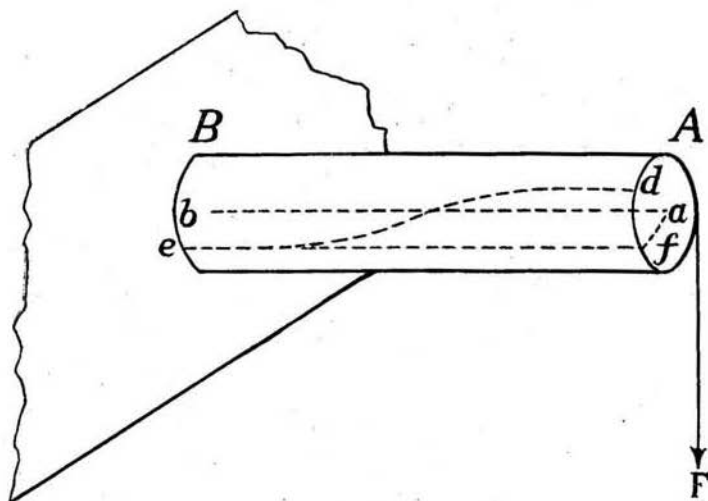


Fig. 94.

in which

S_t''' is the torsional stress per unit of area at a unit's distance from the axis of the bar,

S is the torsional stress per unit of area in the extreme fibre of a section,

r is the distance in inches from the axis of the bar to the extreme fiber of a section,

M_t is the torsional moment in in. lbs., and

I_p is the polar moment of inertia of the section in ins.⁴

In Fig. 94, which is reproduced from page 37, Fiebigger's Civil Engineering, ef is a surface fibre before being distorted by the twisting force F and ed is the position taken by this fibre when the force acts on the bar. Every cross-section of the bar except that at the fixed end B is rotated around the axis ba by the twisting force, each section being rotated through a slightly greater angle

than the one next it on the side of the fixed end of the bar. The maximum rotation occurs in the section at the free end, whose plane contains the force, and the rotation of any other section is equal to the maximum rotation multiplied by the distance of the section under consideration from the fixed end of the bar divided by the length of the bar. If the force did not act at the end of the bar the maximum rotation would still occur in the section whose plane contains the force, and the rotations of the other sections nearer the fixed end of the bar would bear the same relations to the maximum rotation as before, providing we substitute for the length of the bar the distance from the section whose plane contains the force to the fixed end. All sections between the force and the free end of the bar would have the same rotation as that of the section whose plane contains the force.

Let L be the length of the bar shown in Fig. 94,
 l , the length of the arc fd , and
 E , the torsional modulus of elasticity, sometimes called
also the *coefficient of torsional elasticity*;

then, as shown on page 40, Fieberger's Civil Engineering,

$$E = \frac{S}{l/L} \quad (2)$$

or substituting for S its value from equation (1) and solving for l/L

$$l/L = \frac{Mr}{I_p E} \quad (3)$$

Since l is the length of the arc fd and r is the distance of the extreme fibre from the axis of the bar, l/r is the measure of the angle, expressed in radians, through which has rotated the section whose plane contains the twisting force. Solving equation (3) for l/r we have

$$l/r = \frac{M L}{I_p E} \quad (4)$$

116. Stresses in Helical Springs. — Let Fig. 95 represent a helical spring resting on a flat surface JJ and subjected to compression by the force C . Since the action line of the force coin-

cides with the axis of the spring it is symmetrically placed with respect to each section of the bar from which the spring is coiled, and the stresses produced by it in all sections will be the same. The part of the bar between the point of application of the force

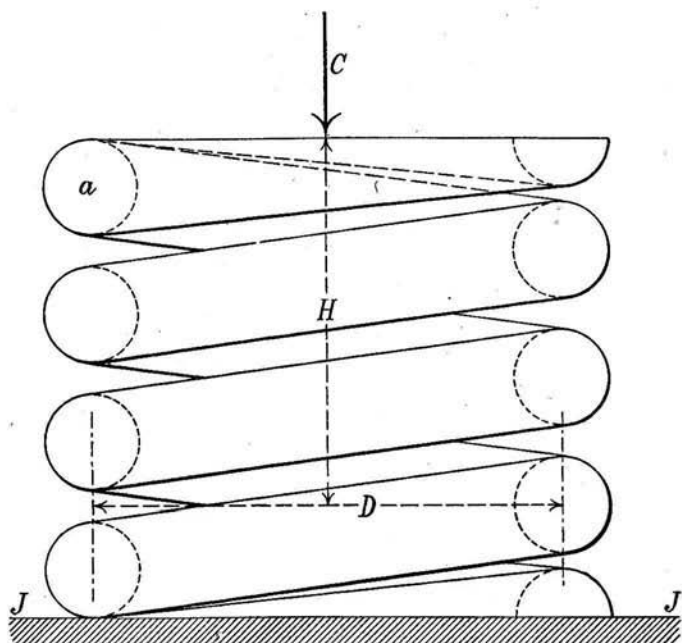


Fig. 95.

and section a is not considered in this discussion as it is an end coil that has been flattened and ground to provide an end surface perpendicular to the axis of the spring. End coils are called *ineffective* coils; they are not relied upon to add to the spring effect as a whole but merely to transmit the force to the rest of the spring.

Let a be a right section of the bar. As the bar is coiled into a helix this section and every other right section will be slightly inclined from the vertical, and the component of the force C perpendicular to the section will cause a slight bending stress and a slight compressive stress therein; but as these stresses are small they are neglected in spring computations, and the plane of

a right section of the bar is assumed to be parallel to the axis of the spring and to the action line of the force. Since the bar is coiled around the axis of the spring, the plane of section a and of every other right section of the bar, considered as vertical sections, will contain the force which, under the assumption made, can produce no bending or compressive stress in the sections. A shearing stress in the sections is produced by the force but it is slight in comparison with the torsional stress and is on this account also neglected. As the force has a lever arm with respect to the axis of the coiled bar it has a torsional moment with respect to it and the corresponding maximum torsional stress in section a is, from equation (1),

$$S = \frac{Mr}{I_p} = \frac{CDr}{2I_p} \quad (5)$$

in which C is the intensity of the force, D is the mean diameter in inches of the helix formed by the coiled bar, r is the distance in inches from the axis of the bar to the extreme fibre of the section, and I_p is the polar moment of inertia of the section in inches.⁴ It is evident that the stress in a will be transmitted from section to section of the bar until the part resting on the flat surface is reached and, since these cross-sections are all alike, equation (5) gives the maximum torsional stress S that occurs in any part of the spring.

117. Fundamental Equations Relating to Helical Springs. — Let l be the length of the arc through which the end of the extreme fibre at section a would rotate around the axis of the bar if straight under the action of the force, and let L be the developed length of the portion of the bar between section a and the point where it rises above the supporting surface JJ , the end coils not being considered. Since a cross-section of the end coil, which rests on the supporting surface and is ground flat where it bears against that surface, cannot rotate under the action of the force, the section at the point where the bar rises above the supporting surface and where the effective part of the spring commences, may be considered as the fixed end of the bar.

The angle θ through which section a would rotate around the axis of the bar if straight under the action of the force is

$$\theta = l/r \text{ radians,}$$

and any other section at a distance x from the fixed end, measured along the developed length of the bar, would rotate around the axis of the bar if straight through an angle

$$\theta' = \frac{l}{r} \times \frac{x}{L} \text{ radians.}$$

A section adjoining the latter and at a distance from it of dx , measured along the developed length of the bar toward section a , would rotate around the axis of the bar if straight through an angle

$$\theta'' = \frac{l}{r} \times \frac{x + dx}{L} \text{ radians,}$$

and relatively to the latter it would rotate through an angle

$$d\theta = \frac{ldx}{rL} \text{ radians,} \quad (6)$$

and this is also the angle through which the one section would rotate relative to the other under the action of the force where the bar is coiled into a helical spring for the axis of the bar may be considered straight in this case for any infinitesimal length dx .

Let H , Fig. 95, be the vertical distance from the center of this section to the point of application of the force C . Then, since the rotation of any section around the axis of the bar will cause every point of the spring above it to move in the arc of a circle whose center coincides with that of the section or with the projection of this center on the plane of rotation of the point, the displacement of the point of application due to the rotation of the one section under consideration past the other through an angle $d\theta$ around the axis of the bar is

$$ds = \{H^2 + (D/2)^2\}^{\frac{1}{2}} d\theta \quad (7)$$

This displacement may be resolved into horizontal and vertical components dh and dv , respectively, as follows:

$$dh = Hd\theta \quad (8)$$

$$\text{and} \quad dv = \frac{D}{2} d\theta \quad (9)$$

As each cross-section rotates around the axis of the bar through an angle $d\theta$ with respect to that immediately adjoining it and

nearer the supporting surface, the point of application of the force will undergo successively the component displacements given in equations (8) and (9), it being understood in this connection that the distance H is a variable. If we consider only the horizontal components of the displacements it will be seen that, as we pass around one complete coil of the spring from any point on the bar to a corresponding point immediately above it, the horizontal components neutralize each other and we may write

$$\int_0^L H d\theta = 0$$

The vertical components of the displacements, however, are seen to be cumulative so that the total displacement of the point of application under the action of the force is in the direction of the axis of the spring and equal to

$$\int_0^L \frac{D d\theta}{2} = \int_0^L \frac{Dl dx}{2rL} = \frac{D}{2} \theta = \frac{Dl}{2r} \quad (10)$$

from which we may deduce *that the total displacement of the point of application of the force, or the total compression of the spring caused by the force, is a movement in the direction of the axis of the spring equal to the mean radius of the coil multiplied by the total angle, expressed in radians, through which the section at the free end of the effective portion of the bar rotates under the action of the force.*

Calling the compression of the spring $N'\Delta$, in which N' is the number of effective coils and Δ is the compression per coil, we have

$$N'\Delta = \frac{Dl}{2r}$$

and substituting for l/r its value from equation (4) and for M_t in the latter equation its value $CD/2$

$$N'\Delta = \frac{CD^2L}{4EI_p} \quad (11)$$

The developed length of the effective part of the bar is approximately $N'\pi D$, and substituting this value for L in equation (11) and solving for Δ we have as the compression per coil

$$\Delta = \frac{\pi CD^3}{4EI_p} \quad (12)$$

Solving equation (5) for C we have

$$C = \frac{2SI_p}{Dr} \quad (13)$$

Equations (11), (12), and (13) are the fundamental equations relating to helical springs. Equations (11) and (12) give the total compression (or extension) and the compression (or extension) per coil, respectively, caused by a force C ; and equation (13) gives the force C which will cause a maximum stress S in the fibers of the bar from which the spring is coiled. By giving to S in equation (13) the maximum value permissible in connection with the quality of the material, the maximum force which the spring can support without undergoing a permanent distortion can be determined. The compression corresponding to this maximum force is then obtained from equation (11) or (12).

Equations (11), (12), and (13) have been deduced under the supposition that the force compresses the spring but they are equally applicable when the force extends it. In the latter case special means have to be provided for holding the spring at one end and to enable the force to be applied in the line of the axis of the spring at the other end. Supposing these means to have been provided for the spring shown in Fig. 95, it is evident that if the force C acts upward it will extend the spring producing in the bar from which it is coiled a torsional stress whose intensity will be the same as when the force acts downward. The angle l/r through which section a rotates under the action of the force when it acts upward will also have the same numerical value as when the force acts downward, but the direction of rotation of the section around the axis of the bar will be opposite in the two cases.

118. Helical Springs Coiled from Bars of Circular Cross-Section. — For a circular cross-section the value of I_p is $\pi d^4/32$ ins.⁴, d being the diameter of the bar. Substituting this value of I_p in equation (13) it becomes

$$C = \frac{.3927 Sd^3}{D} \quad (14)$$

Substituting the value of I_p in equations (11) and (12) they become, respectively,

$$N' \Delta = \frac{8 CD^2 L}{E \pi d^4} \quad (15)$$

and
$$\Delta = \frac{8 CD^3}{E d^4} \quad (16)$$

Equations (14), (15), and (16) are the formulas generally used in computing the strength and compression or extension of helical springs coiled from bars of circular cross-section.

119. Helical Springs Coiled from Bars of Rectangular Cross-Section.—As first shown by Saint-Venant, an eminent French investigator, a plane section whose axes are unequal becomes a warped surface when subjected to great torsional strain, and its polar moment of inertia is not then equal to the sum of its moments of inertia about two axes in its plane perpendicular to each other passing through its center of gravity. Reuleaux states that the polar moment of inertia of a rectangle when subjected to great torsional strain is

$$I_p = \frac{h^3 b^3}{3 (h^2 + b^2)} \text{ ins.}^4$$

and that the distance from the center of gravity to the point of the section most distant from it is

$$r = \frac{hb}{\sqrt{h^2 + b^2}} \text{ ins.}$$

in which h and b are the sides of the rectangle expressed in inches. Substituting these expressions for I_p and r in equation (13), we have

$$C = \frac{2 S h^3 b^3 \sqrt{h^2 + b^2}}{D 3 (h^2 + b^2) h b} = \frac{2 S}{D} \times \frac{h^2 b^2}{3 \sqrt{h^2 + b^2}} \quad (17)$$

and substituting the expression for I_p in equations (11) and (12) they become, respectively,

$$N' \Delta = \frac{3 CD^2 L}{4 E} \times \frac{h^2 + b^2}{h^3 b^3} \quad (18)$$

and
$$\Delta = \frac{3 \pi CD^3}{4 E} \times \frac{h^2 + b^2}{h^3 b^3} \quad (19)$$

Equations (17), (18), and (19) are Reuleaux's formulas for helical springs coiled from bars of rectangular cross-section.

Equations (14) to (19), inclusive, are used by the Ordnance Department, U. S. Army, in the design of counter-recoil springs. Since D is the mean diameter of the helix, the outer diameter of a helical spring coiled from a bar of circular cross-section is $D + d$ and its inner diameter is $D - d$. The outer diameter of a helical spring coiled from a bar of rectangular cross-section is $D + b$ and its inner diameter is $D - b$, b being the side of the rectangle that is perpendicular to the axis of the spring.

120. Requirements to be Fulfilled by a Counter-Recoil Spring. — **Nomenclature.** — In the design of a counter-recoil spring the principal requirements to be fulfilled are the following:

- (a) The spring must have sufficient power to return the gun into battery with certainty at any elevation for which the carriage is designed.
- (b) Its length when the gun is in battery, called its assembled height, must not be inconveniently great.
- (c) It must be capable of sustaining indefinitely a compression corresponding to its assembled height and of being repeatedly compressed through a further distance somewhat greater than the length of recoil of the gun, without suffering permanent deformation.
- (d) The outer diameter of the coiled spring must not be inconveniently great.
- (e) Its inner diameter must be large enough to permit the spring to pass over the spring piston-rod, hydraulic recoil cylinder, or other part on which the spring is assembled.

Let

C be the maximum capacity of the spring in pounds, that is, the maximum force which it can exert or which can act against it without causing the maximum stress in it to exceed a permissible amount.

T be its capacity in pounds at assembled height.

Δ be the compression per coil in inches.

Δ_r be the remaining compression per coil at assembled height in inches.

N' be the number of effective coils in the spring.

N be the total number of coils in the spring.

$N'\Delta$ be the total compression of the spring in inches.

$N'\Delta_r$ be the total remaining compression at assembled height in inches.

l be the length of recoil in inches.

A be the assembled height of the spring in inches.

H be the solid height of the spring in inches.

F be the free height of the spring in inches.

P be the number of sections of the spring.

S be the maximum permissible stress, which will be taken as 100000 lbs. per sq. in.

E be the torsional modulus of elasticity, equal to 12600000 lbs. per sq. in.

α be the maximum angle of elevation of the gun.

W be the weight in pounds of the recoiling parts.

B be the friction in pounds of the packing around the piston-rod of the hydraulic cylinder.

f be the coefficient of starting friction.

121. Design of Counter-Recoil Springs Coiled from Bars of Circular Cross-Section. — In order that the spring may be capable of returning the gun into battery with certainty after it has been fired at the maximum angle of elevation α , the force T which it must be capable of exerting at assembled height must be at least equal to the component of the weight of the recoiling parts parallel to the surface of a plane inclined at an angle α with the horizontal plus the friction due to the component of the weight of the recoiling parts normal to this plane plus the friction of the packing around the piston-rod, or

$$T = W \sin \alpha + fW \cos \alpha + B \quad (20)$$

The force which a counter-recoil spring is capable of sustaining or exerting varies directly with its compression so that the force to which it is subjected at the end of recoil is necessarily greater than that to which it is subjected at assembled height; and by varying the design of the spring the ratio C/T can be varied within wide limits. In order, however, that the assembled height of the spring shall be as small as possible it is necessary that it shall be compressed at the end of recoil until

the coils are nearly in contact, when the force to which it is subjected will be nearly the maximum which it is capable of sustaining without injury.

The force T which the spring must exert as its assembled height having been determined from equation (20), it remains to determine the force C , or maximum capacity of the spring. The movement of the spring required between the loads T and C will be fixed and somewhat greater than the length of recoil. We will place it equal to $l + e$ where e may be 1 inch or more. With the fixed movement, fixed load T at assembled height, and fixed value of D , the mean diameter of the coils, we will now determine the relation between C and T that will result in a minimum solid height and therefore a minimum assembled height of the spring.

It is important to have the solid and assembled heights a minimum for the reason that a saving in weight and cost of spring cylinders, piston rods, etc., is thus effected.

From equation (16) we have for the load C ,

$$N' \Delta = \frac{8 N' C D^3}{E d^4},$$

and for the load T ,

$$N' (\Delta - \Delta_r) = \frac{8 N' T D^3}{E d^4}.$$

Subtracting the latter equation from the former, we have,

$$N' \Delta_r = \frac{8 N' d D^3}{E d^5} (C - T).$$

Considering all coils effective, we have,

$$N' \Delta_r = l + e = \text{a constant.}$$

$$N' d = H = \text{solid height of spring.}$$

Substituting these values and the value of d^5 obtained from equation (14) in the above equation, we obtain

$$H = \frac{E (l + e)}{\pi S^{\frac{5}{8}} D^{\frac{4}{3}}} \frac{C^{\frac{2}{3}}}{C - T}.$$

To get the values of C for which H is a minimum we differentiate H with respect to C and place the differential coefficient

equal to 0. This gives, finally, neglecting the constant coefficient,

$$\frac{dH}{dC} = \frac{5/3 C^{\frac{3}{2}} (C - T) - C^{\frac{5}{2}}}{(C - T)^2} = 0$$

which makes $C = 2.5 T$. This is the condition that should be assumed if it is desired to keep the mean diameter of the coils a fixed amount.

By similar reasoning it may be shown that if instead of assuming D constant, we assume $D + d$, or the outside diameter of the spring constant, the solid height will be a minimum when $C = 2 T$.

This is the condition that must be assumed when it is desired to design a spring to work inside a cylinder whose diameter has been fixed by other considerations.

If we assume $D - d$, or the inside diameter of the spring constant we obtain $C = 3 T$, as the condition for minimum solid height. This is the condition that must be assumed when it is desired to design a spring to work around a rod whose diameter has been fixed by other considerations.*

Assuming the first of the above conditions we have,

$$C = 2.5 T = 2.5 (W \sin \alpha + fW \cos \alpha + B) \quad (21)$$

Since the force which a spring is capable of exerting varies directly as its compression we may write

$$\Delta : \Delta - \Delta_r :: C : T$$

or

$$\Delta_r = \Delta - \frac{T}{C} \Delta = .6 \Delta \quad (22)$$

In order that the spring shall not be compressed at the end of recoil to its solid height, that is, until one coil bears solidly against the next, the total remaining compression at assembled height $N' \Delta_r$ should be somewhat greater than the length of recoil. The excess of $N' \Delta_r$ over the length of recoil has varied somewhat in different designs but it will be sufficient under ordinary circumstances to take it equal to one inch. In order, however, that

* The above demonstration as to the proper ratio $\frac{C}{T}$ under various conditions was prepared by Lt. Col. W. H. Tschappat, Ord. Dept., U. S. A.

the equations may be of general application this excess will be represented by e , and we may write

$$N' \Delta_r = .6 N' \Delta = l + e$$

or

$$N' = \frac{5(l + e)}{3 \Delta}. \quad (23)$$

122. Solid Height of Spring Column. — The solid height of a spring is its length when it is compressed until each coil bears solidly against those adjoining it. It has already been explained that the end coils of a counter-coil spring are considered to be ineffective. By examination of Fig. 95 it will be seen that in order to obtain surfaces at the ends of the spring which are truly perpendicular to its axis it is necessary, in addition to closing the ends of the end coils into contact with the adjoining coils, to grind away part of each end coil, the amount ground away being dependent on the amount the end coil is flattened down and on the ratio of the distance between the coils at assembled height to the dimension of the bar parallel to the axis of the spring. While no general rule can be given to cover every case, it will be assumed in this discussion that one-half of each end coil is ground away, commencing with a very light cut at the section adjoining the effective coil and increasing the cut gradually until the end of the end coil is reduced to a comparatively thin edge. Under this assumption the space taken up by each end coil when the spring is compressed to its solid height is only about one-half of that taken up by an effective coil, and the effect of the end coils on the solid height of the spring may be obtained by considering that only half a coil at each end is ineffective but that its volume is equal to that of an effective half coil.

Because of the difficulty of manufacturing very long helical springs it is frequently necessary to make a counter-recoil spring in two or more sections that are placed end to end in the spring cylinder with pieces of metal called separators between them.

If P be the number of sections of the spring, d the diameter of the coiled bar, and t the thickness of each separator, the solid height of the spring column, under the assumption that one-half a coil at each end of a spring section is ineffective and that the

volume of an ineffective half coil is the same as that of an effective half coil, will be

$$H = (N' + P) d + (P - 1) t \quad (24)$$

123. Assembled Height of Spring Column.— Since the total remaining compression of the spring column at assembled height is to be e inches greater than the length of recoil we may write

$$A = (N' + P) d + (P - 1) t + l + e \quad (25)$$

124. Free Height of Spring Column.— The free height of the spring column is its solid height plus the total compression which it is capable of undergoing, or

$$F = (N' + P) d + (P - 1) t + N' \Delta$$

and substituting for N' in the last term of this expression its value from equation (23)

$$F = (N' + P) d + (P - 1) t + \frac{5(l + e)}{3} \quad (26)$$

125. Introduction of Values of Constants in Equations (14) and (23).— Replacing S in equation (14) by its value, 100000 lbs. per sq. in., and solving for d , we have

$$d = [8.46864 - 10] C^{\frac{1}{3}} D^{\frac{1}{3}} \quad (27)$$

Substituting for Δ in equation (23) its value from equation (16) and for E in the latter equation its value, 12600000 lbs. per sq. in.

$$N' = \frac{[6.41913] (l + e) d^4}{CD^3} \quad (28)$$

the figures in brackets in equations (27) and (28) being the logarithms of the numbers.

126. Order of Procedure.— In designing a counter-recoil spring column the length of recoil and the value of C , equation (21), are fixed by the construction of the carriage, which also restricts within comparatively narrow limits the outer diameter of the spring and, consequently, its mean diameter. Moreover the assembled height of the spring column must not be inconveniently great. In the solution of the problem a convenient value of D is first selected for trial. With this value of D and the fixed value of C , the diameter of the bar from which the spring is to be coiled is obtained from equation (27), and this in con-

nection with the assumed value of D determines the inner and outer diameters of the spring. If these are satisfactory the required number of effective coils is next obtained from equation (28) after substituting therein the fixed values of C and l , the desired value of e , the assumed value of D , and the resulting value of d . The number of effective coils will determine how many spring sections are required in the spring column, and this having been decided and a suitable value for the thickness of the separators assumed, the solid height of the column is given by equation (24), the assembled height by equation (25), and the free height by equation (26).

If the assembled height thus determined is too great another value for D is selected and the assembled height again determined. The assembled height depends directly on the solid height and the latter upon the diameter of the bar and the number of coils. By reference to equation (27) it is seen that the value of d increases directly as $D^{\frac{1}{2}}$, and by substituting for d in equation (28) its value from equation (27) it will be found that the value of N' varies inversely as $D^{\frac{5}{2}}$. The product $N'd$, which is practically the solid height, therefore varies inversely as $D^{\frac{3}{2}}$. The assembled height for a given length of recoil consequently decreases rapidly with the increase in the mean diameter of the spring.

127. Design of Counter-Recoil Springs Coiled from Bars of Rectangular Cross-Section. — Springs coiled from bars of rectangular cross-section are capable of greater compression for a given solid height than those coiled from bars of circular cross-section and, therefore, when the former are used C is taken equal to $2T$, whence

$$C = 2(W \sin \alpha + fW \cos \alpha + B) \quad (29)$$

and

$$N' \Delta_r = .5 N' \Delta = l + e$$

or

$$N' = \frac{2(l + e)}{\Delta} \quad (30)$$

The dimension of the rectangular cross-section parallel to the axis of the spring being h , the solid height of the spring column, under the assumption made in article 122 as to the ineffective coils, is

$$H = (N' + P)h + (P - 1)t, \quad (31)$$

the assembled height is

$$A = (N' + P)h + (P - 1)t + l + e, \quad (32)$$

and the free height is

$$F = (N' + P)h + (P - 1)t + 2(l + e) \quad (33)$$

Let r equal the ratio b/h , whence $b = rh$. Substituting this value of b in equation (17), replacing S by its value, 100000 lbs. per sq. in., and solving for h , we have

$$h = [8.39203 - 10] (1 + r^2)^{\frac{1}{2}} r^{-\frac{2}{3}} C^{\frac{1}{3}} D^{\frac{1}{3}} \quad (34)$$

Substituting for Δ in equation (30) its value from equation (19) and for E and b in the latter equation their values of 12600000 lbs. per sq. in. and rh , respectively,

$$N' = \frac{[7.02919] (l + e) h^4}{CD^3} \times \frac{r^3}{1 + r^2} \quad (35)$$

For a given value of r the product $N'h$, like the product $N'd$ for springs coiled from bars of circular cross-section, varies inversely as $D^{\frac{1}{3}}$.

By substituting in equation (35) the value of h from equation (34) it will be seen that N' varies approximately inversely as $r^{\frac{1}{3}}$ and the product $N'h$ approximately inversely as $r^{\frac{2}{3}}$. The solid and assembled heights for a given length of recoil will, therefore, decrease rapidly as r increases, and if a value of $r = 4$ or $r = 5$ be assumed the assembled height will be much less than it would for a value of $r = 1$. When $r = 1$, that is, when the bar is of square cross-section, the power and compressibility of the spring are about the same as for one coiled from a bar of circular cross-section. A spring coiled from a bar of rectangular cross-section with a value of r from 4 to 5 has, therefore, a very decided advantage over one coiled from a bar of circular cross-section in that its assembled height for a given length of recoil is much less than that of the latter spring. The disadvantage of springs coiled from bars of rectangular cross-section has been the difficulty of their manufacture and until within a comparatively few years much trouble has been experienced in getting satisfactory springs of this type from manufacturers. The practical limit for r in connection with a suitable mean diameter of the spring

seems at present to be about 5, this value being used in the counter-recoil springs of the 3-inch field carriage.

The method of designing counter-recoil springs to be coiled from bars of rectangular cross-section is identical in principle with that described for springs coiled from bars of circular cross-section, the only difference being that a value for r must first be selected. Ordinarily this value will be taken as large as practicable, at present not exceeding about 5.

128. Telescoping Springs. — To reduce the assembled height of the spring column when the length of recoil is great, particularly if it is desired to use springs coiled from bars of circular cross-section, telescoping springs are used, as in the case of the 5-inch barbette carriage, model of 1903, the 4.7-inch siege carriage, model of 1906, the 6-inch siege howitzer carriage, model of 1908, etc.

The principle of the telescoping spring is shown in Fig. 96. A is the spring cylinder attached to a non-recoiling part of the

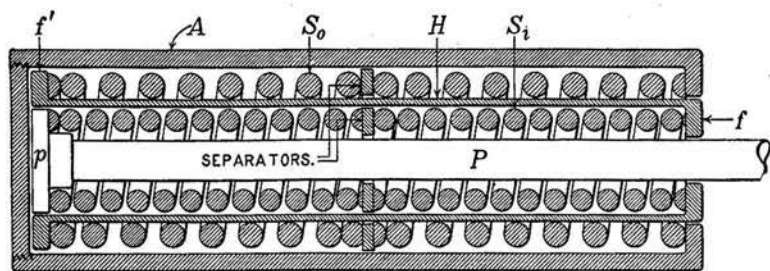


Fig. 96.

carriage, P is the spring piston-rod attached to the gun, S_o is the outer and S_i the inner spring, or spring column in case there is more than one spring section, and H is the stirrup connecting the inner and outer spring columns. The stirrup is a hollow cylinder of steel with an inward projecting flange f at its rear end and an outward projecting flange f' at its front end. When the gun recoils it draws the spring piston-rod with it, compressing the spring column S_i between the spring piston p and the inner flange f of the stirrup. At the same time the pressure on the flange f is communicated through the stirrup and its outer flange f' to the outer spring column S_o , compressing it also. Owing to

the compression of the outer spring column the rear ends of the stirrup and inner spring column will move out of the opening provided for the purpose in the spring cylinder, but the rear end of the outer column rests against the end of the spring cylinder and cannot move during recoil. The sum of the compressions of the two columns is equal to the length of recoil of the gun. It is evident that the number of spring columns connected by stirrups can be increased to three or more, if desired, with consequent decrease in the assembled height for a given length of recoil. The forces exerted by the spring columns must be equal to each other at all times.

In a special case such as that of the 5-inch barbette carriage, model of 1903, to be described later, it may not be advisable to permit the rear ends of the stirrup and inner spring column to be drawn through an opening in the spring cylinder during recoil. When this is so the assembled height of the inner column must be such that it can, when the gun is in battery, be held by the stirrup away from the rear end of the spring cylinder at a distance at least equal to the amount by which the outer column is compressed during recoil plus the thickness of the rear flange of the stirrup, for otherwise the end of the stirrup would strike the rear end of the cylinder during recoil.

129. Design of Telescoping Springs. — As in the case of non-telescoping springs the mean diameters of the springs are first selected for trial, keeping in view the limitations imposed by the design of the carriage. A convenient mean diameter for the outer spring would be selected and the mean diameter of the inner spring made as great as possible under the circumstances, taking into consideration the necessary thickness of wall of the stirrup and the clearances required between it and the springs. To allow for variations in the diameters of the springs during manufacture and for the bulging outward of the springs, which is likely to occur when they are compressed nearly to their solid height, it is well to allow a clearance of about .3 in. between the outer diameter of the inner spring and the inner diameter of the stirrup and between the outer diameter of the outer spring and the inner diameter of the spring cylinder. The thickness of wall of the stirrup and the thickness of its flanges are determined in accordance with the principles discussed in Chapter IV. The

mean diameters of the springs having been selected, the diameters of the bars from which they must be coiled are next obtained from equation (27).

In the ordinary case the assembled heights of the inner and outer spring columns will be equal. Let the subscript 1 be used to distinguish the symbols pertaining to the outer spring column and the subscript 2 to distinguish those pertaining to the inner column, and let $l_1 + e/2$ and $l_2 + e/2$ be the total remaining compressions at assembled height of the outer and inner spring columns, respectively. Then we may write from equation (25)

$$(N'_1 + P_1) d_1 + (P_1 - 1) t_1 + l_1 + e/2 = (N'_2 + P_2) d_2 + (P_2 - 1) t_2 + l_2 + e/2$$

or, since P and t will ordinarily be the same in the outer and inner spring columns,

$$(N'_1 + P) d_1 + l_1 = (N'_2 + P) d_2 + l_2 \quad (36)$$

From equation (28), noting that $l + e$ becomes in this case $l_1 + e/2$ for the outer spring column and $l_2 + e/2$ for the inner,

$$N'_1 = \frac{[6.41913] (l_1 + e/2) d_1^4}{CD_1^3}$$

and

$$N'_2 = \frac{[6.41913] (l_2 + e/2) d_2^4}{CD_2^3}$$

and substituting these values in equation (36) it becomes

$$\left\{ \frac{[6.41913] (l_1 + e/2) d_1^4}{CD_1^3} + P \right\} d_1 + l_1 = \left\{ \frac{[6.41913] (l_2 + e/2) d_2^4}{CD_2^3} + P \right\} d_2 + l_2 \quad (37)$$

We also have

$$l_1 + l_2 = l \quad (38)$$

In equations (37) and (38) C and l are known from the construction of the carriage, P and e are also known, their values having been decided upon earlier, D_1 and D_2 have been assumed and d_1 and d_2 calculated from equation (27). The only unknown quantities, therefore, are l_1 and l_2 and their values can be obtained by the solution of the equations. Having obtained the

values of l_1 and l_2 the corresponding values of N'_1 and N'_2 are obtained from equation (28) and the solid, assembled, and free heights from equations (24), (25), and (26), respectively. As a check on the accuracy of the work the assembled heights should be equal. If the assembled height of the spring columns thus determined is not satisfactory other values for D_1 and D_2 would be selected and the assembled height re-determined and so on. It is of course not essential that the assembled heights of the inner and outer columns shall be equal, and the total required compression may if desired be divided between the columns in any arbitrary manner. For a given length of recoil, however, the least assembled height is obtained when it is the same for both columns.

130. Design of Non-Telescoping Springs Assembled One Within the Other. — If it is desired to decrease the assembled height somewhat without increasing the outer diameter of the spring, the amount of the desired decrease not being so considerable as to require the use of telescoping springs, it can be done by placing one spring within the other as shown in Fig. 97,

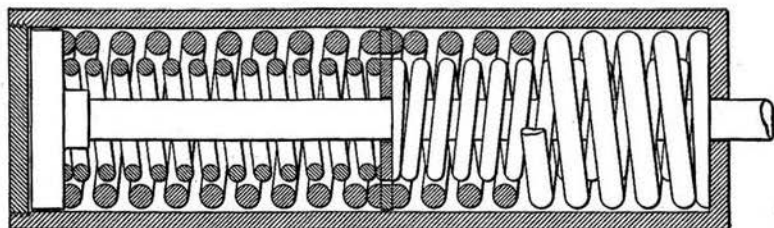


Fig. 97.

in which case each spring column sustains a part only of the total load. When springs are assembled in this manner the inner and outer springs must be coiled in opposite directions so that if one breaks the broken bar will not in untwisting be caught between the coils of the other.

In the equations that follow the symbols relating to the outer spring column will be distinguished by the subscript 1 and those relating to the inner column by the subscript 2. From Fig. 97 it is seen that the assembled heights of the inner and outer spring columns must be equal, and, if a minimum assembled height is

desired, the total remaining compressions at assembled height and the solid heights of the columns should also be equal. Since the solid heights are to be equal we may write from equation (24)

$$(N'_1 + P_1) d_1 + (P_1 - 1) t_1 = (N'_2 + P_2) d_2 + (P_2 - 1) t_2$$

In this equation we may neglect the terms $P_1 d_1$, $P_2 d_2$, $(P_1 - 1) t_1$, and $(P_2 - 1) t_2$ for P may be taken the same in both columns and the slight difference in assembled height due to the difference in the diameter of the bars in the end coils of the two columns can be made up by the difference in thickness of the separators in the columns, if any are used, or by the manufacturer in a number of ways, by increasing slightly the number of coils in the inner spring or by increasing slightly the pitch of the coils at assembled height, it being understood that the manufacturer is not limited by the specifications to the number of coils but only to the strength of the springs at assembled and solid heights, to inner and outer diameters, and to maximum solid heights which must not be exceeded.

Making these omissions, we have

$$N'_1 d_1 = N'_2 d_2$$

and substituting for N'_1 and N'_2 their values from equation (28) and for d_1 and d_2 in the resulting expression their values from equation (27) we have

$$\frac{[6.41913] (l_1 + e) \{ [8.46864 - 10] C_1^{\frac{1}{2}} D_1^{\frac{1}{2}} \}^5}{C_1 D_1^3} = \frac{[6.41913] (l_2 + e) \{ [8.46864 - 10] C_2^{\frac{1}{2}} D_2^{\frac{1}{2}} \}^5}{C_2 D_2^3}$$

and reducing; since $l_1 + e = l_2 + e$,

$$C_1^{\frac{3}{2}} / D_1^{\frac{5}{2}} = C_2^{\frac{3}{2}} / D_2^{\frac{5}{2}} \text{ or } C_1 / D_1^2 = C_2 / D_2^2 \quad (39)$$

We also have

$$C_1 + C_2 = C \quad (40)$$

For any required value of C the values of C_1 and C_2 can be obtained from equations (39) and (40) after the values of D_1 and D_2 have been decided upon. D_1 is first selected and D_2 is made as nearly equal to it as possible keeping in mind a required clearance

of about .3 in. between the inner diameter of the outer spring and the outer diameter of the inner spring.

If there are three or more springs assembled one within the other we may write

$$C_1/D_1^2 = C_2/D_2^2 = C_3/D_3^2, \text{ etc.}$$

and

$$C_1 + C_2 + C_3 + \text{etc.} = C$$

from which the values of C_1 , C_2 , C_3 , etc., can be determined after the values of D_1 , D_2 , D_3 , etc., have been decided upon.

Having obtained the values of C_1 , C_2 , etc., the diameter of the bar from which each spring is to be coiled, the number of effective coils, the solid height, assembled height, and free height of each spring or spring column can be determined from equations (27), (28), (24), (25), and (26), respectively. Any adjustment of the thicknesses of the separators or other measures to make the assembled heights of the spring columns exactly the same can now be decided upon.

131. Measures for Decreasing Assembled Height also Applicable to Springs Coiled from Bars of Rectangular Cross-Section. — In the discussions of telescoping springs, and of non-telescoping springs assembled one within the other, it has been assumed that the bars from which the springs are coiled are of circular cross-section because special measures for decreasing the assembled heights of such springs are more often necessary than for springs coiled from bars of rectangular cross-section. The principles and methods discussed are, however, equally applicable to springs of the latter type.

132. Counter-Recoil Springs for the 5-Inch Barbette Carriage, Model of 1903. — The 5-inch barbette carriage, model of 1903, is provided with two spring cylinders and two sets of springs acting together on the gun through the spring piston-rods and a spring yoke, which is a cross piece bearing against the rear of the recoil-band lug and carried on the recoil piston-rod. A spring piston-rod is fastened to each end of the spring yoke. With this arrangement each set of springs has to exert but one-half of the force required to return the gun into battery after recoil has ended.

The spring cylinders were designed primarily for springs coiled from bars of rectangular cross-section but as difficulty had been

experienced in obtaining satisfactory springs of this type it was decided to allow springs coiled from bars of circular cross-section to be used if desired. As the assembled height of a spring column of the latter springs assembled in the ordinary way is greater than the length of the spring cylinder it was decided to provide for telescoping springs; and further, as it was desirable that the spring cylinder be capable of receiving either type of spring, an opening could not be provided in the cylinder to allow the rear ends of the stirrup and inner telescoping spring to be drawn through it during recoil as is ordinarily the case when telescoping springs are used. It was, consequently, necessary, in order to prevent the stirrup from striking the rear end of the cylinder during recoil, to shorten it and the inner spring column so as to leave a space between them and the rear end of the spring cylinder when the gun is in battery somewhat greater than the amount by which the outer spring is compressed during recoil.

Fig. 98 shows a spring cylinder of this carriage containing springs coiled from bars of rectangular cross-section and also a cylinder containing telescoping springs coiled from bars of circular cross-section.

133. Example 1.—Let it be required to design a counter-recoil spring coiled from a bar of rectangular cross-section for use in the 5-inch barbette carriage, model of 1903. The following data are known from the construction of the carriage, viz.:

Weight of recoiling parts = 12632 lbs.

Friction of packing around recoil piston-rod = 220 lbs.

Length of recoil = 13 ins.

Maximum angle of elevation = 15° .

Coefficient of starting friction = .25.

Number of spring cylinders = 2.

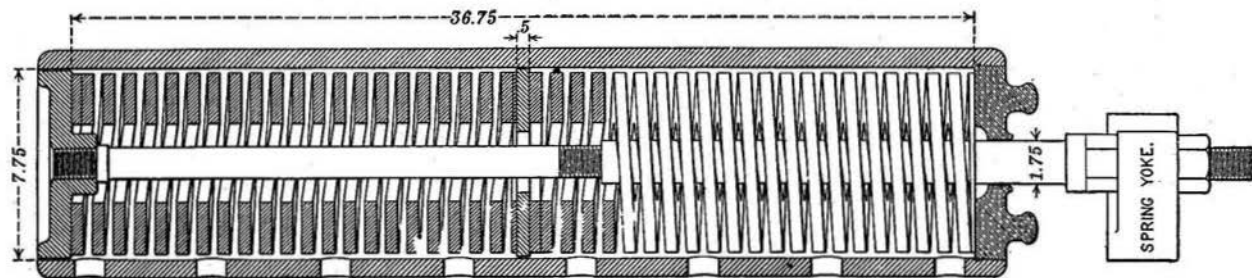
Assume values of $e = 1$ in., $r = 4.25$, and $D = 5.225$ ins.

From equation (29), since there are two spring cylinders,

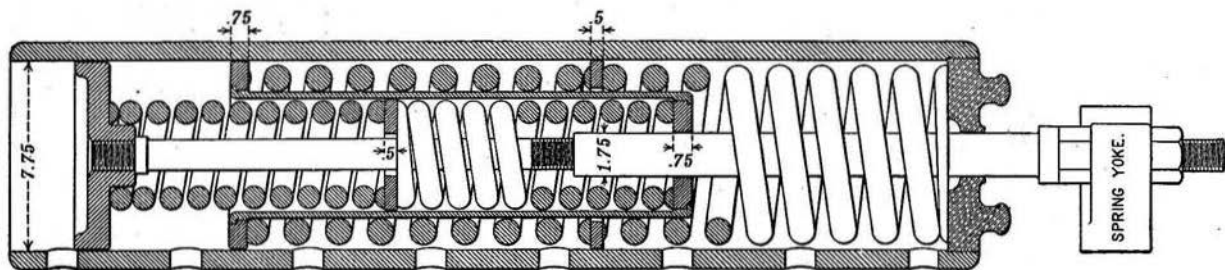
$$C = \frac{2}{2} (12632 \sin 15^\circ + .25 \times 12632 \cos 15^\circ + 220) = 6540 \text{ lbs.}$$

From equation (34)

$$h = [8.39203 - 10] \{1 + (4.25)^2\}^{\frac{1}{2}} (4.25)^{-\frac{2}{3}} (6540)^{\frac{1}{3}} (5.225)^{\frac{1}{3}} = .499 \text{ in.}$$



Springs Coiled from Bars of Rectangular Cross-Section.



Telescoping Springs Coiled from Bars of Circular Cross-Section.

Fig. 98. —Spring Cylinders and Springs of 5-Inch Barbette Carriage, Model of 1903.

From equation (35)

$$N' = \frac{[7.02919] 14 (.499)^4}{6540 (5.225)^3} \times \frac{(4.25)^3}{1 + (4.25)^2} = 40.07 \text{ effective coils.}$$

On account of the number of coils the spring will be divided into two sections and the thickness of the separator will be taken as .5 in. The solid height is, then, from equation (31),

$$H = (40.07 + 2) \times .499 + .5 = 21.49 \text{ ins.}$$

The assembled height is, from equation (32),

$$A = 21.49 + 14 = 35.49 \text{ ins.,}$$

and, since the length of the space available for the spring column in the cylinder is 36.75 ins., the assembled height is satisfactory. The free height is, from equation (33),

$$F = 21.49 + 2 \times 14 = 49.49 \text{ ins.}$$

The dimension of the bar perpendicular to the axis of the spring is

$$b = rh = 4.25 \times .499 = 2.121 \text{ ins.}$$

The outer diameter of the coil is

$$D + b = 5.225 + 2.121 = 7.346 \text{ ins.}$$

and, since the inner diameter of the spring cylinder is 7.75 ins., the clearance is sufficient. The inner diameter of the coil is

$$D - b = 5.225 - 2.121 = 3.104 \text{ ins.}$$

and, since the larger diameter of the spring piston-rod is only 1.75 ins., the inner diameter of the spring is satisfactory.

Compare these results with the data given in the specifications for the 5-inch barbette carriage, model of 1903, noting that allowance is therein made for a total remaining compression of the spring column at assembled height equal to 15 ins. In order that the cross-section of the coiled bar should conform to a commercial size the spring just designed would in practice be made of a bar having a cross-section of .5 in. $\times$ 2.125 ins.

134. Example 2. — Let it be required to determine the assembled height of a column of counter-recoil springs for the 5-inch barbette carriage, model of 1903, the springs being coiled from

bars of circular cross-section and having such an outer diameter that they can be used in the existing spring cylinder provided the assembled height is not too great.

From equation (21), since there are two spring cylinders,

$$C = \frac{2.5}{2} (12632 \sin 15^\circ + .25 \times 12632 \cos 15^\circ + 220) = 8175 \text{ lbs.}$$

Assuming $D = 6.29$ ins. we have, from equation (27),

$$d = [8.46864 - 10] (8175)^{\frac{1}{3}} (6.29)^{\frac{1}{3}} = 1.095 \text{ ins.}$$

The outer diameter of the spring is, therefore,

$$6.29 + 1.095 = 7.385 \text{ ins.}$$

and, since the inner diameter of the cylinder is 7.75 ins., the clearance is sufficient.

From equation (28), taking e as 1 in.,

$$N' = \frac{[6.41913] 14 (1.095)^4}{8175 (6.29)^3} = 25.97 \text{ effective coils.}$$

Dividing the spring into two sections and taking the thickness of the separator as .5 in., the assembled height is, from equation (25),

$$A = (25.97 + 2) 1.095 + .5 + 14 = 45.13 \text{ ins.}$$

As the length of the space available for the spring column in the cylinder is only 36.75 ins. springs coiled from bars of circular cross-section cannot be used therein unless special measures are taken to diminish their assembled height.

135. Example 3. — Let it be required to determine the assembled height of a column of counter-recoil springs for this carriage formed by placing springs of smaller diameter inside the larger springs, the springs being coiled from bars of circular cross-section and acting directly on the spring piston.

Assume the mean diameter of the outer spring to be 6.45 ins., from which it follows that to allow for proper clearance between the inner and outer springs the mean diameter of the inner should not exceed about 4.45 ins.

From equation (39)

$$C_2 = \left(\frac{4.45}{6.45} \right)^2 C_1 = .476 C_1$$

and since $C = 8175$ lbs. as before, we have, from equation (40),

$$C_1 + C_2 = 1.476 C_1 = C = 8175 \text{ lbs.}$$

or

$$C_1 = 5539 \text{ lbs.}$$

From equation (27)

$$d_1 = [8.46864 - 10] (5539)^{\frac{1}{3}} (6.45)^{\frac{1}{3}} = .969 \text{ in.}$$

and from equation (28), taking $e = 1$ in.,

$$N'_1 = \frac{[6.41913] 14 (.969)^4}{5539 (6.45)^3} = 21.80 \text{ effective coils.}$$

Dividing the spring column into two sections and taking the thickness of the separator as .5 in., the assembled height of the outer column (and of the inner column also) is, from equation (25),

$$A_1 = (21.80 + 2) \times .969 + .5 + 14 = 37.56 \text{ ins.}$$

By comparison with the assembled height of a column of springs coiled from bars of circular cross-section and assembled in the ordinary way it will be seen that a material reduction in assembled height has been made, but this height is still greater than that of a column of springs coiled from bars of rectangular cross-section and assembled in the ordinary way. Although the assembled height is greater than the length of the space available for the spring column in the cylinder, the difference is so small that by using springs coiled from bars of slightly smaller diameters and assuming a slightly higher value for the permissible torsional stress, springs could be obtained that, when assembled in this way, would fit in the spring cylinder of this carriage and function satisfactorily. Such springs would, however, require greater care in their production by the manufacturer.

136. Example 4.—Let it be required to design telescoping springs for this carriage to be coiled from bars of circular cross-section, and with the condition that the spring cylinder must not be altered in any respect to the end that either springs coiled from bars of rectangular cross-section or telescoping springs coiled from bars of circular cross-section may be used therein at will.

Since the spring cylinder must not be altered, provision cannot be made for drawing the rear ends of the stirrup and the inner

spring column through an opening in the cylinder during recoil, and a space must be left between the rear end of the stirrup and the rear end of the cylinder, when the gun is in battery, that is at least equal to the total remaining compression of the outer spring column at assembled height. As the inner spring is smaller and, therefore, less expensive than the outer, the inner spring column will be given the greatest possible assembled height consistent with the conditions of the problem.

Let the total remaining compression at assembled height of the outer spring column be $l_1 + .5$ and the total remaining compression at assembled height of the inner spring column be $l_2 + .5$, and let

$$l_1 + .5 + l_2 + .5 = l + 1 = 14 \text{ ins.}$$

Then the assembled height of the inner spring column is, from equation (25),

$$A_2 = (N'_2 + P_2) d_2 + (P_2 - 1) t_2 + l_2 + .5$$

and this height plus $l_1 + .5$ plus the thickness of the flange at the rear end of the stirrup must equal the length of the space available for the spring column in the cylinder, which is 36.75 ins. Taking the thickness of the flanges of the stirrup as .75 in., we may write

$$(N'_2 + P_2) d_2 + (P_2 - 1) t_2 + l_2 + .5 + l_1 + .5 + .75 = 36.75 \text{ ins.}$$

or

$$(N'_2 + P_2) d_2 + (P_2 - 1) t_2 = 36.75 - 14 - .75 = 22 \text{ ins.}$$

which expresses the fact that the solid height of the longest inner spring column that can be used in the cylinder under the assumed conditions is 22 ins.

The mean diameter of the outer springs and the diameter of the bars from which they are coiled will be taken as 6.29 ins. and 1.095 ins., respectively, as before; and to allow for a proper thickness of the wall of the stirrup and for the proper clearances between the springs and the stirrup, the mean diameter of the inner springs will be taken as 3.4 ins., which will also make the inner diameter of these springs sufficiently large to enable them to pass easily over the spring piston-rod.

From equation (27)

$$d_2 = [8.46864 - 10] (8175)^{\frac{1}{3}} (3.4)^{\frac{1}{3}} = .891 \text{ in.}$$

and taking $P_2 = P_1 = 2$ and $t_2 = t_1 = .5$ we have

$$(N'_2 + 2) \times .891 + .5 = 22 \text{ ins.}$$

or $N'_2 = 22.13$ effective coils.

From equation (28)

$$l_2 + .5 = \frac{22.13 \times 8175 \times (3.4)^3}{[6.41913] (.891)^4} = 4.298 \text{ ins.}$$

The amount of compression to be provided by the outer spring column is, therefore,

$$14 - 4.298 = l_1 + .5 = 9.702 \text{ ins.}$$

and the number of effective coils in the outer column is, from equation (28),

$$N'_1 = \frac{[6.41913] (9.702) (1.095)^4}{8175 (6.29)^3} = 18.0$$

The assembled height of the outer column is, therefore, from equation (25),

$$A_1 = 20 \times 1.095 + .5 + 9.702 = 32.10 \text{ ins.}$$

and since this is less than the length of the available space in the spring cylinder diminished by the thickness of the front flange of the stirrup, equal to $36.75 - .75 = 36$ ins., the design of telescoping springs just made is satisfactory for use in the spring cylinder of the 5-inch barbette carriage, model of 1903, designed primarily for counter-recoil springs coiled from bars of rectangular cross-section.

137. Example 5. — Let it be required to determine the assembled height of a set of telescoping springs for this carriage under the assumptions that the assembled heights of the inner and outer columns are equal, that the rear ends of the stirrup and inner spring column may be drawn through an opening in the spring cylinder during recoil, and that the diameter of the cylinder is the same as at present.

The values of D and d will be taken the same as for the telescoping springs already designed which are as follows:

$$\begin{array}{ll} D_1 = 6.29 \text{ ins.} & d_1 = 1.095 \text{ ins.} \\ D_2 = 3.40 \text{ ins.} & d_2 = .891 \text{ in.} \end{array}$$

Let the total remaining compression at assembled height of the outer spring column be $l_1 + .5$ and of the inner spring column $l_2 + .5$, and let $l_1 + l_2 = 13$ ins., the length of recoil required. Then from equation (37), assuming $P = 2$,

$$3.0311 l_1 + 3.2056 = 5.5878 l_2 + 4.076$$

and substituting in this equation for l_2 its value $13 - l_1$, and solving for l_1 we have

$$l_1 = 8.53 \text{ ins. and } l_2 = 4.47 \text{ ins.}$$

From equation (28)

$$N'_1 = \frac{[6.41913] (9.03) (1.095)^4}{8175 (6.29)^3} = 16.75 \text{ effective coils.}$$

and

$$N'_2 = \frac{[6.41913] (4.97) (.891)^4}{8175 (3.40)^3} = 25.59 \text{ effective coils.}$$

From equation (25)

$$A_1 = 18.75 \times 1.095 + .5 + 9.03 = 30.06 \text{ ins.}$$

and

$$A_2 = 27.59 \times .891 + .5 + 4.97 = 30.05 \text{ ins.}$$